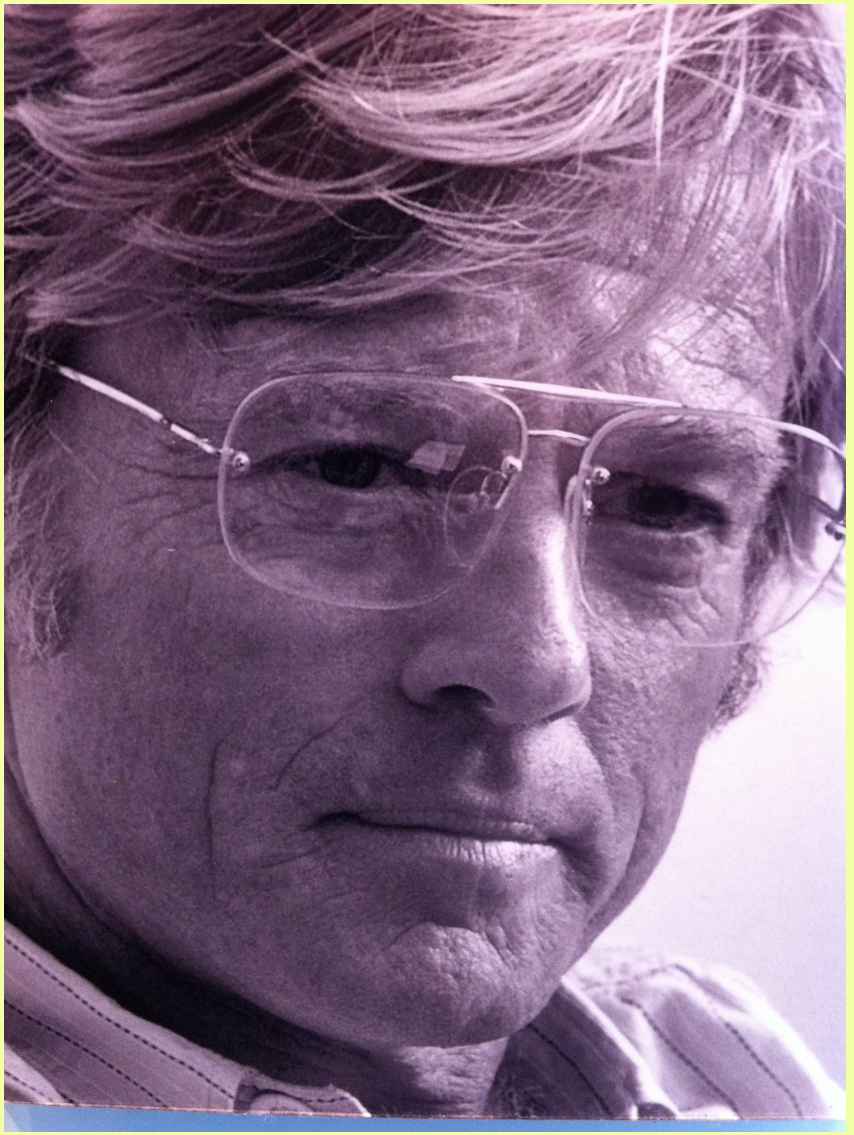
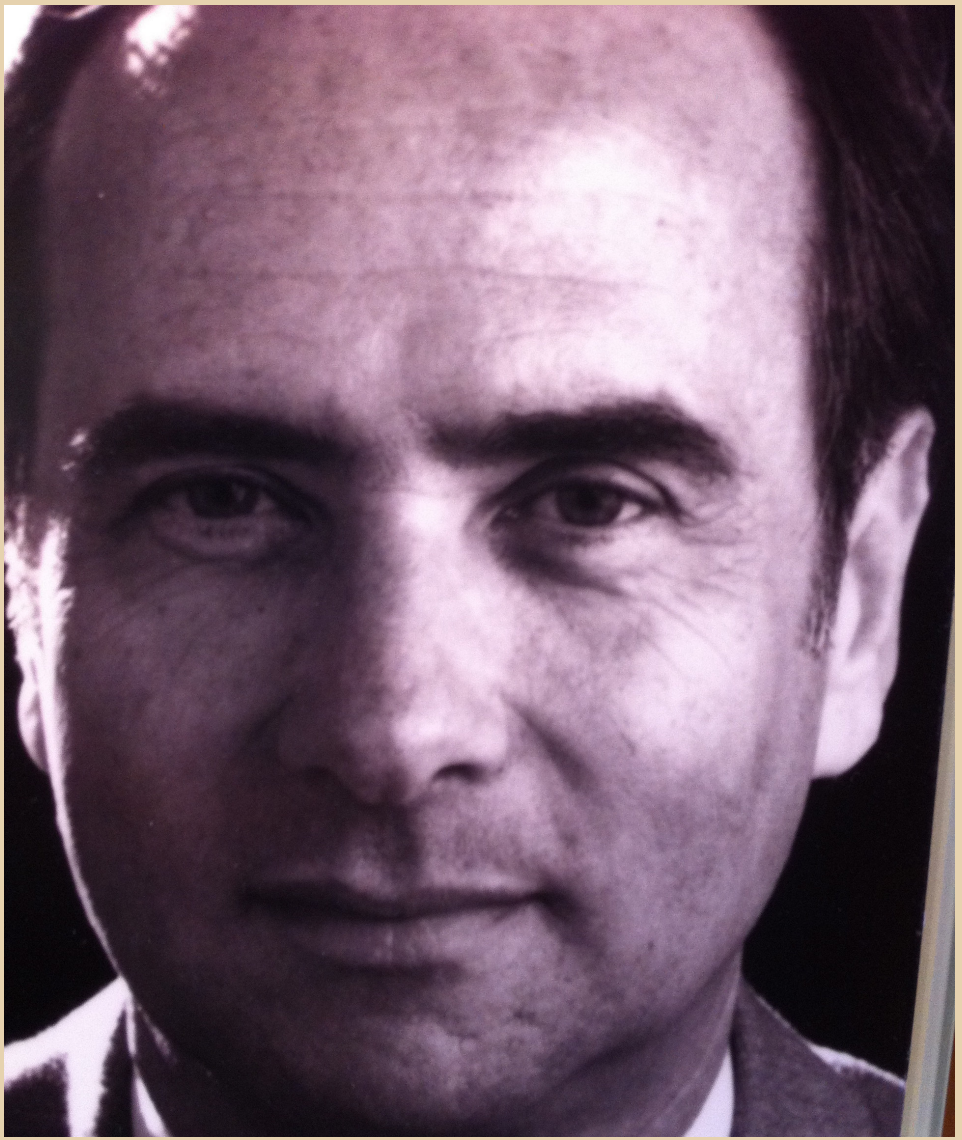
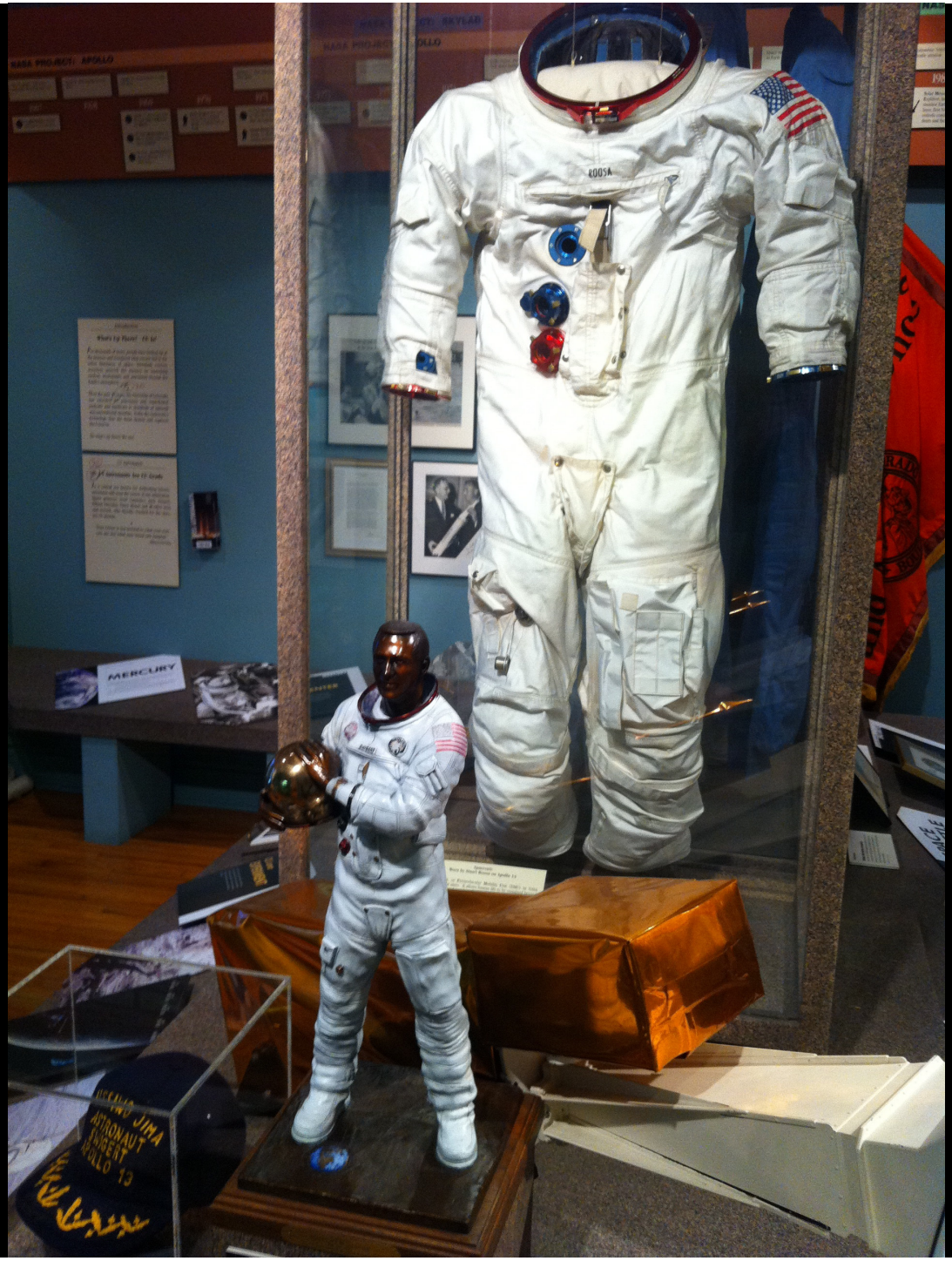
RME3

Glenn Miller





Theodore Maiman


MERCURY
ASTRONAUT
APOLLO 13

INTEGRATING STEM

Science, Technology, Engineering, and Math Education at Colorado

Informed Classroom Practice: Progress and Challenges in Math Education

David C. Webb
University of Colorado at Boulder

3rd International RME Conference

September 23, 2011

What is a learning trajectory?

Learning Trajectories (LT)

- (How) do LTs promote new understandings?
- (How) do LTs motivate new commitments?
- (How) do LTs inform classroom practice?

How do LTs promote new understandings?

Mapping Mathematical Representations

1. Task centric – instructional materials
2. Organization of student products & responses
3. Frameworks that generalize student activity

* Models, tools and strategies are elevated and valued as important objects for math reasoning

Children’s Mathematics

From Carpenter, Fennema, Franke, Levi & Empson (1999). Heinemann.

Cognitively Guided Instruction

- Origins in research on how students learn arithmetic
- Examples of student reasoning
- With video focusing mostly on students doing mathematics
- Significant contribution is a robust framework for teachers (and parents!!!) that articulates student reasoning

Children’s Mathematics

From Carpenter, Fennema, Franke, Levi & Empson (1999).Heinneman.

How do LTs promote new understandings?

To what degree do they support teacher noticing, observing, and decision making?

Interpretation

Interpretation

Interpretation

How do learning trajectories motivate new commitments?

Knowledge and understanding of mathematics is not enough…

What is the motivation to…

- see instructional materials as a resource and not a script?
- adapt and possibly design sequences of tasks and questions?
- to enact **their** trajectories with **their** students?


MATH
Fractions → Decim
Example A: 6/10 six tenth
0.6
Example B: 4/5 = 8/10 = 0.8
Example C: 3/4 = 3 ÷ 4 = 0.75
Example D: Use a calc
Money/ Decimals
Fraction Dollars
0.25
0.50
Multiplication
Menu

Consider: Lesson Study

- Collaborative planning
- Observation followed by intentional revision and improvement
- Student engagement and learning drives analysis of observed acts of teaching

Perhaps…

- Teachers' authentic inquiry motivates how they interact with students
- The need to observe and listen to students motivates the way class is organized
- Confidence in adaptation and design, leads to informed instr. decision making

How do learning trajectories inform classroom practice?

Consider…

- Learning trajectories help us answer several questions:
  - What objectives should we establish?
  - Where do we start?
  - How do we know where to go next?
  - How do we get there?

Doug Clements and Julia Sarama (2004). Learning trajectories in early mathematics: Sequences of acquisition and teaching.
http://literacyencyclopedia.ca/index.php?fa=items.show&topicId=270

Furtak (2009). *Formative assessment for secondary science teachers*

- Formative assessment… consists of three steps that can be phrased as questions:
  - Where are you going?
  - Where are you now?
  - How are you going to get there? (p. 3)

Conjecture:

The design, study and use of learning trajectories is the professional enactment of formative assessment

“With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon”

- Deborah Ball, 1990

Instructional Dilemma

- Where are you going? (goal)
- Where are you now? (assessment)

- How are you going to get there? ? ? ? ? ? ? ?

Instructional Dilemma

- How are you going to get there?
  - One Path: Quickest, most direct, efficient
  - Research Base, RME, Student Centered..
    - So many paths, so little time

Instructional Dilemma

- How are you going to get there?
  - One Path: Quickest, most direct, efficient

    Did you really “get there”?
  - So many paths, so little time

Instructional Dilemma

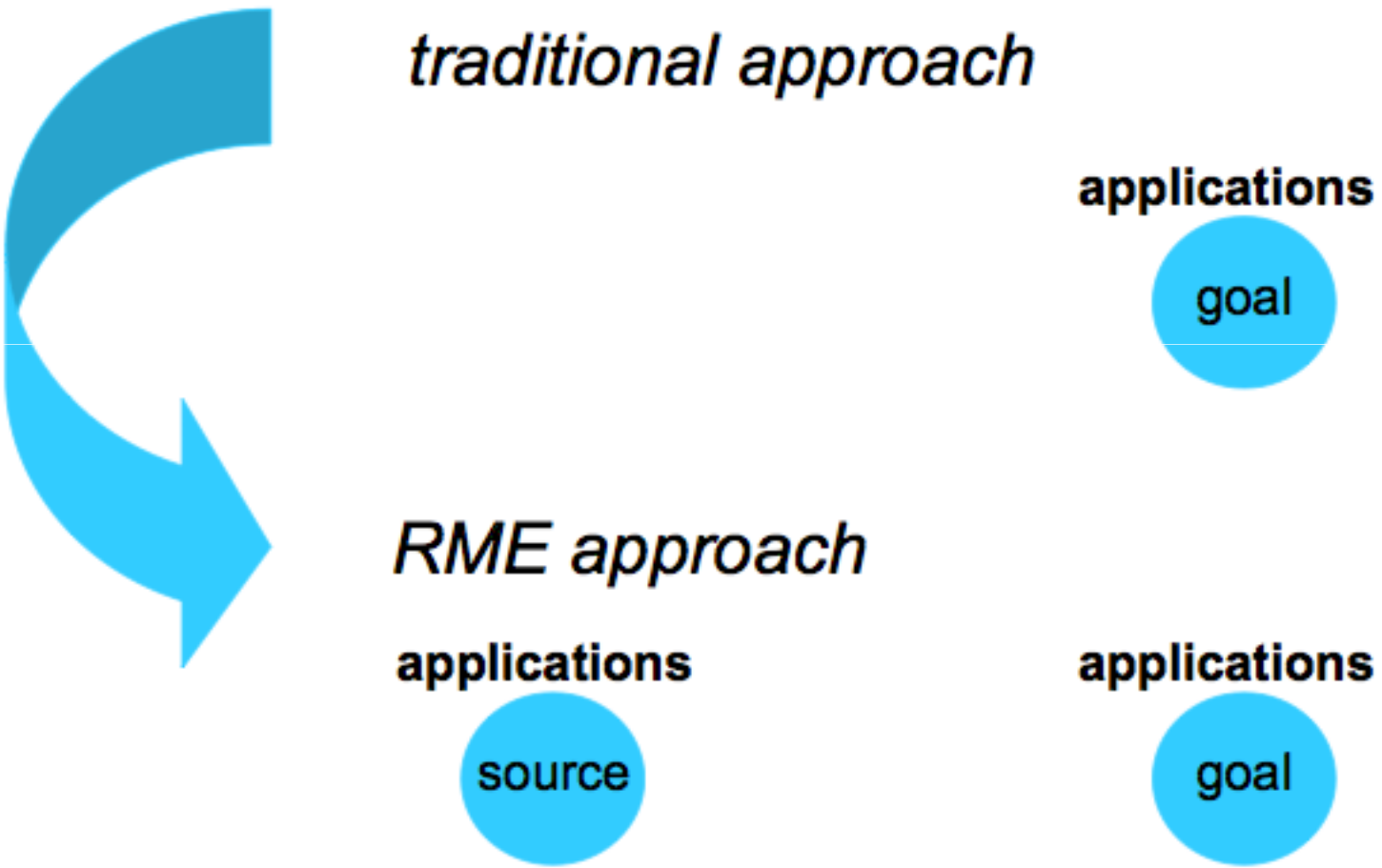
- How are you going to get there?
  - One Path: Quickest, most direct, efficient

    Did you really “get there”?
  - So many paths, so little time

    Which path(s) can actually

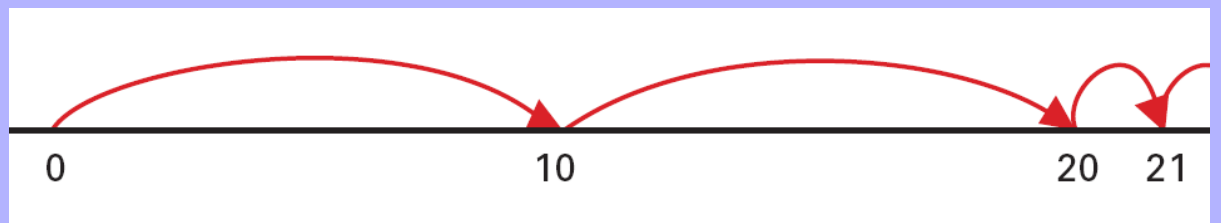
    get you where you need to go?

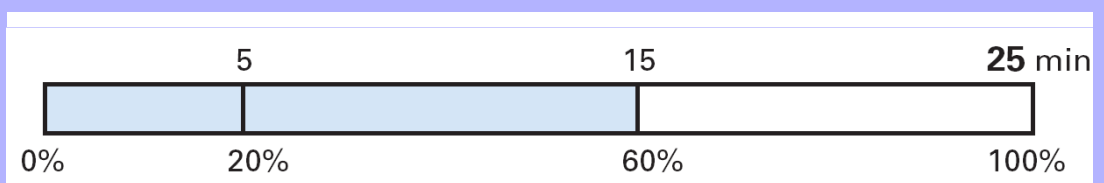
RME-guided path



van den Heunel-Panhuizen, M. (2009). Keynote presentation at the 2nd International RME conference. Boulder, CO.

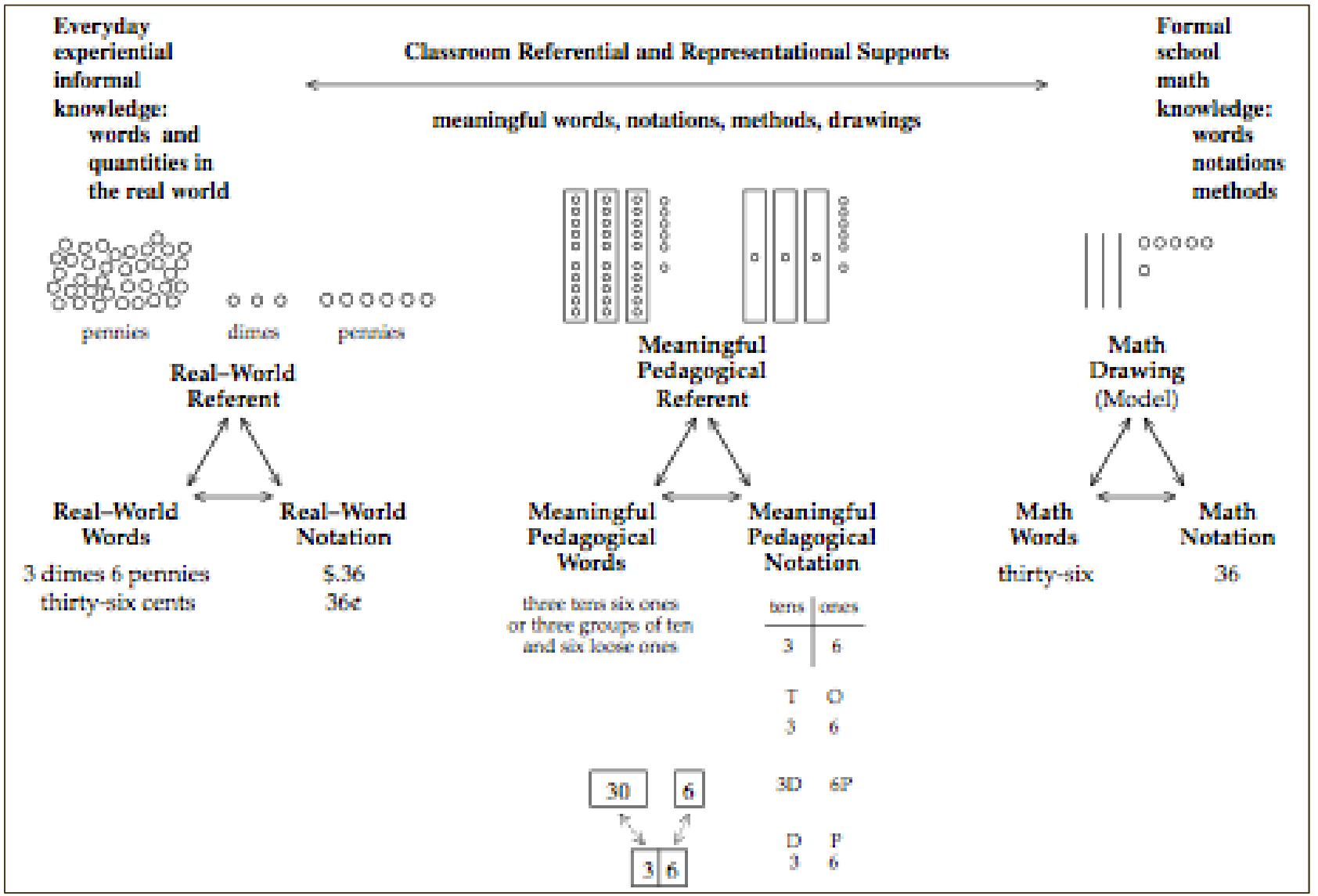
Models & Tools



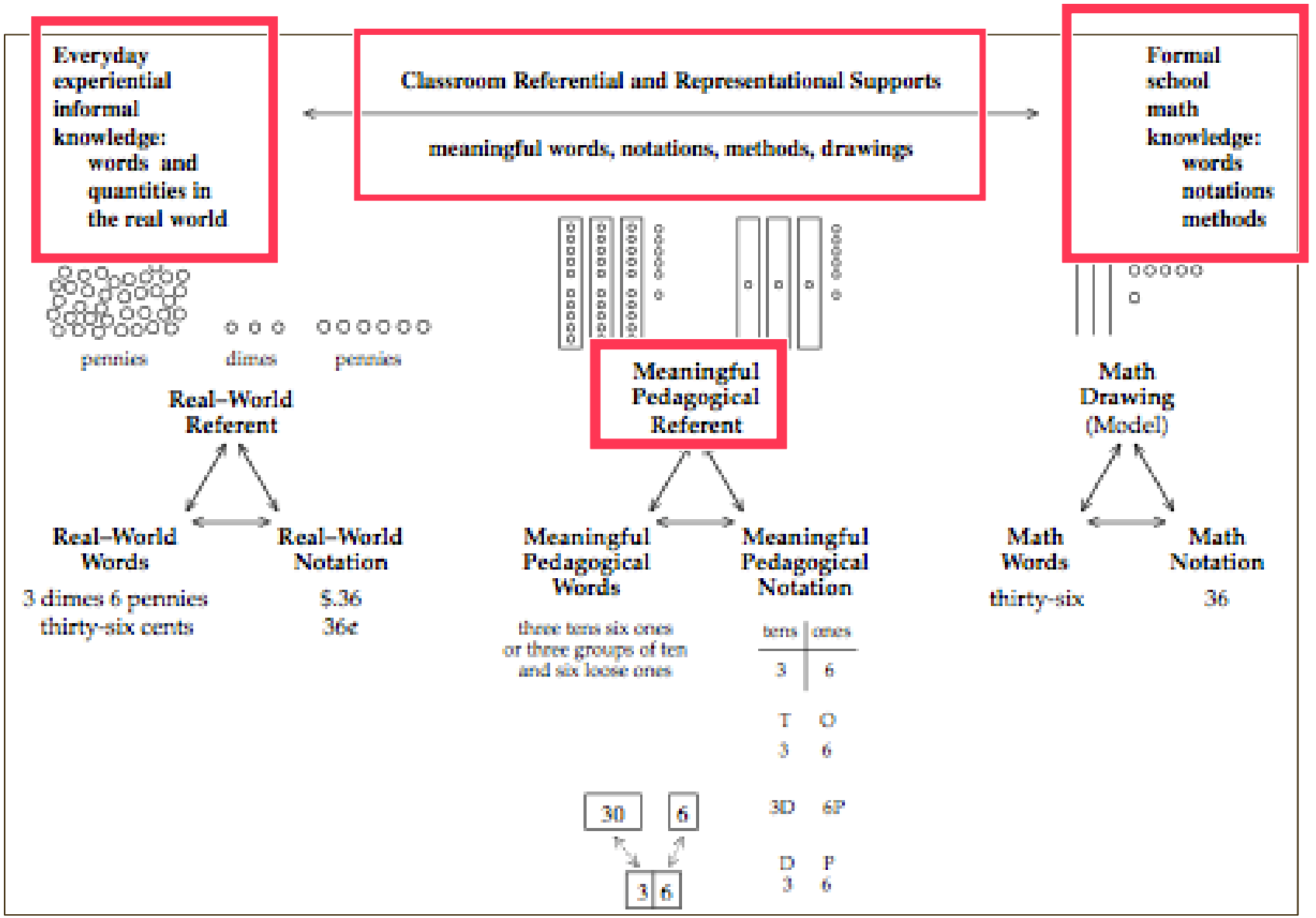


ORDER	DRINK	FRIES	FISH	TOTAL
1	2	1	2	$8.80
2	1	—	1	$3.60
3	3	1	1	$7.40
4				
5				
6				
7				

Minutes	10	20	80
Miles	$\frac{1}{2}$	1	4



From: Fuson, Kalchman & Bransford. (2005). Mathematical Understanding: An Introduction. In M. S. Donovan and J. D. Bransford (Eds.), *How Students Learn: Mathematics in the Classroom* (pp 217-256). Washington, DC: National Academy Press.



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Typology of reasoning

1) Formal school math knowledge: words, notations, methods

2) Classroom referential and representational supports… meaningful pedagogical referent

3) Everyday, experiential, informal knowledge… words and quantities in the real world

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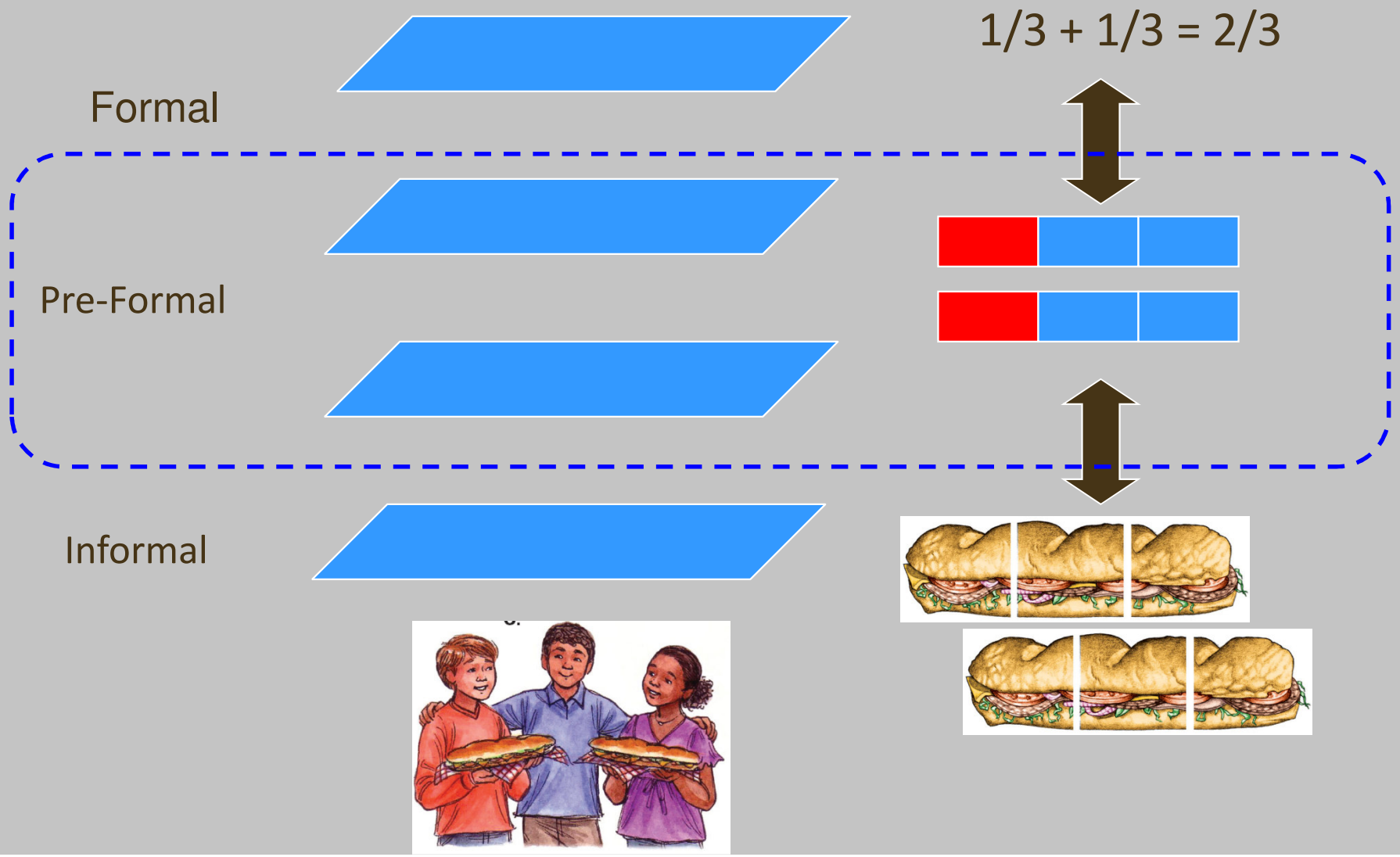
Progressive Formalization



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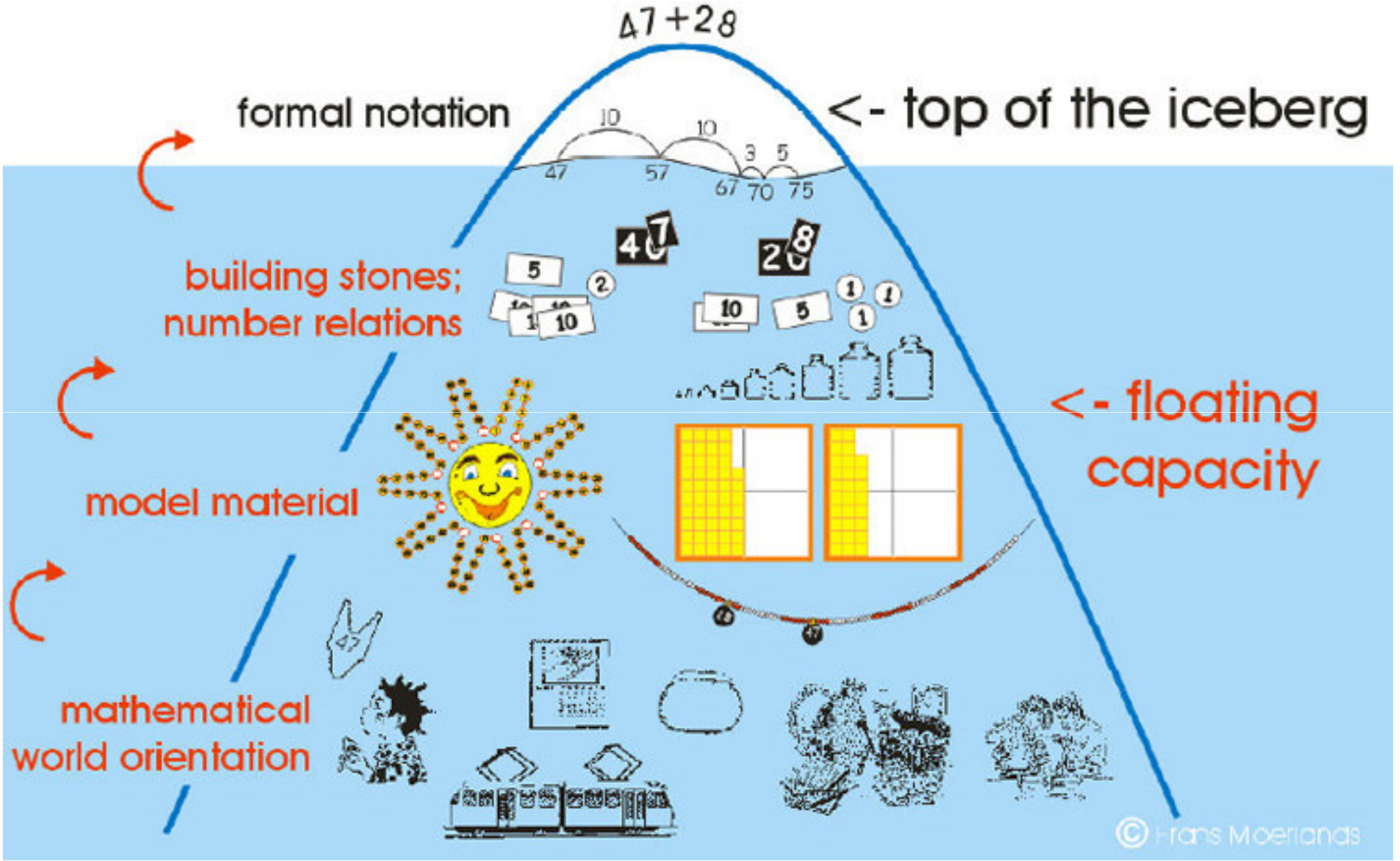
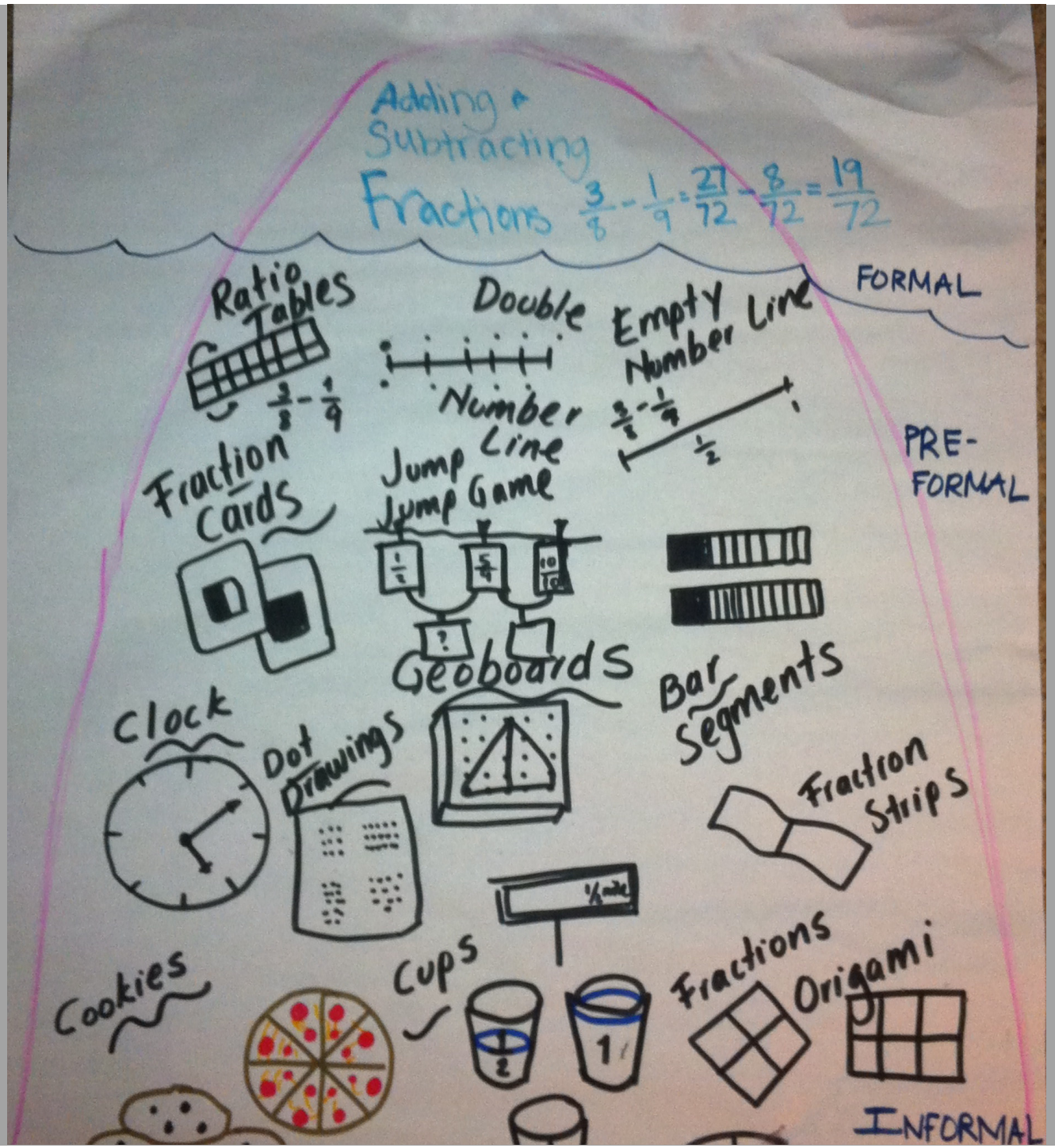
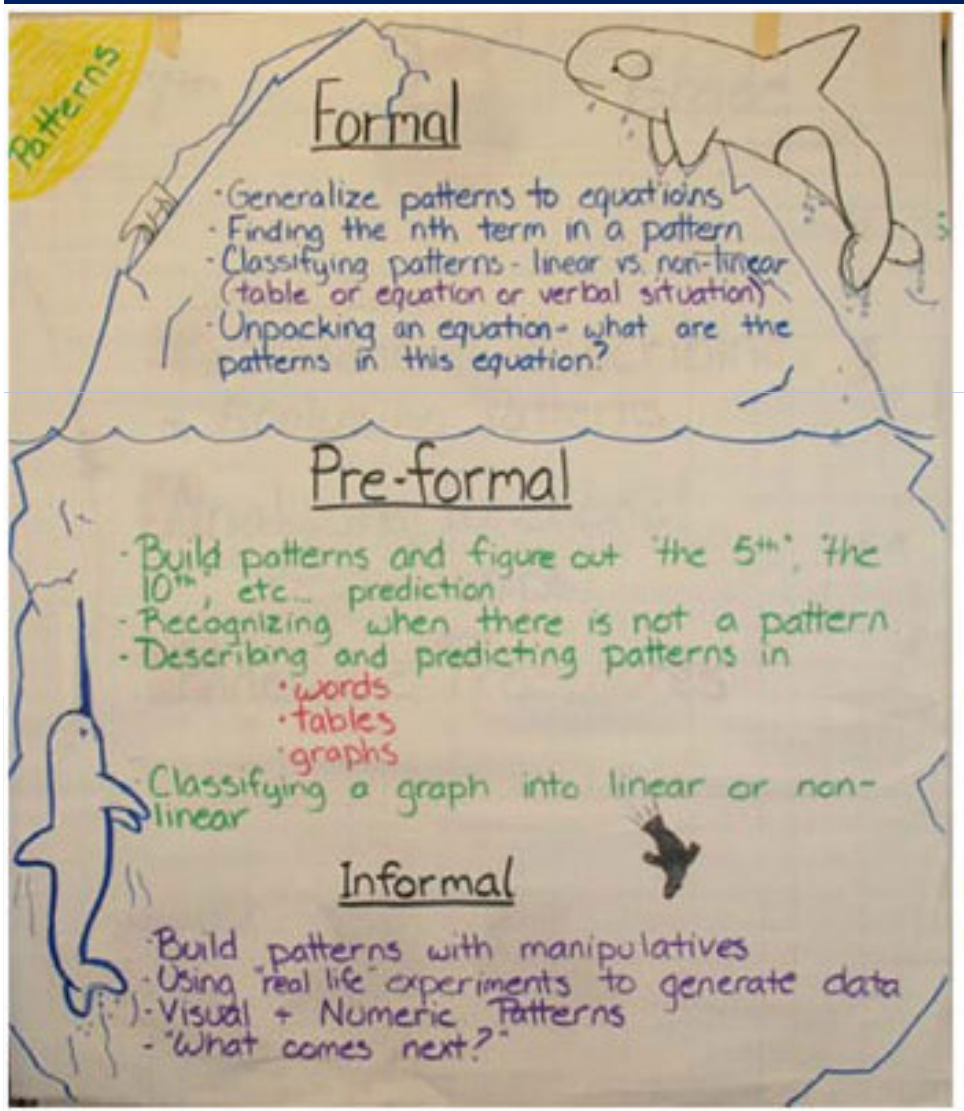
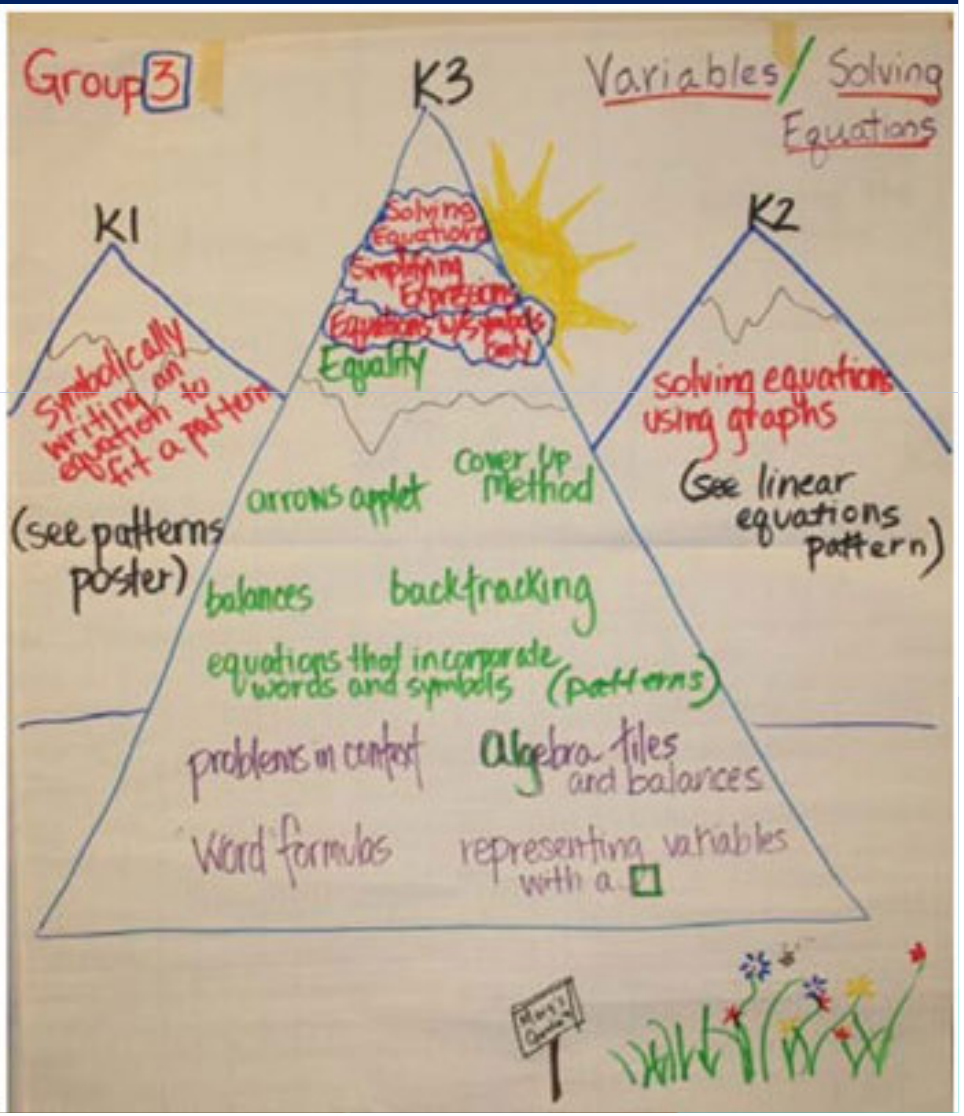

47+28
formal notation
<- top of the iceberg
building stones;
number relations
model material
mathematical
world orientation
<- floating
capacity
© Hans Moerlands


Adding + Subtracting Fractions $\frac{3}{8} - \frac{1}{9} = \frac{27}{72} - \frac{8}{72} = \frac{19}{72}$
FORMAL
Ratio Tables
$\frac{3}{8} - \frac{1}{9}$
Double Number Line
Empty Number Line
$\frac{3}{8} - \frac{1}{9}$
$\frac{1}{2}$
PRE-FORMAL
Fraction Cards
Jump Jump Game
Geoboards
Bar Segments
Clock
Dot Drawings
Fraction Strips
Cookies
Cups
Fractions
Origami
INFORMAL

Identifying informal & pre-formal representations



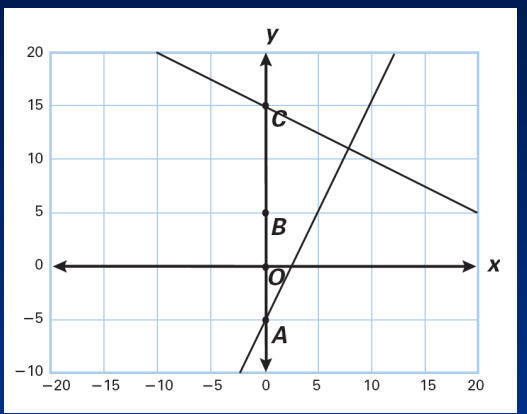


$$3A + 2P = \$9.20$$
$$1A + 2P = \$5.20$$

From: Math in Context.
Comparing Quantities.
Britannica.

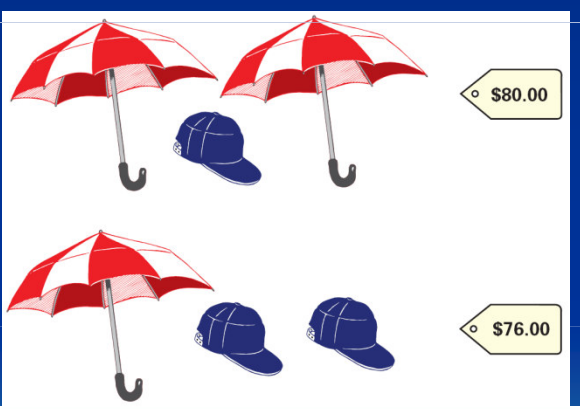
ORDER	TACO	SALAD	DRINK	TOTAL
1	2	4	--	$10
2	1	2	3	$8
3	3	--	3	$9
4	1	2	--	
5	1	--	1	
6	2	2	1	
7	4	2	3	
8				
9				
10				

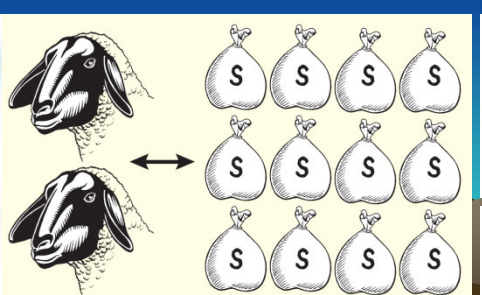
÷3



Costs of Combinations (in cents)

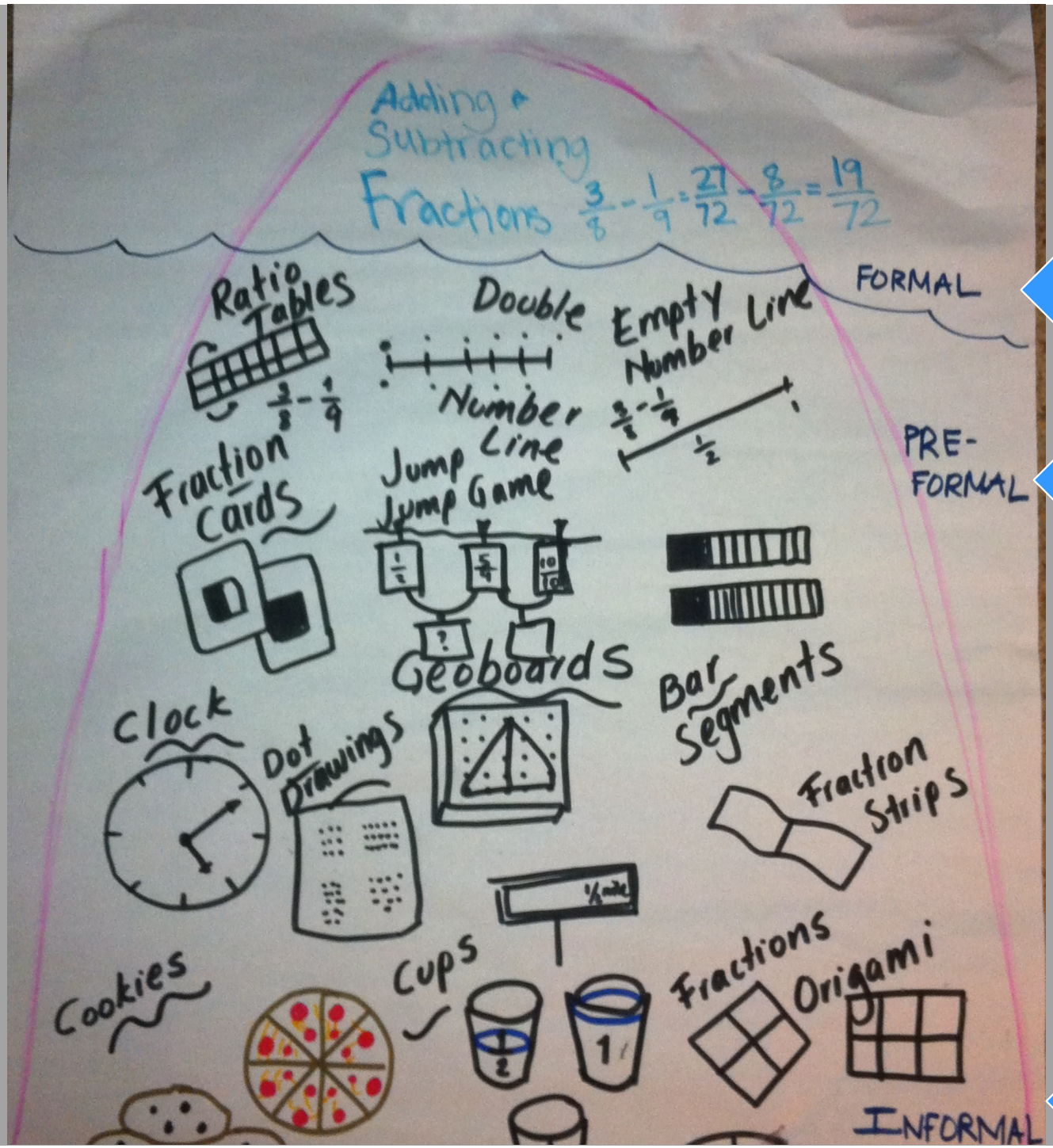
Number of Pencils \ Number of Erasers	0	1	2	3	4	5	6	7	8
7			155						305
6		115	140	165					
5	75	100	125	150	175	200			
4	60	85	110	135	160	185	210		
3	45	70	95	120	145	170	195		
2	30	55	80	105	130	155	180		
1	15	40	65	90	115	140	165	190	
0	0	25	50	75	100	125	150	175	





Identifying informal & pre-formal representations

- Drawing from personal and professional experiences with the math topic
- Review of their instructional resources
- Opportunities for teachers to solve and discuss math problems (at their own level) and reflect on the strategies they used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Cookies
Cups
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Origami
FORMAL
PRE-FORMAL
INFORMAL

Over the past several decades we have witnessed a consistency in the research narrative describing how students encounter, make sense of, and structure contexts that can be mathematized

RME (1971 +) …

CGI (1983 +) …

Math in the City (1995 +) … Cobb et al (90s)

Math in Context (1995 +)…

Clements & Sarama (mid 90s +).. Fuson et al (90s +)

Rasmussen and colleagues (2000+)

And we have also been gifted with a wealth of ideas -- the models, tools and strategies that could be introduced to students to support their reasoning in a way that honors student sense making

Constraints? Challenges?

- Reduced Opportunities for Teacher Learning (OTL) and access to research
- Reduced OTL through collaboration
- District and school level curricular policy that constrains teacher flexibility

Designing PD and classrooms that support teachers-as-designers

- To what degree are teachers invited to critique and adapt trajectories in current materials?
- …participate in design of trajectories?
- To what degree does our professional development engage teachers in instructional (re)design and new mathematical learning?

BPEME

SystemsEq

Solving Systems of Equations using the Substitution Method

LESSON PLAN
(Introduces 3 variable problems and equations)
- use the different combinations of 3 chickens of different sizes and their total weights problem and solve informally
- use the notebook method for the same problem
- translate notebook method into the concepts / steps used in the elimination method

TERMS / CONCEPTS
- least common multiple

INTRODUCE 2 VARIABLE EQUATIONS
-translate umbrella/cap problem into variables and equations
- translate informal methods used into the concepts / steps used in the elimination method

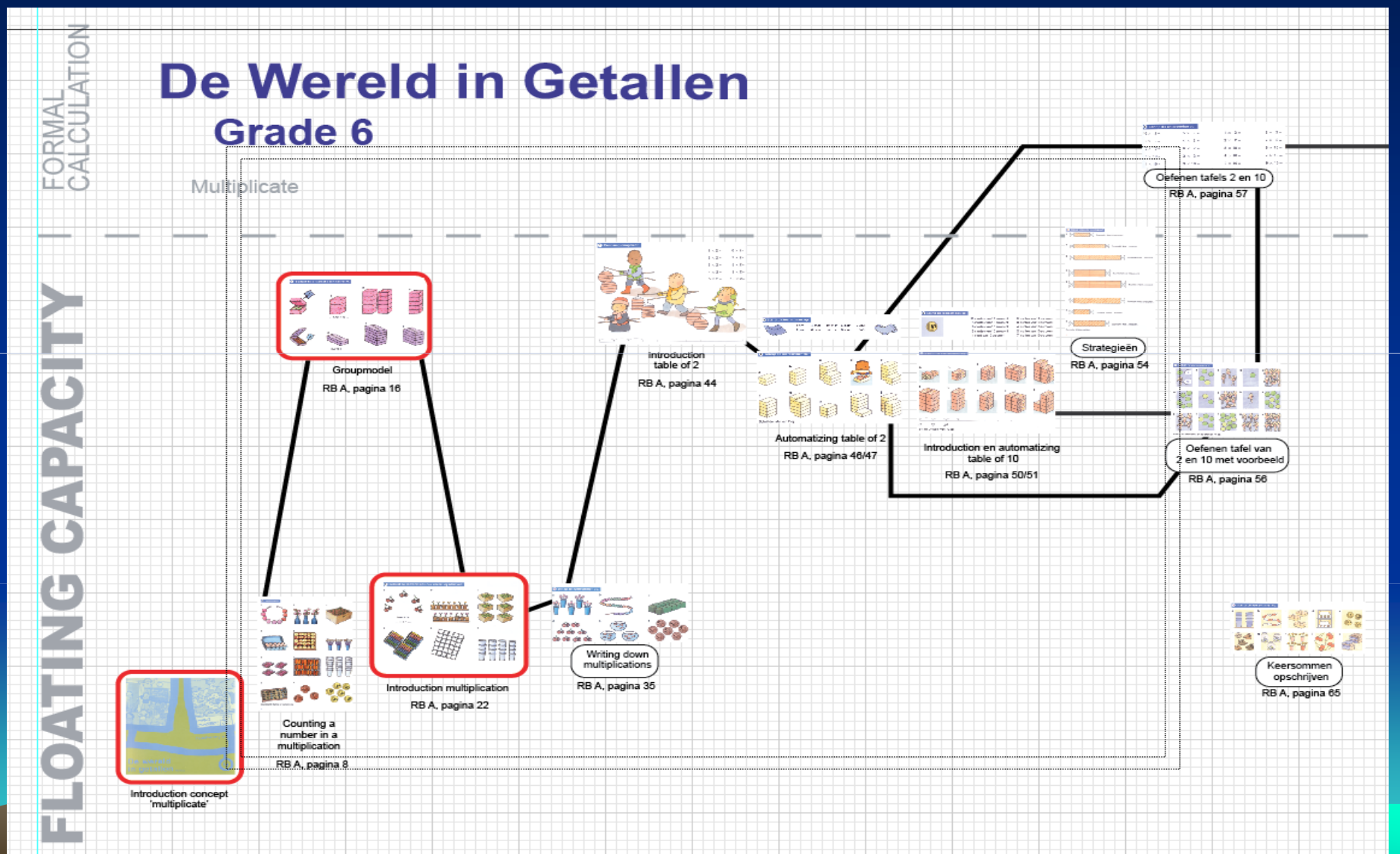
INTRODUCE 2 VARIABLE PROBLEMS
- use the different combinations of umbrella and caps and cost problem (MiC Comparing Quantities p. 17)
- use the hot dogs and lemonade combination problem or the sunglasses and shorts problem below (MiC Comparing Quantities p. 16)

TERMS / CONCEPTS
- coefficient

www.educationaldesigner.org

Issue 2

Progressive (Schematization)



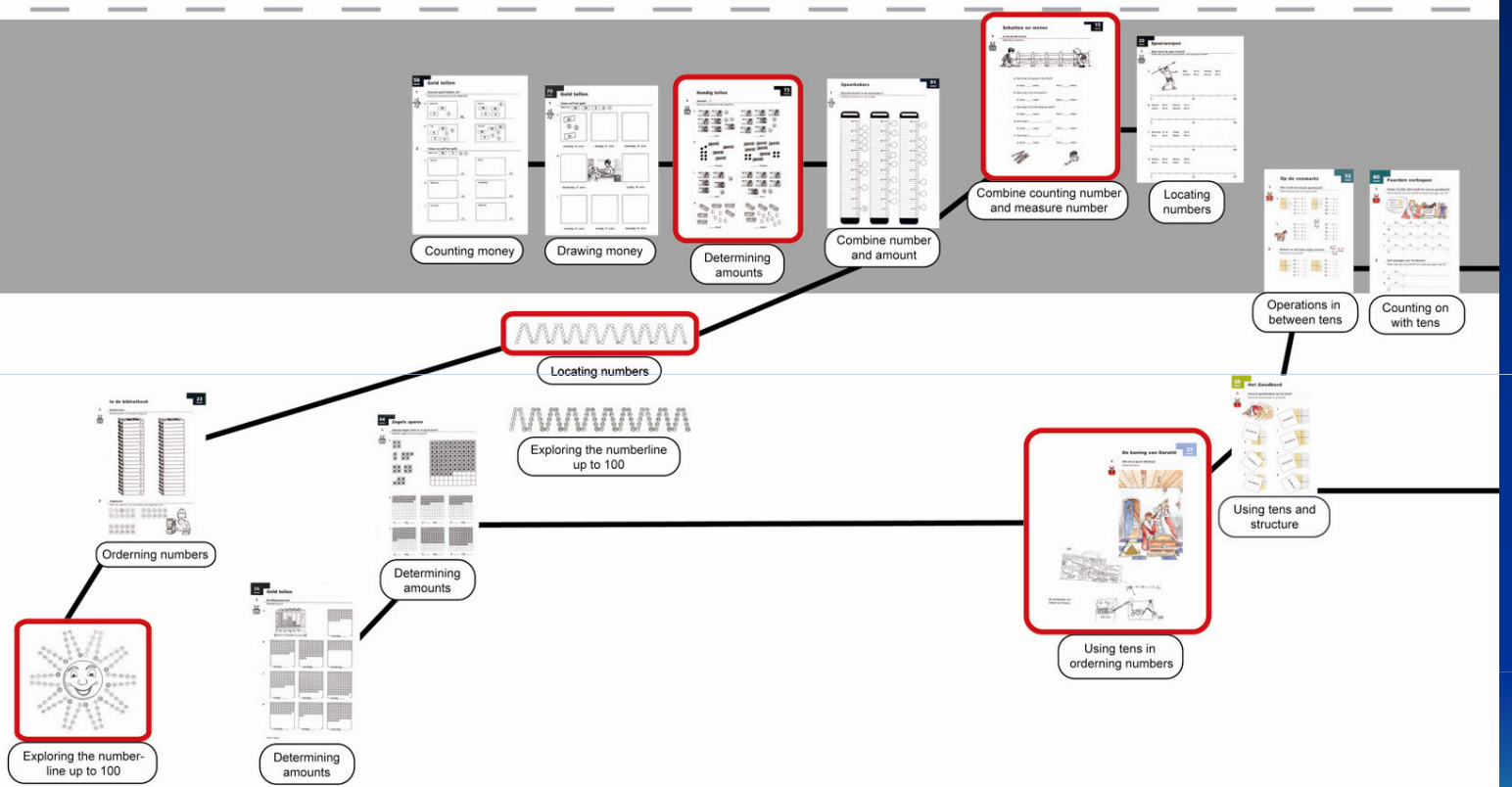
FORMAL ARITHMETIC

Wis en Reken

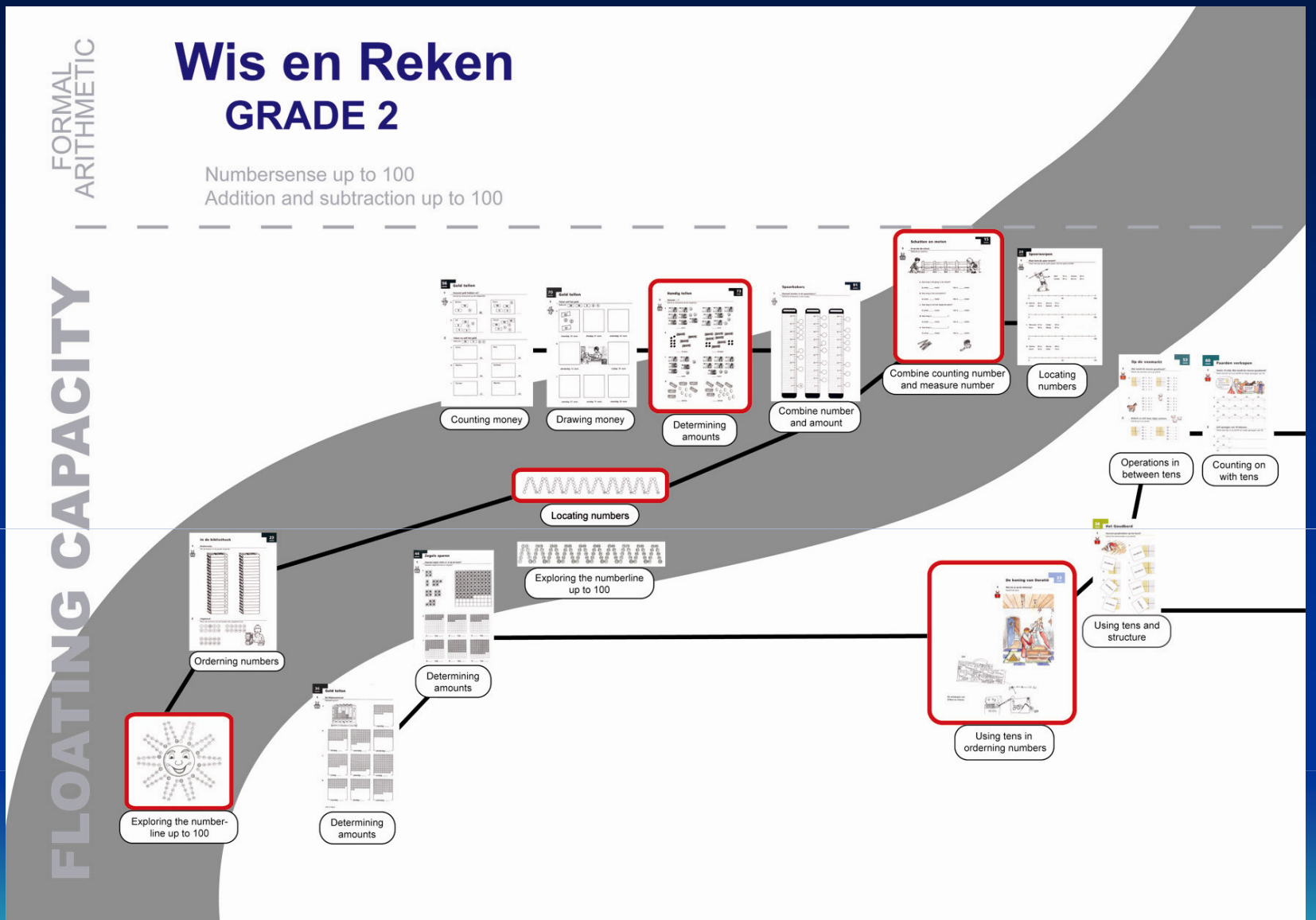
GRADE 2

Numbersense up to 100
Addition and subtraction up to 100

FLOATING CAPACITY



Horizontal


FORMAL ARITHMETIC
Wis en Reken
GRADE 2
Numbersense up to 100
Addition and subtraction up to 100
FLOATING CAPACITY
Counting money
Drawing money
Determining amounts
Combine number and amount
Combine counting number and measure number
Locating numbers
Operations in between tens
Counting on with tens
Locating numbers
Exploring the numberline up to 100
Ordering numbers
Determining amounts
Using tens and structure
Using tens in ordering numbers
Exploring the number-line up to 100
Determining amounts

Diagonal

- Suggestions and recommendations
  - Planning integrates learning line and assessment
  - Opportunities for teacher design
  - Partnerships between researchers and teachers

Learning trajectories

- Opportunity to promote new understandings
  - Focus: student responses, representations, and strategies
- Activities that motivate new commitments?
  - Do teachers have the freedom to critique and adapt instructional materials?
- Design work that informs classroom practice
  - Basis for formative assessment link

What is a learning trajectory?

Questions?

Nederland Panel