FI US
University of Colorado Boulder

Realistic Mathematics Education Conference: Sept 18 - 20, 2015

Beyond 'Flatland' in primary school mathematics education in the Netherlands

Marja van den Heuvel-Panhuizen

Freudenthal Faculty of Social and Behavioural Sciences

Universiteit Utrecht

Freudenthal Faculty of Science

Faculty of Science

Secundary
Education

Faculty of Social and Behavioural Sciences

Early Childhood
Special Education
Primary Education
Vocational Education

Freudenthal Faculty of Social and Behavioural Sciences

Universiteit Utrecht

Freudenthal Faculty of Science

Overview

- A summary of the guiding principles of RME
- A blind spot in RME?
- An online game to introduce early algebra in primary school
- A new project aimed at making the primary school mathematics curriculum more mathematical

Realistic Mathematics Education

"realistic"

- to imagine = ZICH REALISEREN
- meaningful context → *real* world or *fantasy* world

  → formal world of *mathematics*

- still under construction
- over the years different accentuations

Realistic Mathematics Education

Mechanistic Mathematics Education

- teaching is transmission
  * atomized
  * step-by-step
- bare number calculations
- little attention applications (especially not at the start)
- fixed procedures, recipes
- distinct strands
- mostly individual seat work
- much guidance

- activity principle
- reality principle
- level principle
  * various levels of understanding
  * progressive schematization
  * models as bridges
- intertwinement principle
- interactivity principle
- guidance principle

Realistic Mathematics Education

- reality principle

applications

source

applications

target

TIMSS 2003 Study - Grade 8

A scoop holds $\frac{1}{5}$ kg of flour. How many scoops of flour are needed to fill a bag with 6 kg of flour?

Answer: $6 \div \frac{1}{5}$

6×5

30 scoops

International average: 38% got a full credit

US students: 52% got a full credit

NL students: 74% got a full credit

Formal strategy

$$6 \div \frac{1}{5} = 6 \times \frac{5}{1}$$

Informal context-connected strategy

	$\xrightarrow{\times 5}$		$\xrightarrow{\times 6}$
number of scoops	1	5	30
kg	$\frac{1}{5}$	1	6
	$\xrightarrow{\times 5}$		$\xrightarrow{\times 6}$

1 scoop holds $\frac{1}{5}$ kg;
so, 1 kg is 5 scoops
and 6 kg is 6 times 5, is 30 scoops.

Rather than beginning with abstractions or definitions to be applied later,
one must start with rich contexts
that ask for mathematical organization;
or, in other words, one must start with *contexts* that can be *mathematized*.

“What humans have to learn is not mathematics as a closed system, but rather as an activity,
the process of mathematizing reality and if possible even that of mathematizing mathematics.”

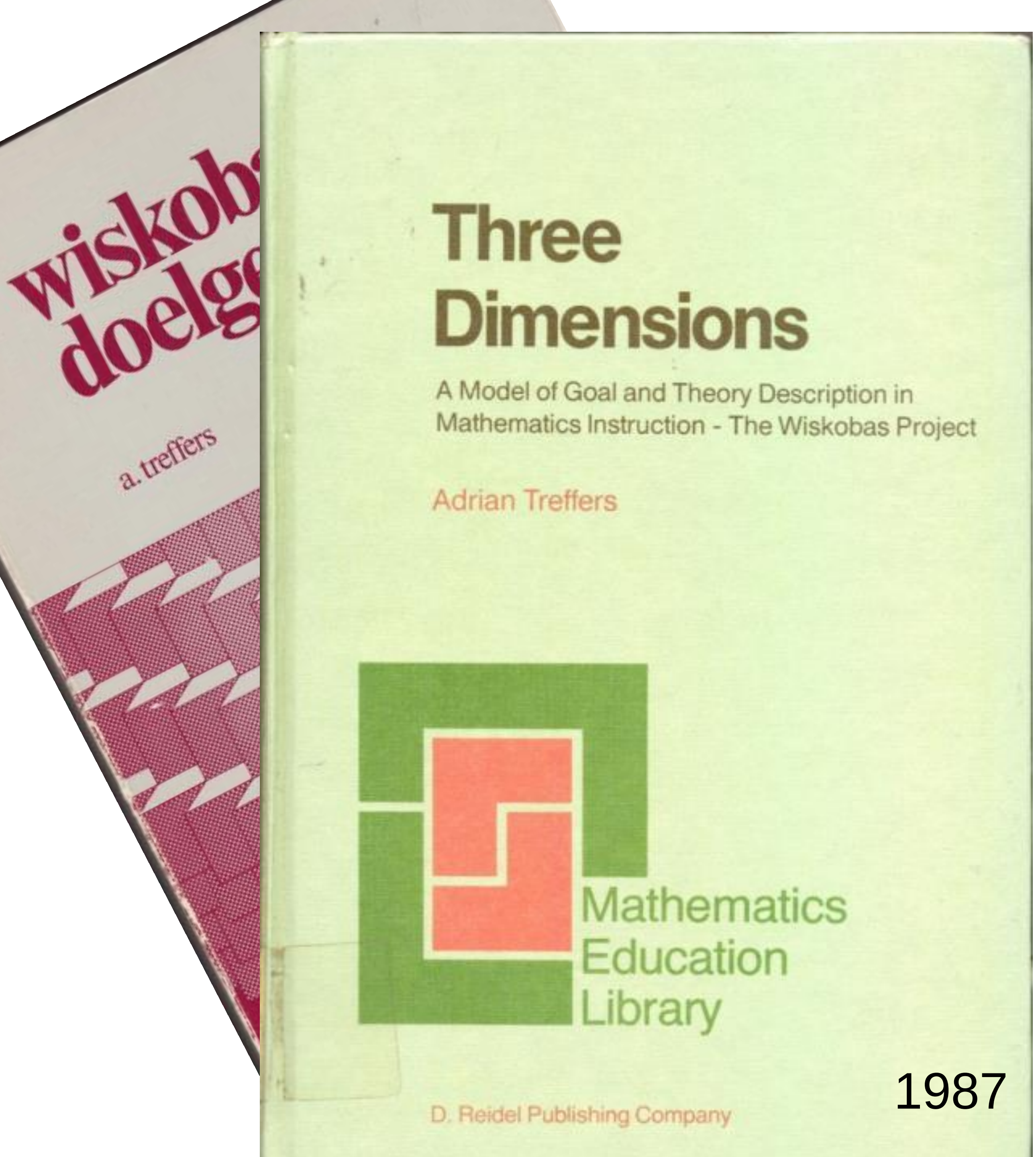
(Freudenthal, 1968)

HANS FREUDENTHAL

WHY TO TEACH MATHEMATICS SO AS TO BE USEFUL

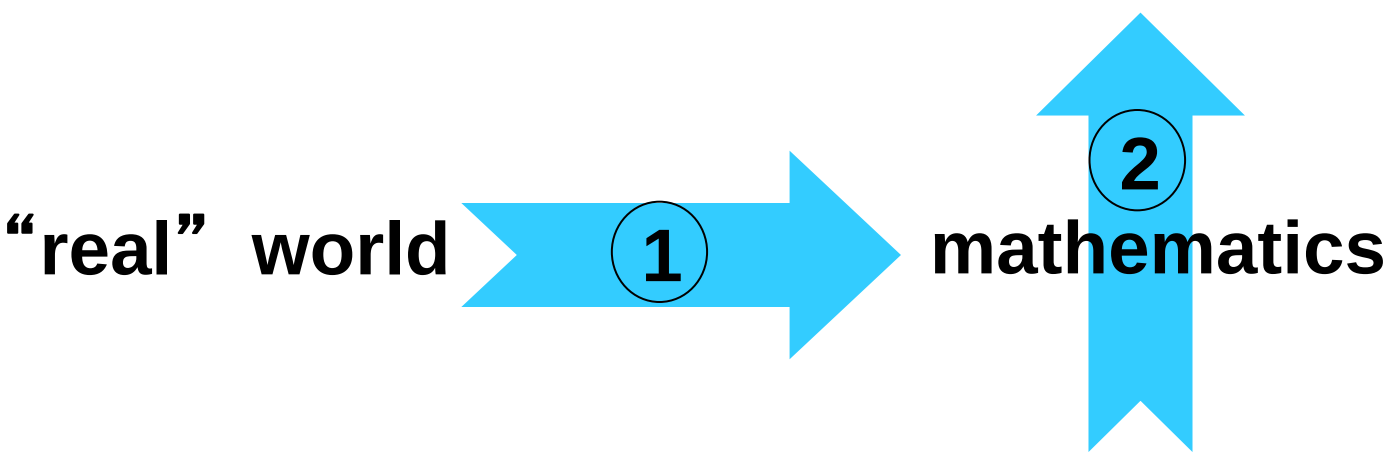
My first task at this moment is to welcome you who have come here from various countries to sacrifice one week of your holidays for the benefit of mathematical education all over the world. I trust this meeting will be as useful as according to the general theme of this conference mathematical education should be held to be. I trust we all will learn as much from each other's experiences and arguments as we like to do and often have done at such opportunities. With great satisfaction I remember the meeting of December 1964 at Utrecht and I hope the few among you who have participated in that conference will share my feelings of gratitude. But whenever I shall remember those pleasant days and evenings, and lively discussions, I will never forget the man whom I met first and last on that occasion, the

Educational Studies in Mathematics 1 (1968), 3-8

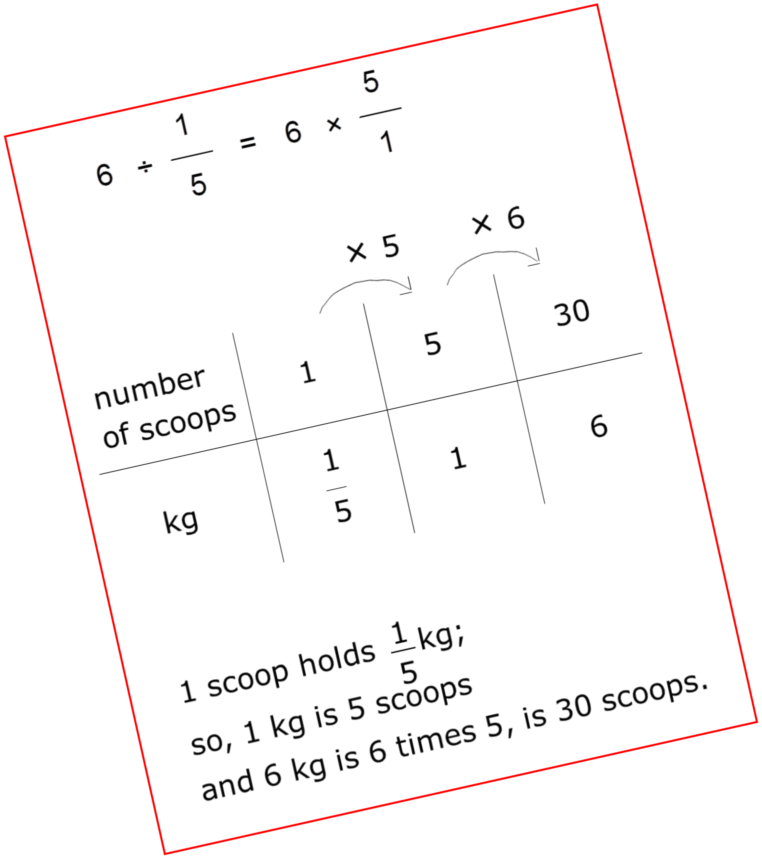

wiskob
doelge
a. treffers
Three
Dimensions
A Model of Goal and Theory Description in
Mathematics Instruction - The Wiskobas Project
Adrian Treffers
Mathematics
Education
Library
D. Reidel Publishing Company

1987

mathematizing



Realistic Mathematics Education



$$6 \div \frac{1}{5} = 6 \times \frac{5}{1}$$

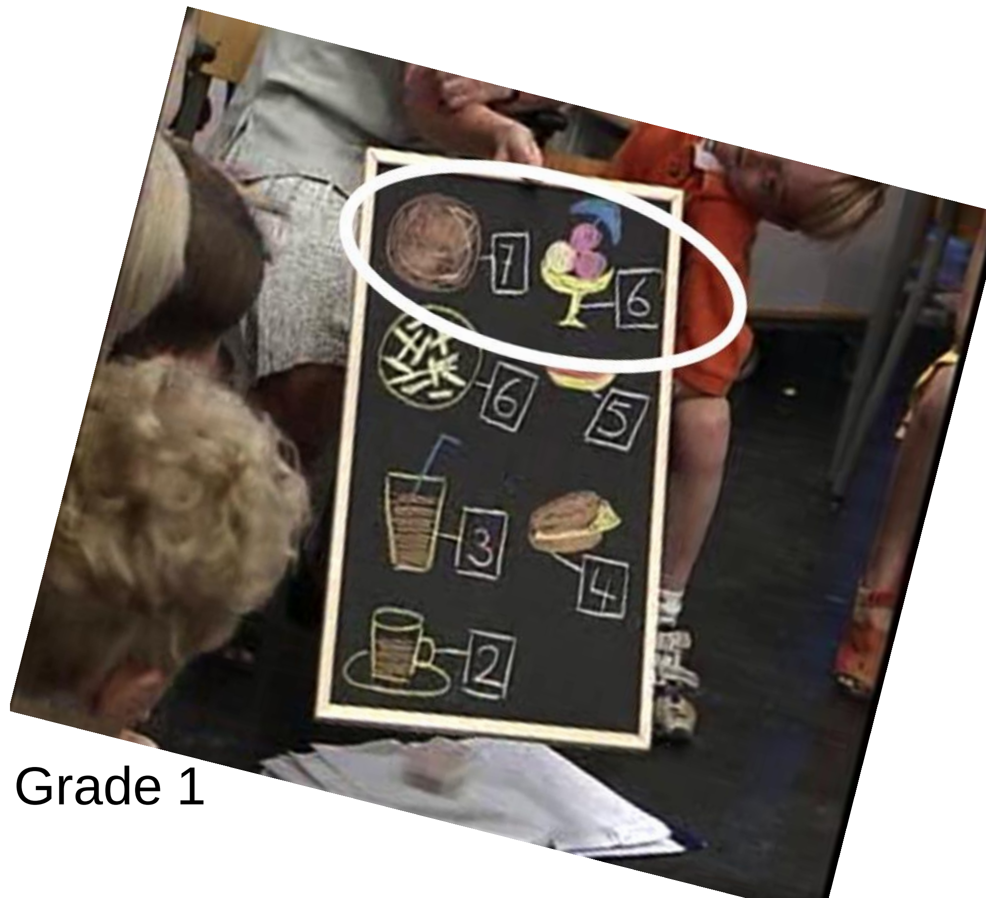
	$\times 5$	$\times 6$	
number of scoops	1	5	30
kg	$\frac{1}{5}$	1	6

1 scoop holds $\frac{1}{5}$ kg;
so, 1 kg is 5 scoops
and 6 kg is 6 times 5, is 30 scoops.

Grade 8

- level principle

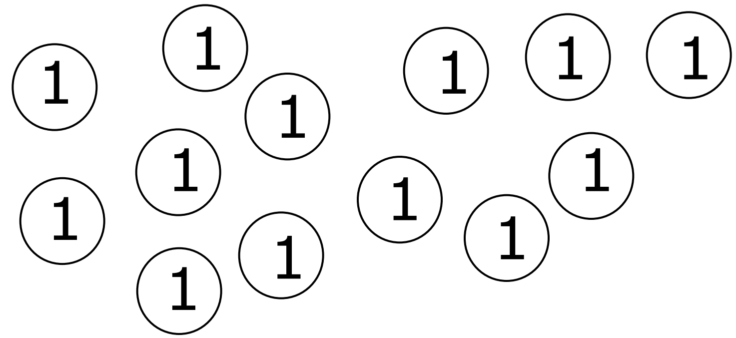
* various levels of understanding
* progressive schematization
* models as bridges



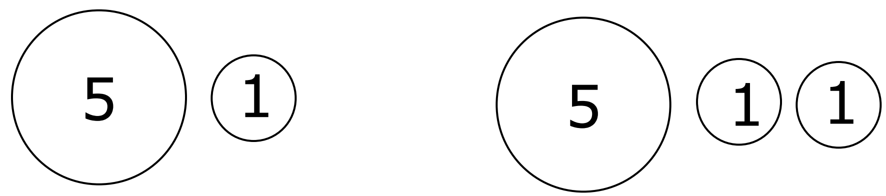
Grade 1

Grade 1

Maureen



Thijs and Nick



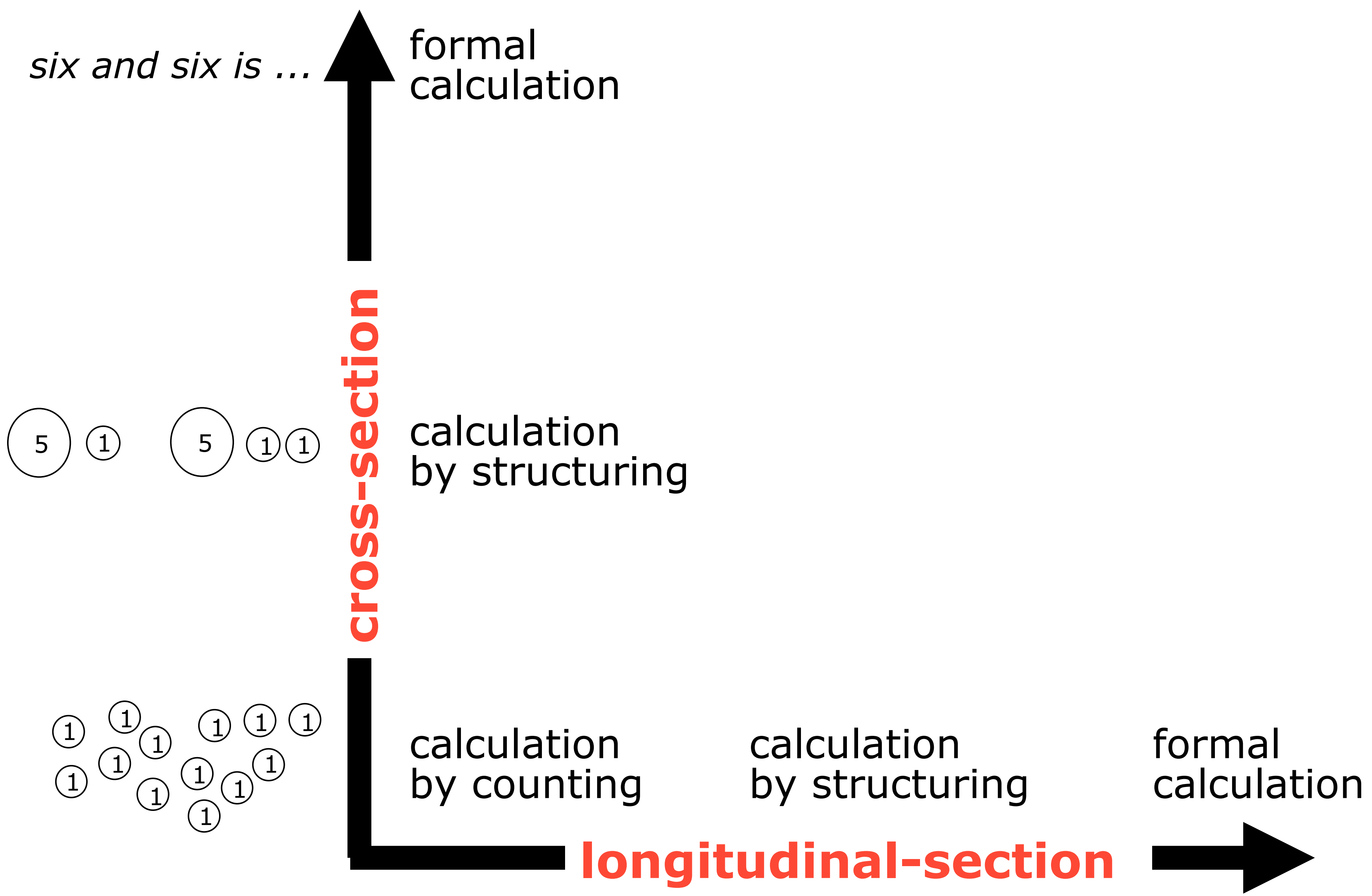
Luuk

"First, put three guilders out of the six to the seven guilders; that makes ten guilders; and three makes thirteen guilders"

Hannah

"Six and six is twelve; and one makes thirteen guilders"

Grade 1



six and six is ...
formal calculation
5 1 5 1 1
calculation by structuring
cross-section
calculation by counting
calculation by structuring
formal calculation
longitudinal-section

Realistic Mathematics Education

- level principle

* various levels of understanding
* progressive schematization
* models as bridges

progressive ‘complexization’ ↔

2 / 6 \ 3 6 0	3 / 72 \ 24 6 12 12 0	5 / 342 \ 68 30 42 40 2

progressive schematization

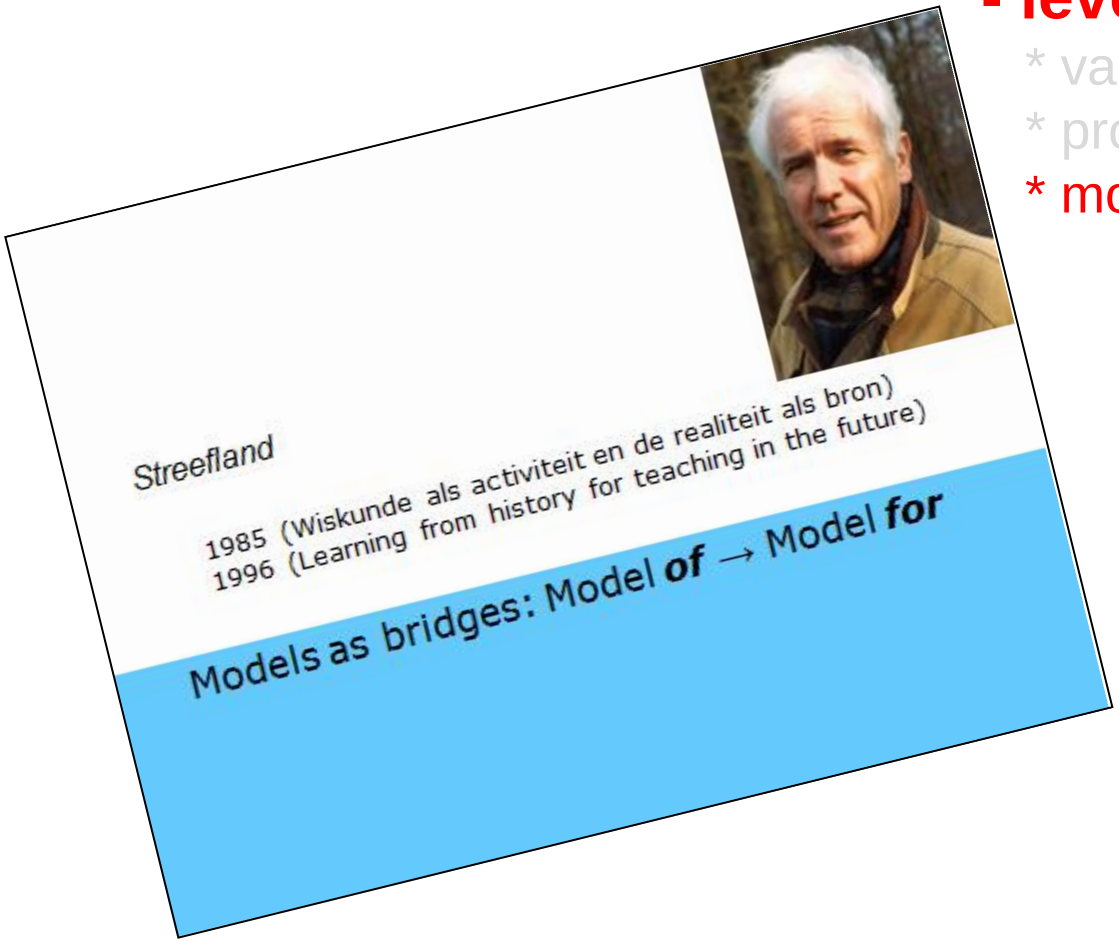
12	6394	
	1200	100
	5194	
	1200	100
	3994	
	1200	100
	2794	
	1200	100
	1594	
	1200	100
	394	
	120	10
	274	
	120	10
	154	
	120	10
	34	
	24	2
	10	532 r. 10

12	6394	
	2400	200
	3994	
	2400	200
	1594	
	1200	100
	394	
	360	30
	34	
	24	2
	10	532 r. 10

12	6394	
	6000	500
	394	
	360	30
	34	
	24	2
	10	532 r. 10

12	6394	532
	60	
	39	
	36	
	34	
	24	
	r. 10	

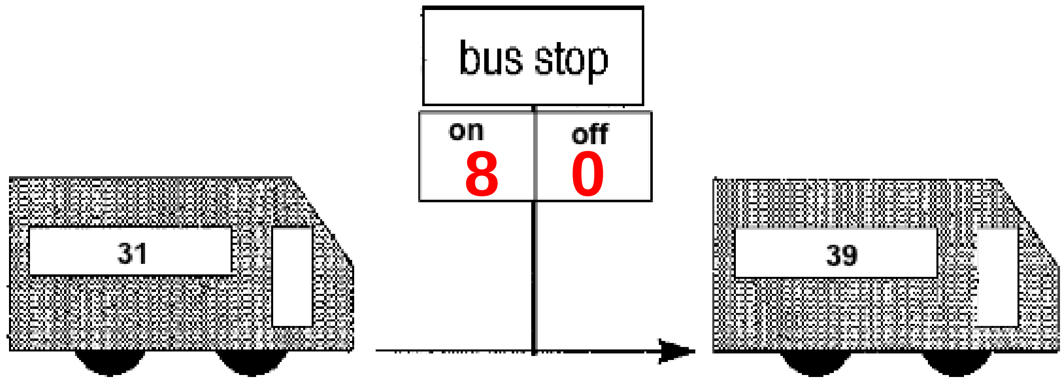
Realistic Mathematics Education

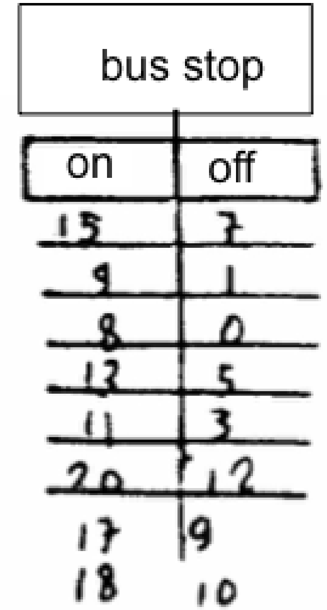


- level principle

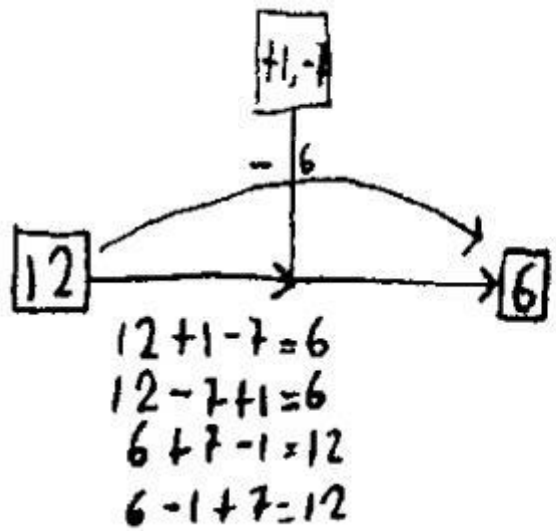
* various levels of understanding
* progressive schematization
* models as bridges

model *of*

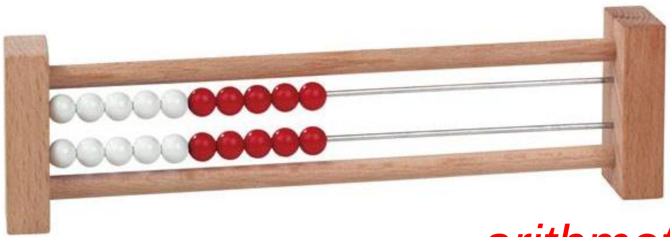




model *for*

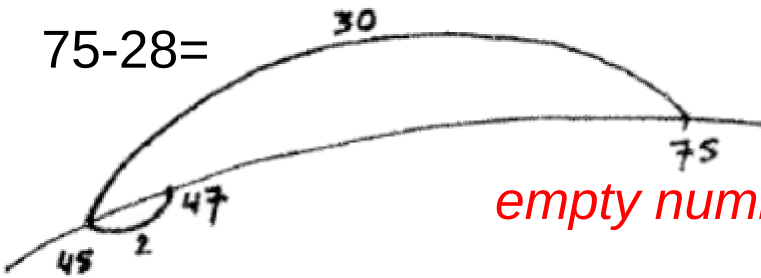


Realistic Mathematics Education



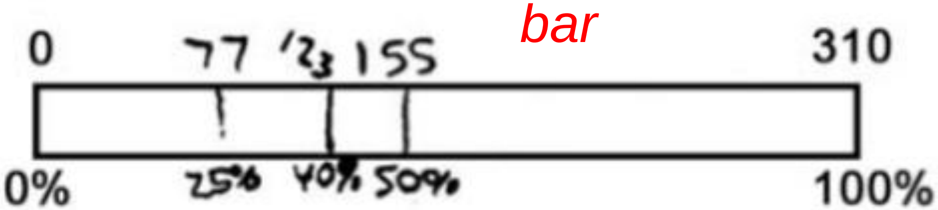
arithmetic rack

75-28=



empty number line

bar



ratio table

	×5	×6	
number of scoops	1	5	30
kg	$\frac{1}{5}$	1	6

- level principle

* various levels of understanding
* progressive schematization
* models as bridges

mechanistic mathematics education

TAAK 41. 85

1. 2/1400\ 4/1600\ 7/2800\ 8/4000\
3/1500\ 9/2700\ 6/4200\ 5/2500\

2.
15—8=	150—80=	130—40=	1400—30=
23—7=	430—60=	360—80=	4700—40=
34—9=	520—90=	940—50=	8400—70=
152—6=	1630—40=	370—80=	6700—90=
394—8=	4720—50=	540—90=	5300—10=

3.
15+8=	150+80=	2347+ 5=	4972+5000=
26+7=	260+70=	1652+ 40=	3286+ 300=
39+5=	580+90=	2382+ 500=	5729+ 60=
157+6=	3750+80=	3785+3000=	1758+ 7=
348+8=	7860+60=	2531+ 18=	2583+ 17=

4.
1208 7 ×	1065 6 ×	1413 7 ×	1829 3 ×	2700 2 ×
123 9 ×	456 8 ×	789 7 ×	903 8 ×	777 6 ×

5. Telkens $2\frac{1}{2}$ erbij: $2\frac{1}{2}$, 5, $7\frac{1}{2}$, ..., ..., ..., ..., 25
Telkens $7\frac{1}{2}$ erbij: $7\frac{1}{2}$, 15, ..., ..., ..., ..., ..., 75
Telkens $12\frac{1}{2}$ erbij: $12\frac{1}{2}$, 25, ..., ..., ..., ..., ..., 125

6. *De helft ervan nemen. Uit het hoofd.*

200	400	600	300	500	700	250	450
100							

Realistic Mathematics Education

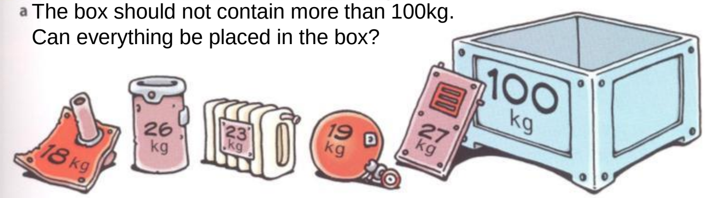
Les 4 Week 4 Blok 1

1 Solve the problems. Do it in a smart way.

22 + ... = 30	13 + ... = 60	33 + ... = 100	2 × ... = 70
72 − ... = 30	79 − ... = 60	133 − ... = 100	... + 25 = 70
38 − ... = 30	2 × ... = 60	... + 66 = 100	170 − ... = 70
3 × ... = 30	... − 5 = 60	... − 30 = 100	... + 59 = 70
... − 15 = 30	... + 28 = 60	... × 25 = 100	... × 7 = 70

2 Estimate.

a The box should not contain more than 100kg.
Can everything be placed in the box?



b Will you get 5 euro deposit money?

€ 0,40 € 3,30 € 1,50 € 0,75

Supermarkt
statiegeld
€ 0,40
€ 1,50
€ 3,30
€ 0,75

3 Will you get more or less than 5 euro deposit money?

Supermarkt statiegeld	Supermarkt statiegeld	Supermarkt statiegeld	Supermarkt statiegeld	Supermarkt statiegeld
€ 0,75	€ 1,25	€ 1,65	€ 0,95	€ 0,05
€ 1,75	€ 0,50	€ 1,35	€ 2,50	€ 2,55
€ 2,25	€ 0,75	€ 0,80	€ 1,45	€ 1,20
	€ 2,00	€ 1,20	€ 0,50	€ 1,95

~1968 2015

mechanistic mathematics education

TAAK 53. 109

1.

20/1480\	30/2190\	40/2160\
50/3400\	60/4860\	20/1640\
90/4680\	70/2170\	80/6560\
70/3710\	80/5120\	60/1620\

2.

430	321	212	203	142
19 ×	28 ×	37 ×	46 ×	55 ×

3.

1458	567	2048	2348	738
2057	3296	372	1356	2367
143	25	59	1854	4
17	4647	5788	2973	815

4. *Aftrekken:*

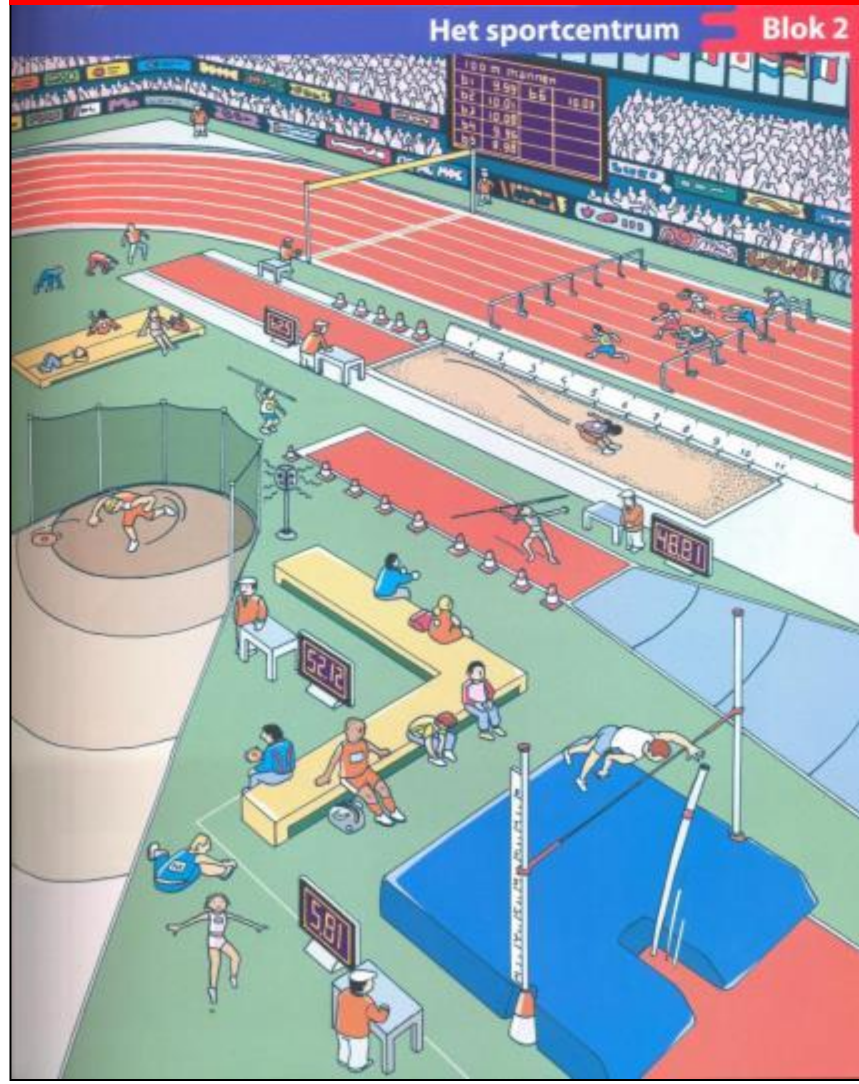
7100	8000	6042	3810	7002
3675	4783	5291	2170	6999

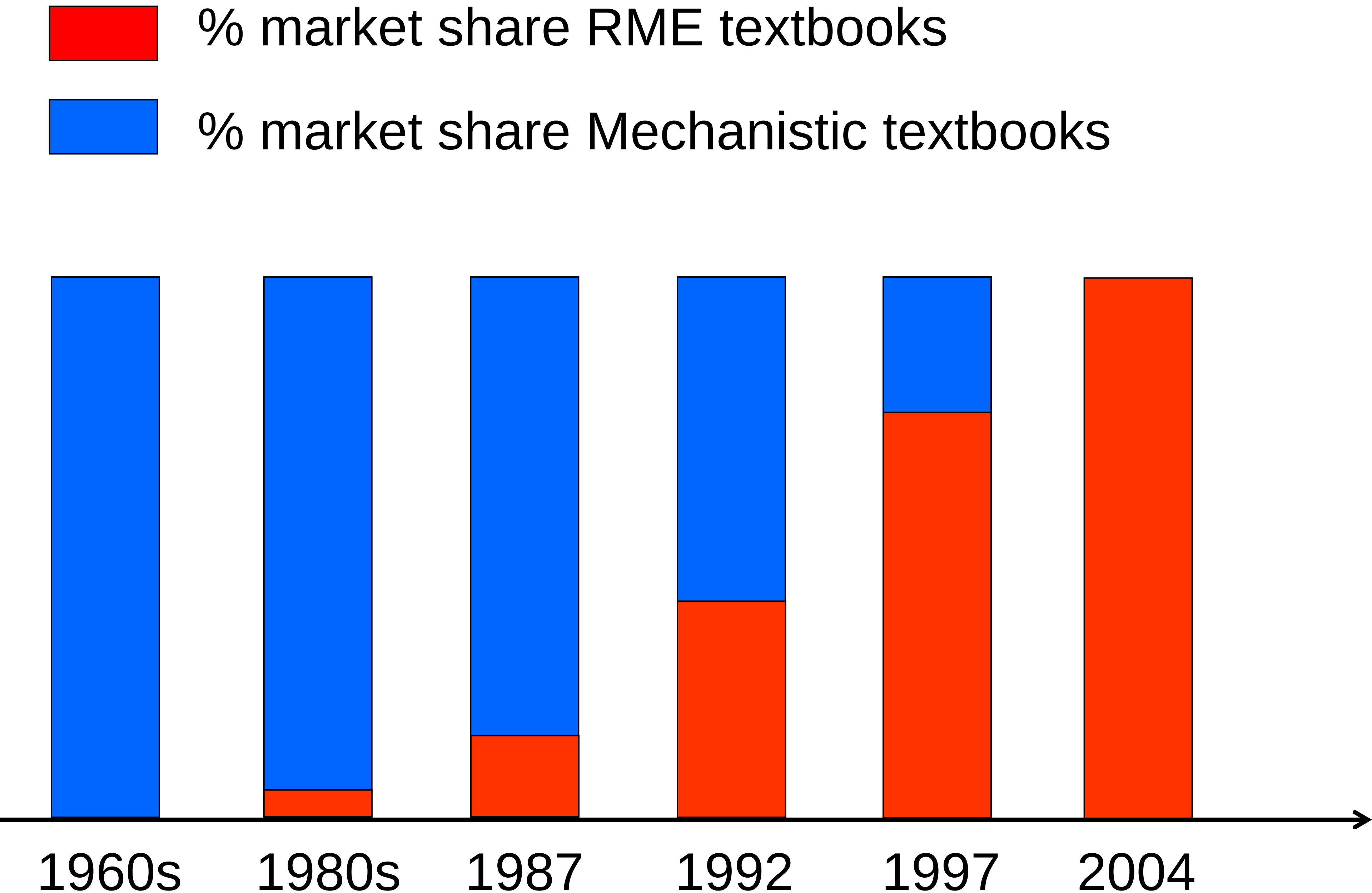
5. Bob, Wim en Koos hebben samen *f* 6.
Bob heeft *f* 0,95 Koos heeft *f* 3,15 Wim heeft *f* . .

6.

1 jaar = . . mnd.	26 mnd. = . . jaar + . . mnd.
3 jaar = . . mnd.	39 mnd. = . . jaar + . . mnd.
6 jaar = . . mnd.	63 mnd. = . . jaar + . . mnd.
5 jaar = . . mnd.	72 mnd. = . . jaar + . . mnd.

Realistic Mathematics Education




% market share RME textbooks
% market share Mechanistic textbooks
1960s
1980s
1987
1992
1997
2004

Overview

- A summary of the guiding principles of RME
- A blind spot in RME?
- An online game to introduce early algebra in primary school
- A new project aimed at making the primary school mathematics curriculum more mathematical

POPO

UK government-funded
World Class Arena programme

NL Survey in 2004

152 high achieving (top 25%) fourth-graders from 22 schools
15 problems from the ***World Class Tests***

Find the number
It is smaller than 100.
If you divide it by 7, there is no remainder.
If you divide it by 3, the remainder is 2.
If you divide it by 5, the remainder is 1.

Show how you solved the problem.

26% correct

Angela is 15 years now andJohan is 3 years.
In how many years will Angela be twice as old as John?

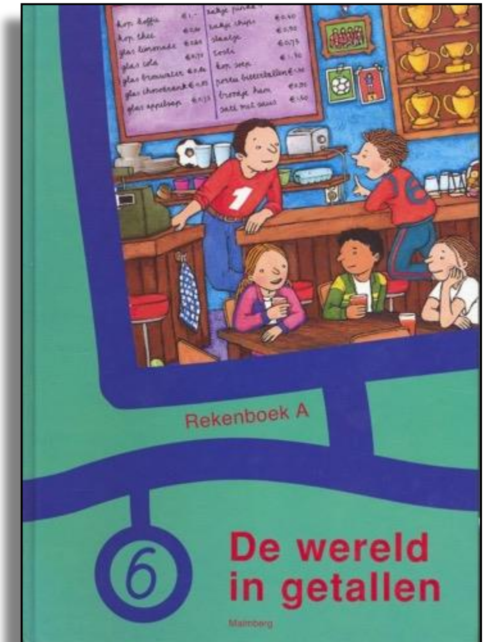
Show how you solved the problem.

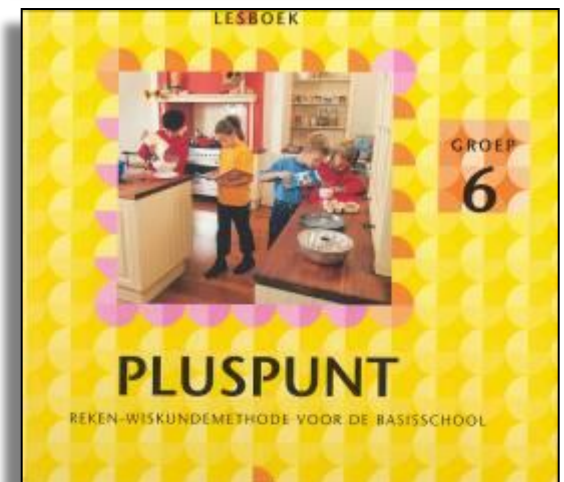
24% correct

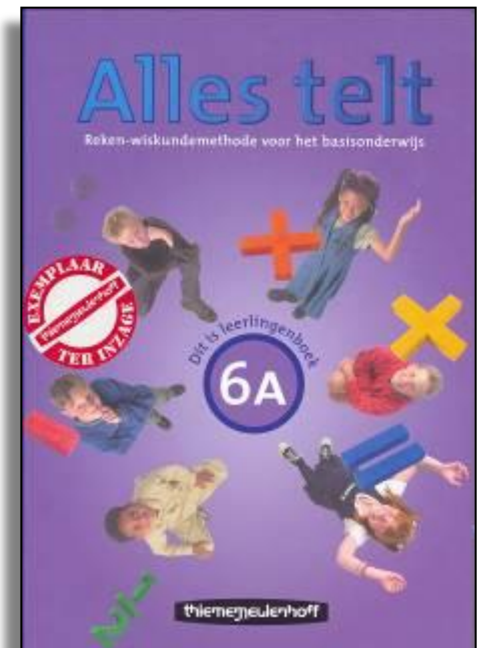
POPO

TEXTBOOK ANALYSIS

to identify **opportunity-to-learn** solving puzzle-like tasks







POPO

Types of taskes

You have a soup cup (300 ml). How can you use it to measure 2100 ml of water?

STRAIGHT-FORWARD TASKS

You have a soup cup (300 ml), a mug (200 ml) and a glass (250 ml). Show different ways in which you can use these containers to measure 1500 ml of water.

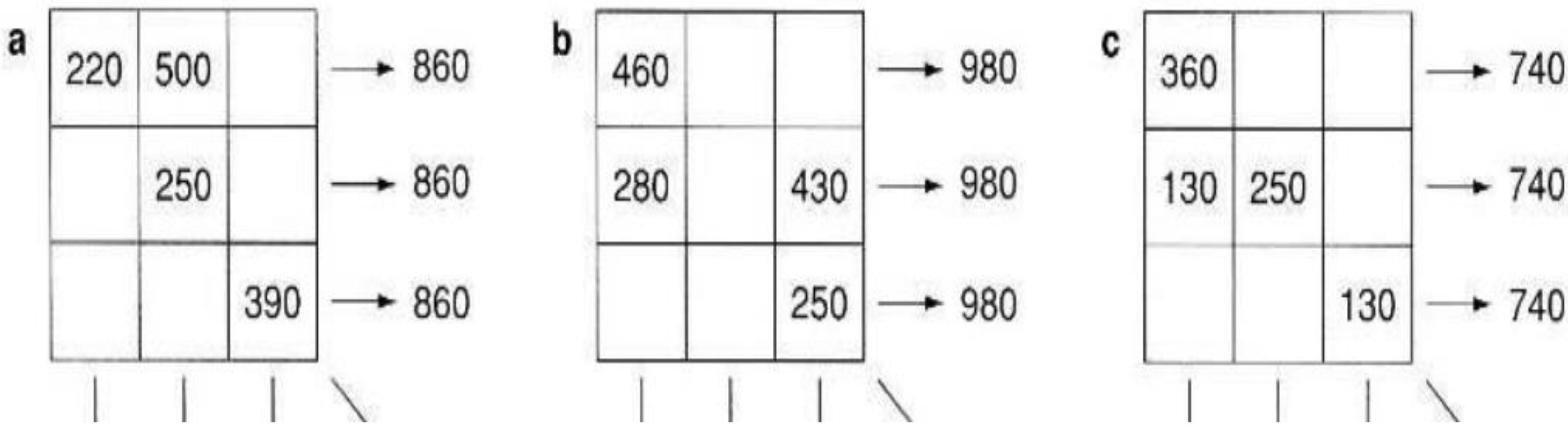
GRAY-AREA TASKS

You have a 5-liter and a 3-liter jug. How can you take 4 liters of water out of the big bowl using these two jugs? You may pour water back into the bowl.

PUZZLE-LIKE TASKS

POPO

GRAY-AREA TASKS



(Pluspunt, Workbook 6, p.15)

POPO

GRAY-AREA TASKS

Pay the exact amount. Try it in at least five ways.
Draw the money.

(De Wereld in Getallen, Arithmetic book 6A, p. 59)

POPO

PUZZLE-LIKE TASKS

1 Three times the same number.

20 − ▼ = ...	30 − ● = ...	25 − ■ = ...	80 − ◆ = ...	37 − ▲ = ...
20 + ▼ = ...	30 + ● = ...	25 + ■ = ...	80 + ◆ = ...	37 + ▲ = ...
20 × ▼ = ...	30 × ● = ...	25 × ■ = ...	80 × ◆ = ...	37 × ▲ = ...
together 160	together 300	together 150	together 560	together 370

(De Wereld in Getallen, Arithmetic book 6A, p. 36)

POPO

PUZZLE-LIKE TASKS

5 Fill in the squares.

Use the numbers 10, 20, 30, 40, 50, 60 each two times.
The middle box shows to total of each row and column.

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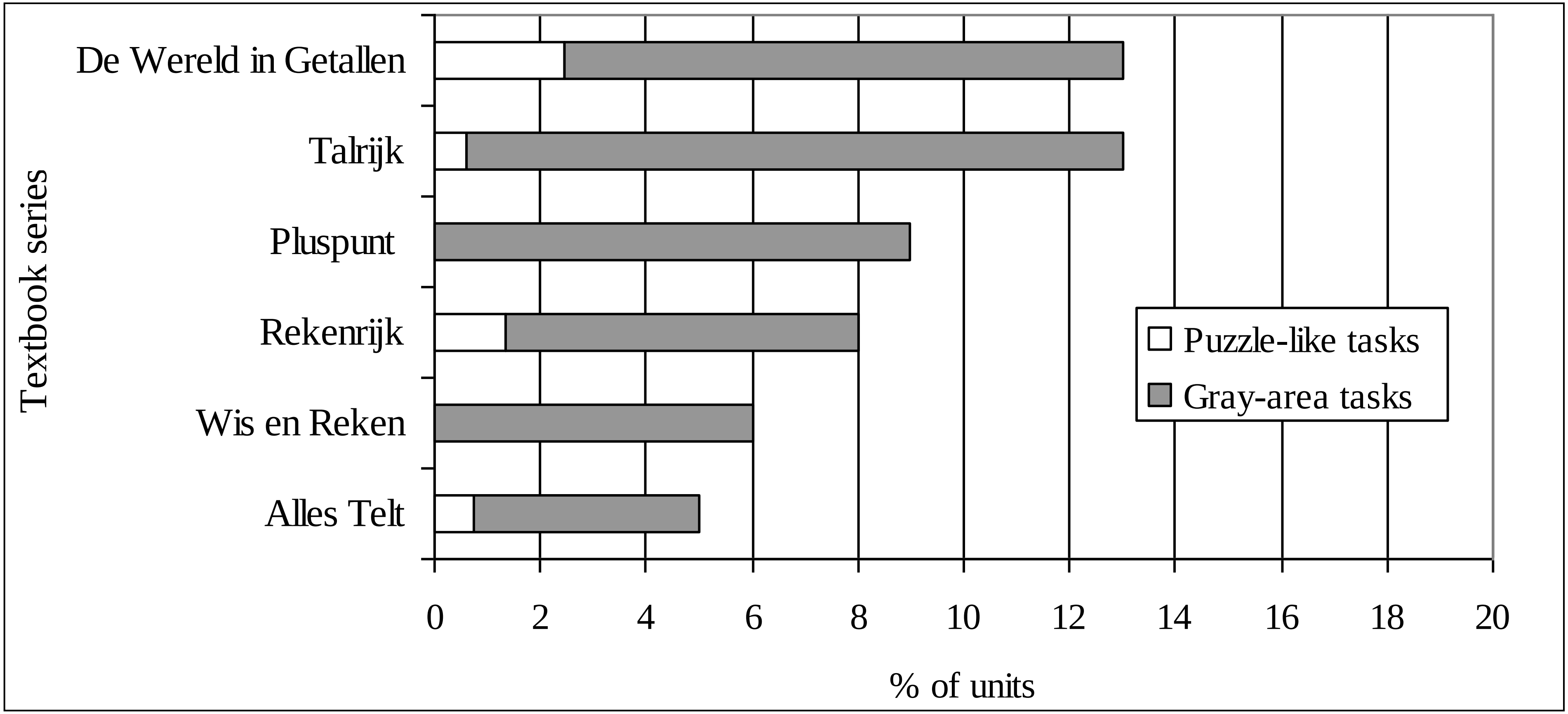
(De Wereld in Getallen, Arithmetic book 6A, p. 67)

POPO

Results textbook analysis



Mediterranean Journal for
Research in Mathematics Education
Vol. 8, 2, 31-68, 2009

Non-Routine Problem Solving Tasks in Primary School Mathematics Textbooks – A Needle in a Haystack

Angeliki Kolovou *, Marja van den Heuvel - Panhuizen ** and Arthur Bakker*

* Freudenthal Institute for Science and Mathematics education, Utrecht University, the Netherlands

** Freudenthal Institute for Science and Mathematics education, Utrecht University, the Netherlands and IQB, Humboldt University, Berlin, Germany

ABSTRACT: *In this paper, we report on a study in which we investigated the nature o numerical problem solving tasks as presented in primary school mathematics textbook in the Netherlands. Although several factors influence what mathematics teachers tea children, there is much evidence that the curriculum and the textbooks are importa determinants of what children are taught and what they learn. Contradicting resu from TIMSS and poor performances of Dutch fourth graders on a test on mathemati ... were the immediate reasons for this textbook analysis study. ... aterials of six mathematics textbook series.*

There are 218 passengers and 191 crew members on a ship. How many people are on the ship altogether?

Answer: 409

% correct
NL
81

Duncan first traveled 4.8 km in a car and then he traveled 1.5 km in a bus. How far did Duncan travel?

- 6.3 km
- B 5.8 km
- C 5.13 km
- D 4.95 km

% correct
NL
73

In a soccer tournament, teams get:

3 points for a win
1 point for a tie
0 points for a loss

Zedland has 11 points.

What is the **smallest** number of games Zedland could have played?

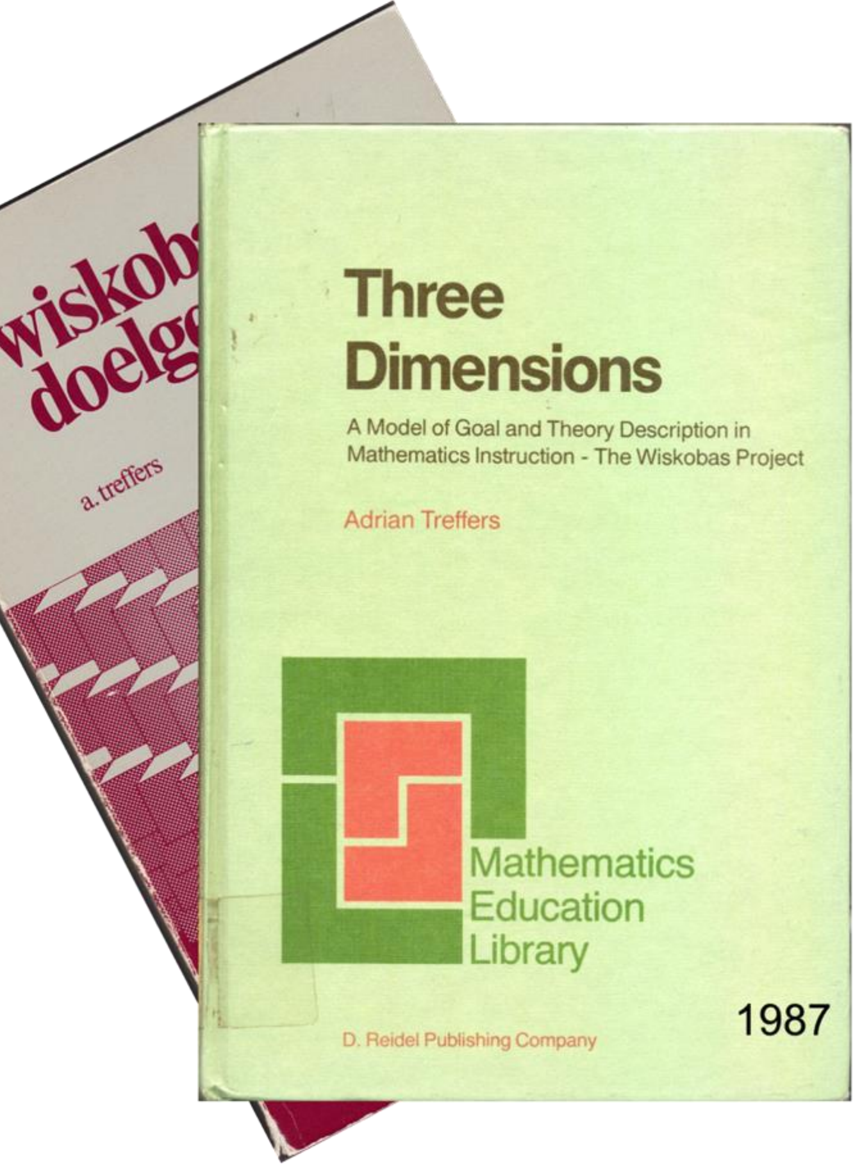
5

Answer: ____________________

% correct
NL
36

Is this typical for Realistic Mathematics Education?

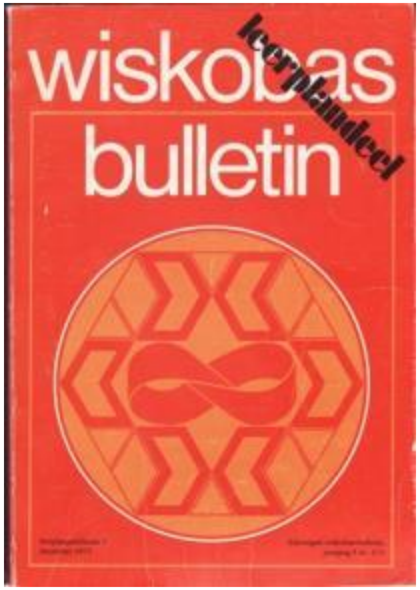
Let's have a look at the source of RME



The new **objectives** of Wiskobas

“In short: the new objectives concerned mathematising, e.g.
generalising,
proving,
schematising,
symbolising,
using models” (p. 21)

“The six **subject areas** from which Wiskobas takes its content and instructional activities […] are:
arithmetic,
measuring,
geometry,
probability and statistics,
relations and functions,
language and logic” (p. 119)

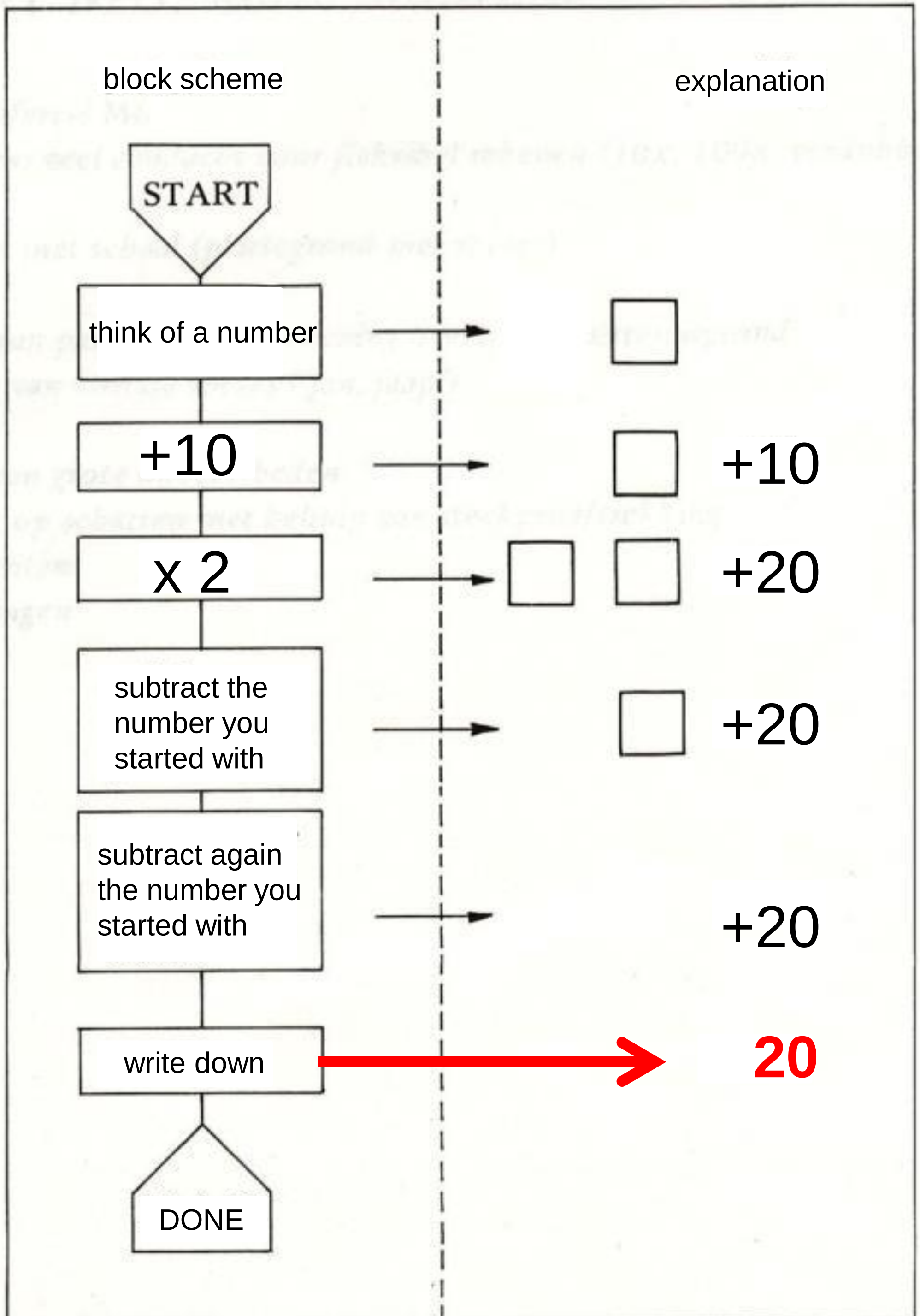


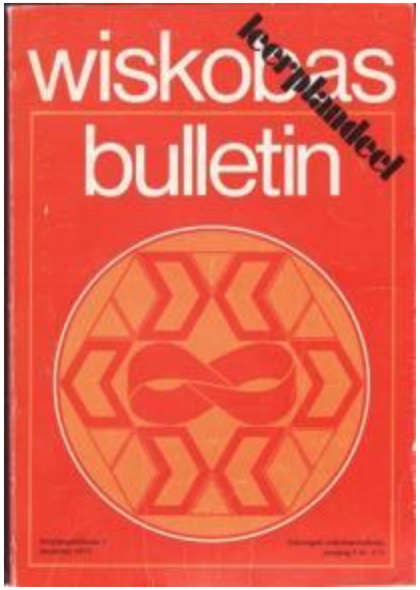
***Machine language* in Wiskobas**

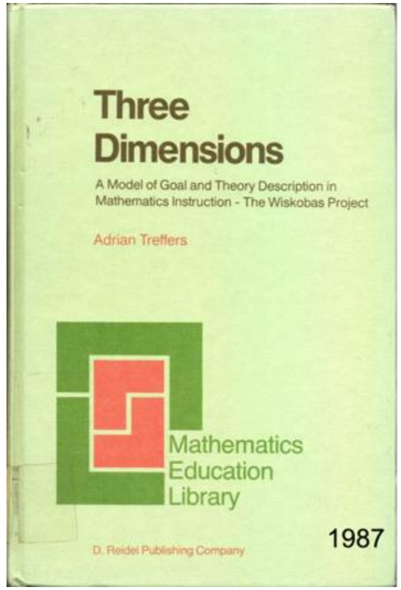
(Scenario M7) *Leerplanpublikatie 2*, 1975

- Block schemes to crack a number game

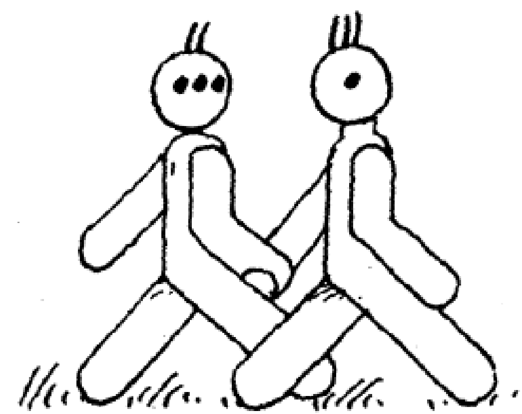
Think of a number,
add 10,
multiply by 2,
subtract the number you started with,
subtract this starting number again,
what number did you get?


block scheme
explanation
START
think of a number
+10
+10
x 2
+20
subtract the number you started with
+20
subtract again the number you started with
+20
write down
20
DONE



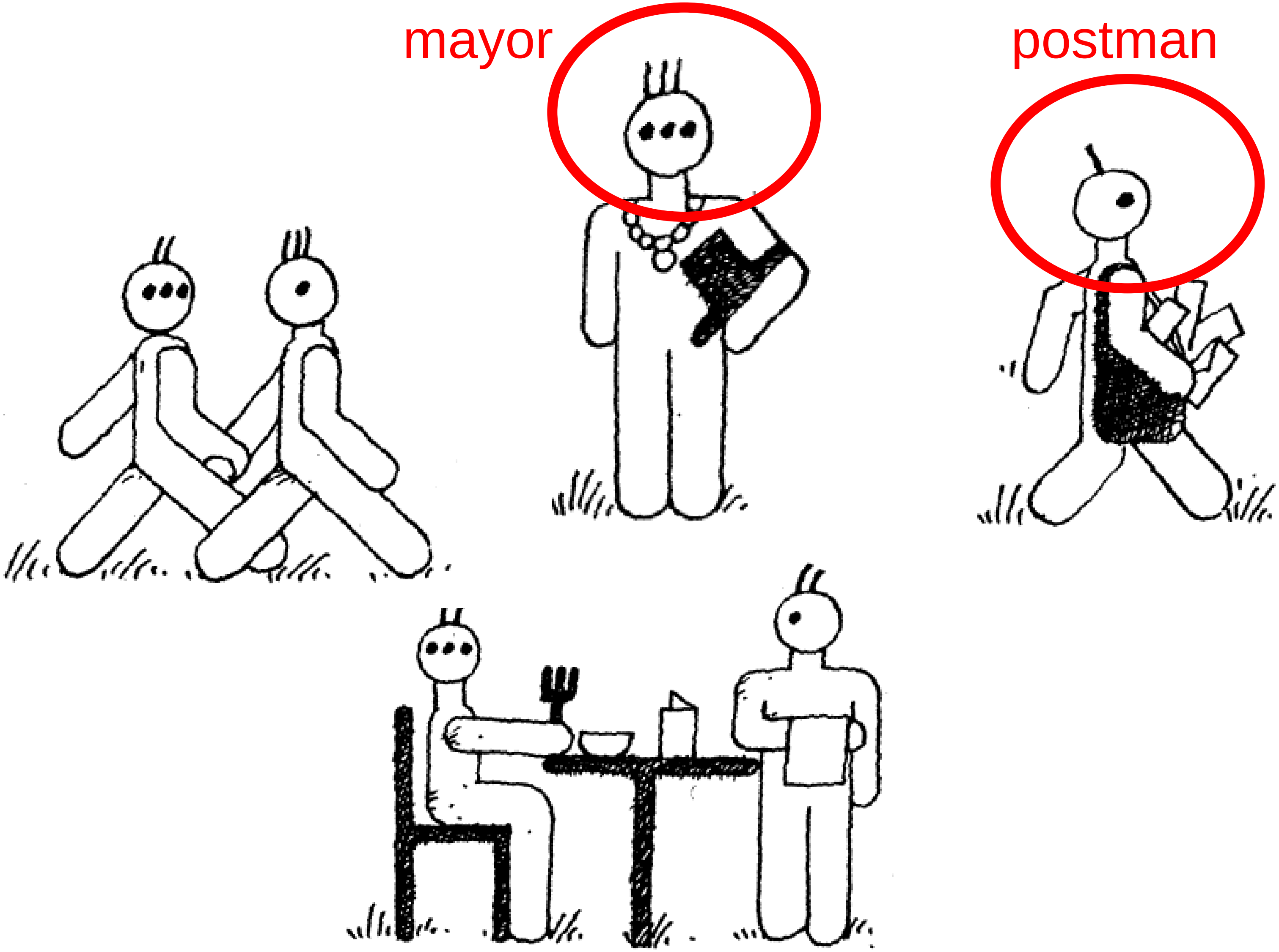


FRECKLEHAM in Wiskobas
(Scenario M2) *Leerplanpublikatie 2*, 1975

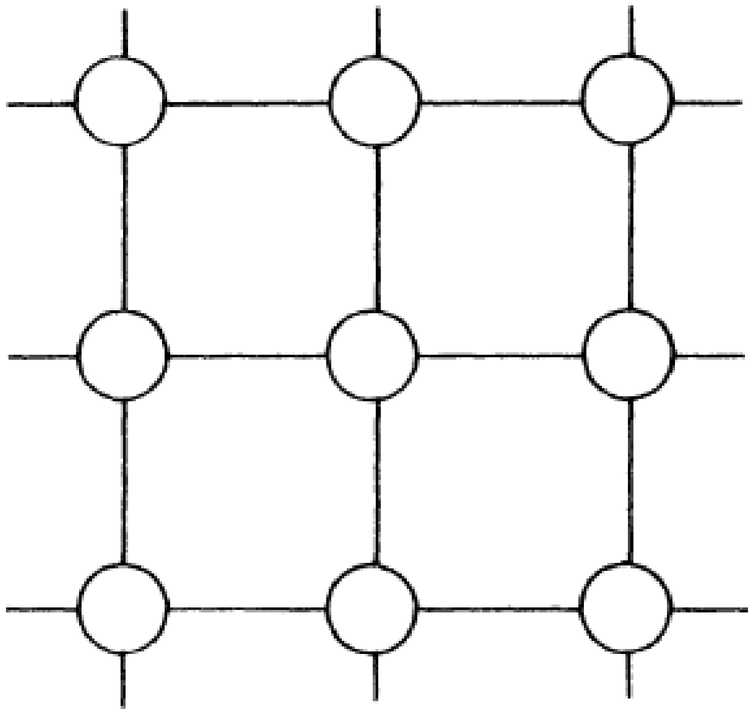


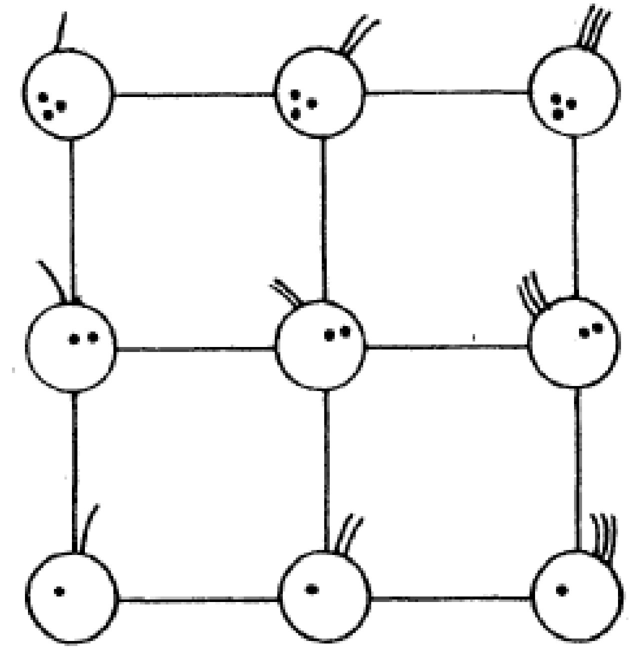
- Visualizing relations
- Reasoning by means of arrow language and using symbols
- Intuitively making use of logical concepts and properties
- Investigating properties of relations (transitivity)

FRECKLEHAM



Map of Freckleham





Where could  live?

Where could  live?

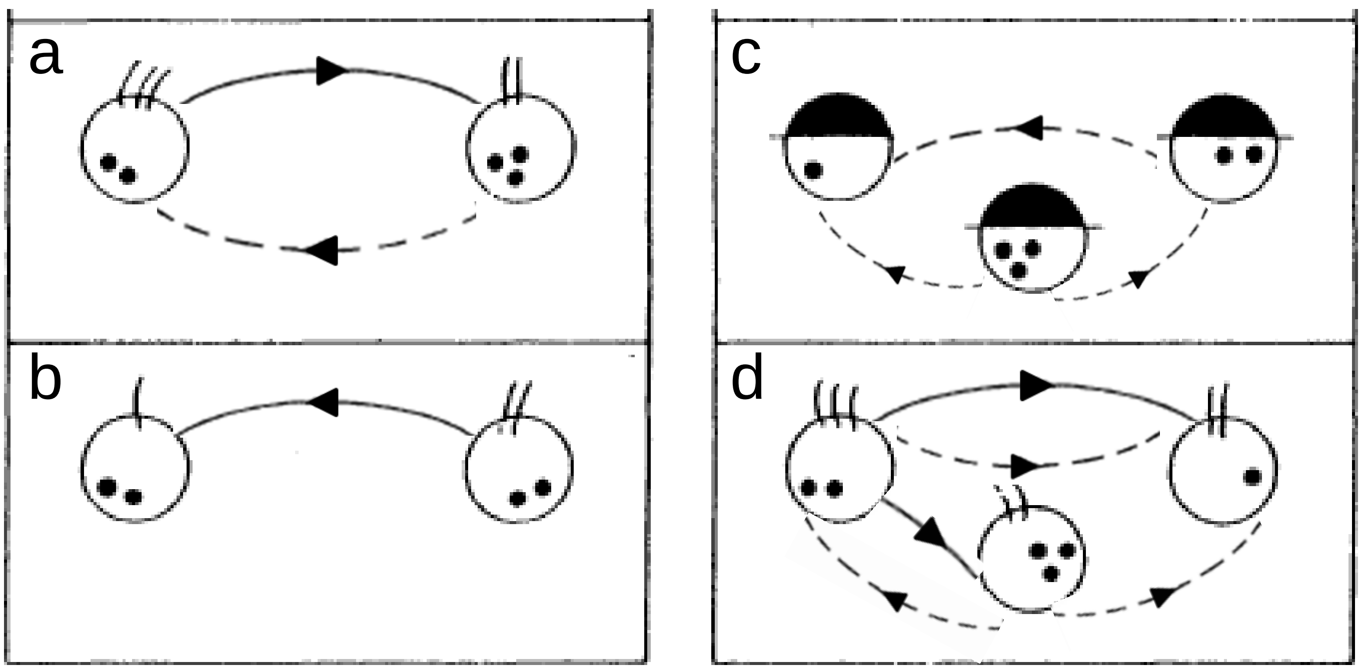
Give names to the streets and avenues.

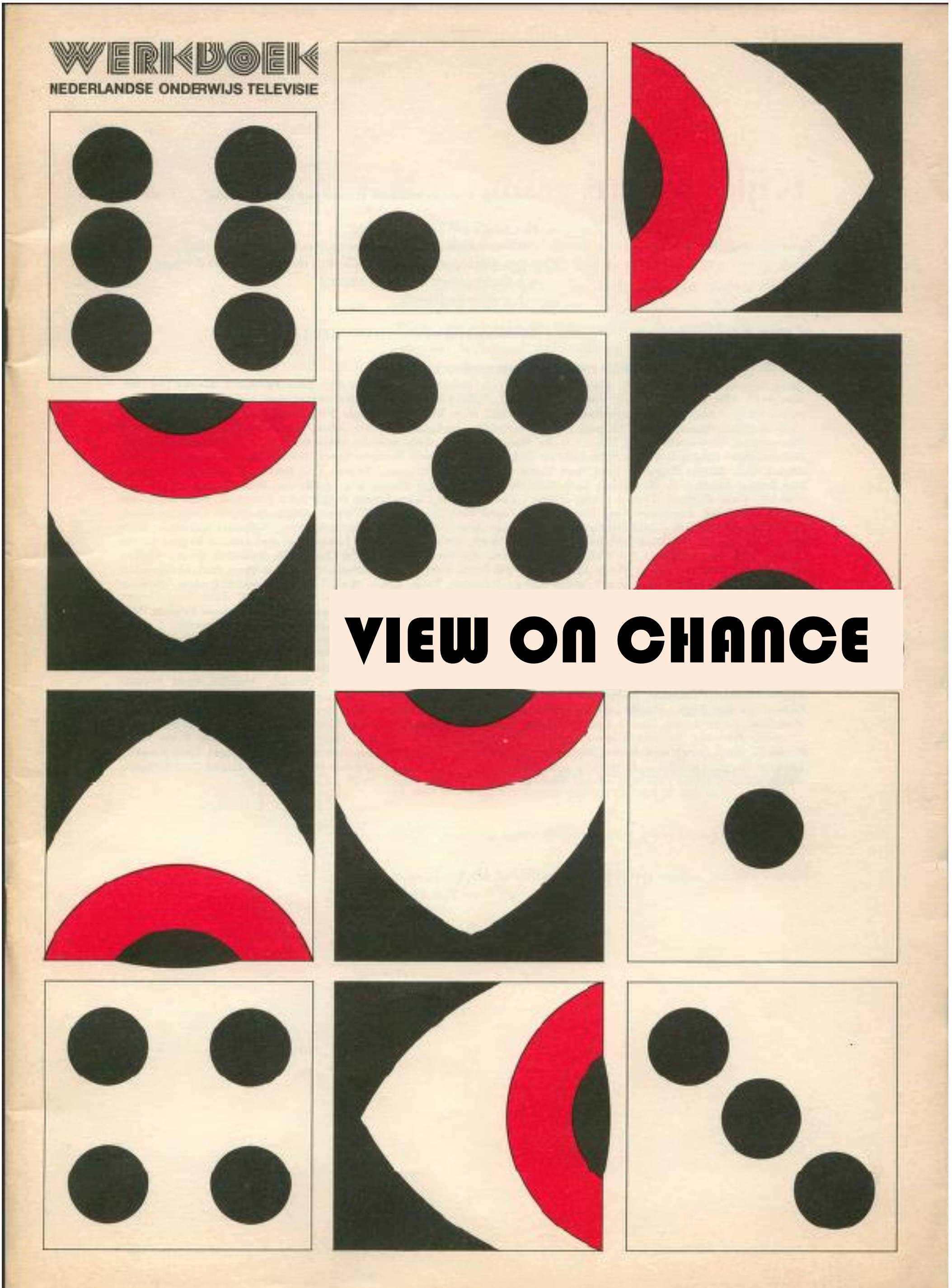
FRECKLEHAM

Greetings

“I you” means “I have more **hairs** than you”

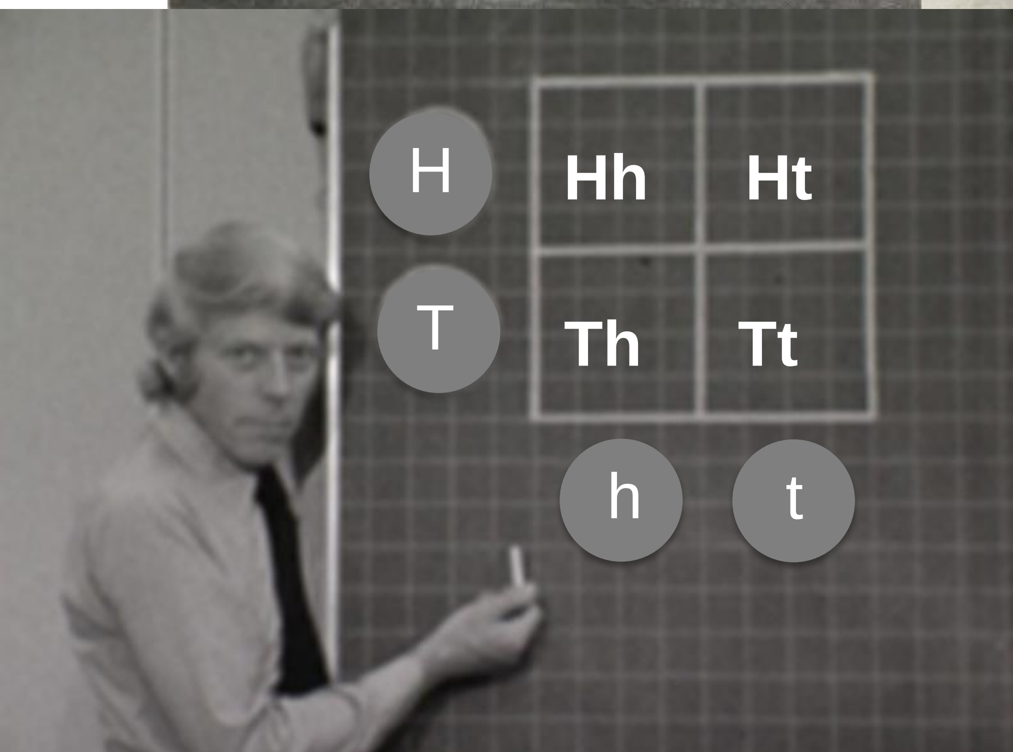
“I you” means “I have more **freckles** than you”





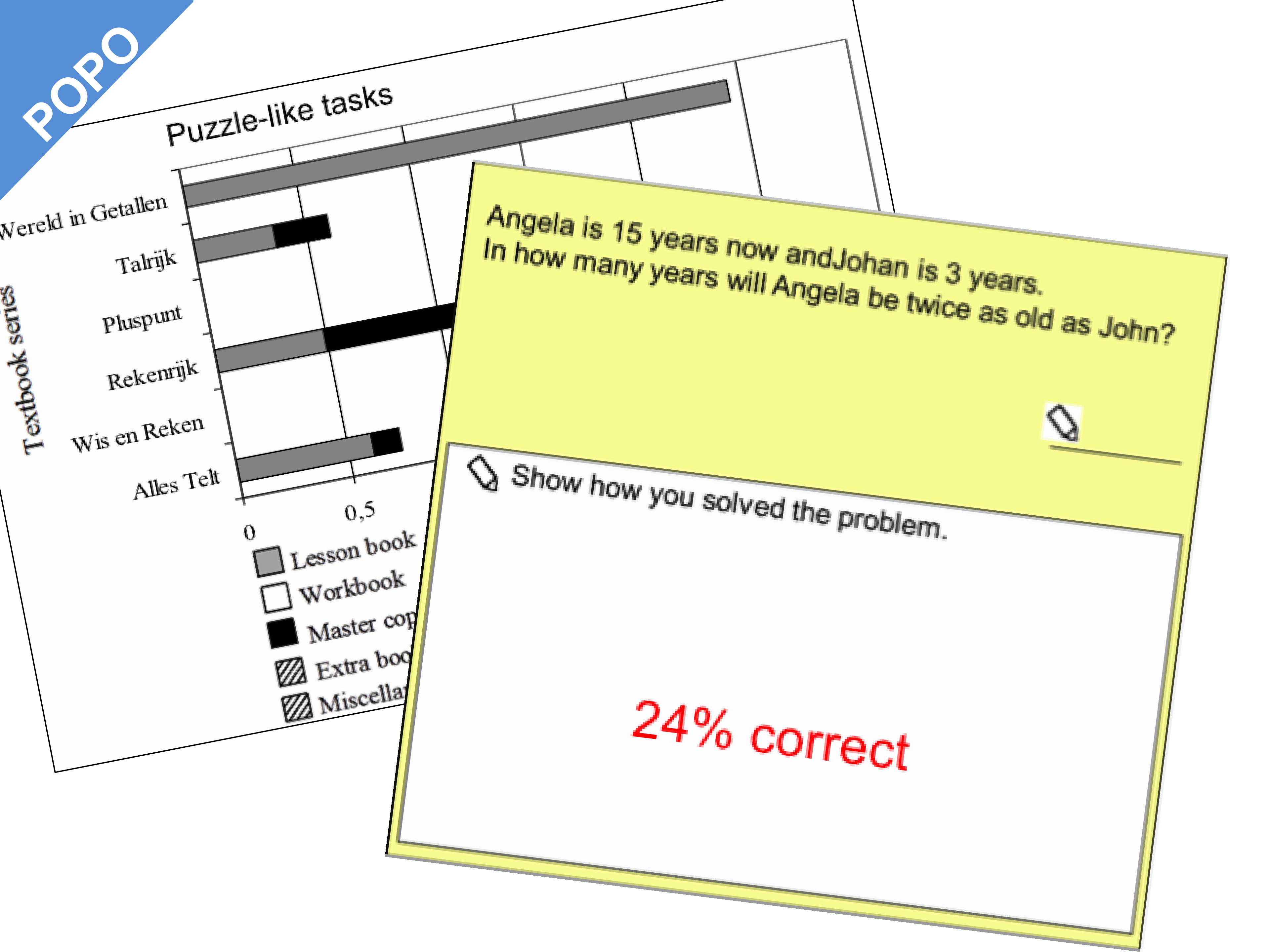
Kijk op kans
Janssen & Goffree,
1972/1973

VIEW ON CHANCE



Overview

- A summary of the guiding principles of RME
- A blind spot in RME?
- An online game to introduce early algebra in primary school
- A new project aimed at making the primary school mathematics curriculum more mathematical



POPO
Puzzle-like tasks
Wereld in Getallen
Talrijk
Pluspunt
Rekenrijk
Wis en Reken
Alles Telt
Textbook series
0
0,5
Lesson book
Workbook
Angela is 15 years now andJohan is 3 years.
In how many years will Angela be twice as old as John?
Show how you solved the problem.
24% correct

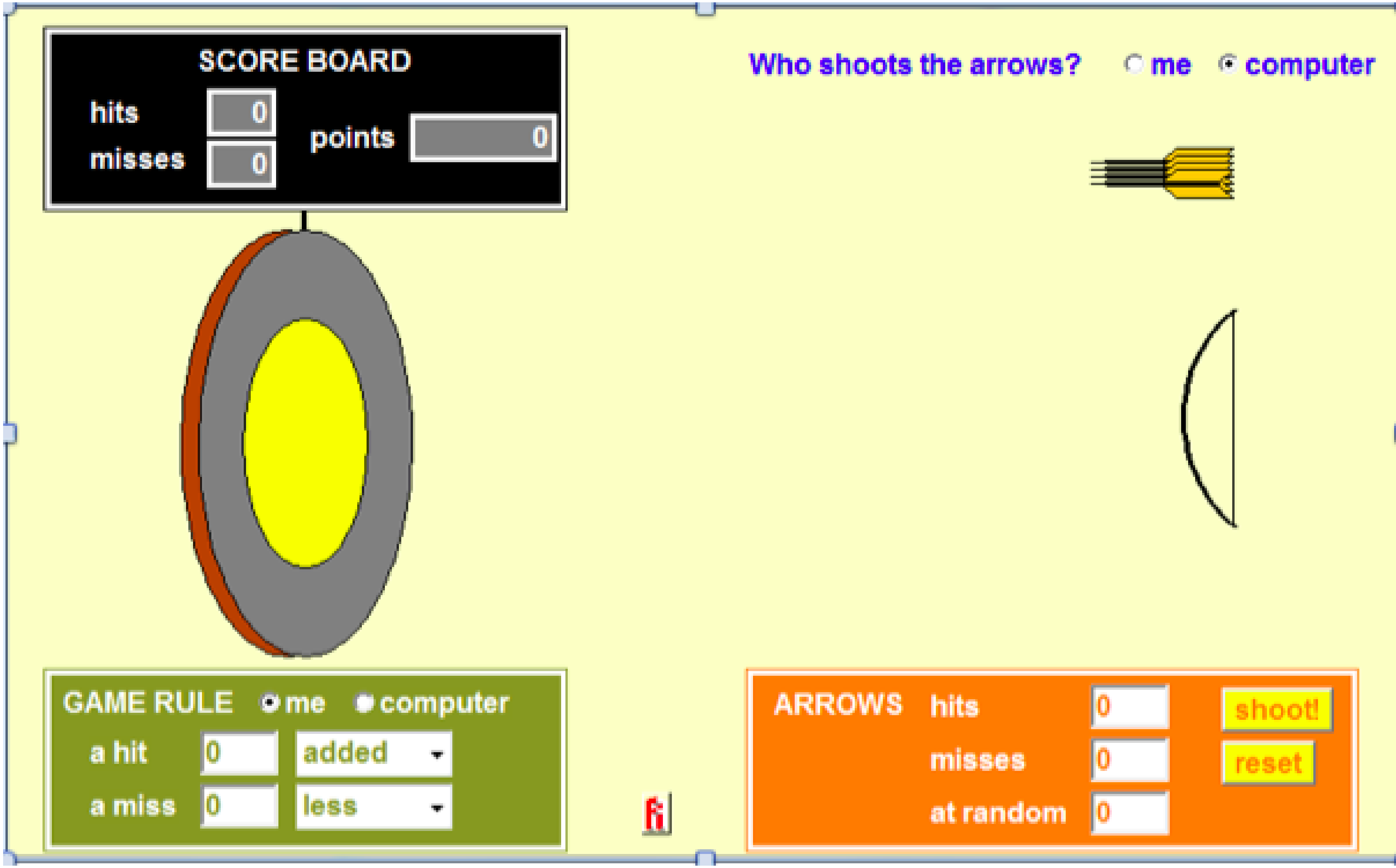
POPO

Angela is 15 years now and Johan is 3 years.
In how many years will Angela be twice as old as John?

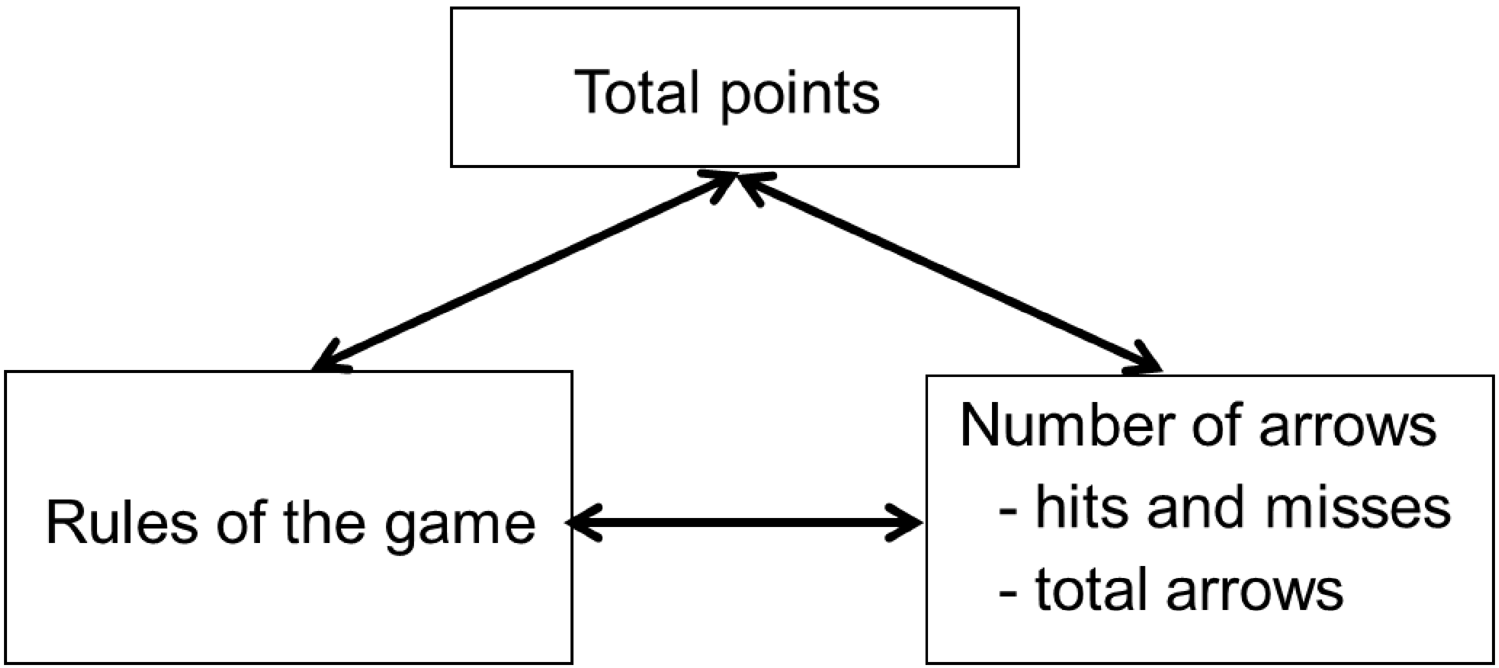
Frequency information used (*N* = 152)				
Age Angela	Age John	Absolute age difference remains the same	Angela older, then John as well	Angela is 2x as old as John
122	120	69	59	63
80%	79%	45%	39%	41%

POPO

Hit the target



Covarying quantities in **Hit the target**



POPO

For every hit: 3 points
For every miss: 1 point is taken away
With **how many hits and misses** do you get 15 points in total?

What is the **game rule** for 15 points, 15 hits, 15 misses?

Are there **other game rules** for 15 points, 15 hits, 15 misses?

What is the **game rule** for 16 points, 16 hits, 16 misses?

Are there **other game rules** for 16 points, 16 hits, 16 misses?

What is the **game rule** for 100 points, 100 hits, 100 misses?

Are there **other game rules** for 100 points, 100 hits, 100 misses?

POPO

Journal for Research in Mathematics Education
2013, Vol. 44, No. 3, 510–549

An Intervention Inclu
Game to Improve Gr
Performance in E

Angeliki Ko
Freudenthal Institute for Science
Utrecht University,

Marja van den Heu
Freudenthal Institute for Science
Utrecht University,

Olaf K
IPN, Leibniz Institute for Scien
Kiel, G

This study investigated whether an interve
236 Grade 6 students' performance in early
rying quantities. An exploratory quasi-exp
posttest-control-group design. Students in
home a number of problems by playing an
online activity. Before and after the interv
problem solving was administered. Statist
of the intervention on posttest performa
mathematical ability, and gender. Althoug
mance on the pretest as well as on the p
intervention. Implications of these result

Key words: Algebra; Gender issues; Inst

Although great significance is at
e.g., Schoenfeld, 1995; Katz, 2007;
National Research Counci

Educ Stud Math
DOI 10.1007/s10649-013-9483-5

Primary school students' strategies in early algebra problem solving supported by an online game

Marja van den Heuvel-Panhuizen · Angeliki Kolovou · Alexander Robitzsch

Abstract In this study we investigated the role of a dynamic online game on students' ear
algebra problem solving. In total 253 students from grades 4, 5, and 6 (10–12 years old) us
the game at home to solve a sequence of early algebra problems consisting of context
problems addressing covarying quantities. Special software monitored the students' onl
working when solving the problems. Before and after the intervention a paper-and-pencil test
early algebra was administered. The data analysis revealed that the online working contribu
to the students' early algebra performance. There was a significant gain in performance ac
all grades. The highest effect was found in grade 6. Out of the three strategy profile clusters
could be distinguished in the whole sample, the cluster dominated by using extreme values
the cluster characterized by the trial-and-error strategy were most influential on the gain in
algebra performance. The students' level of online working, which was defined as a comb
tion of online involvement and strategy use, appeared to have a marginally significant effe
the gain score for the total sample. Per grade there was no significant effect, yet the leve
online working were significantly related to grade. Free playing was mostly performed in
4, looking for answers in grade 5, and exploring relations slightly more in grades 5 and 6.
17 % of the effect of grade on the gain score was mediated by the level of online worki

Online learning environment · Dynamic gam

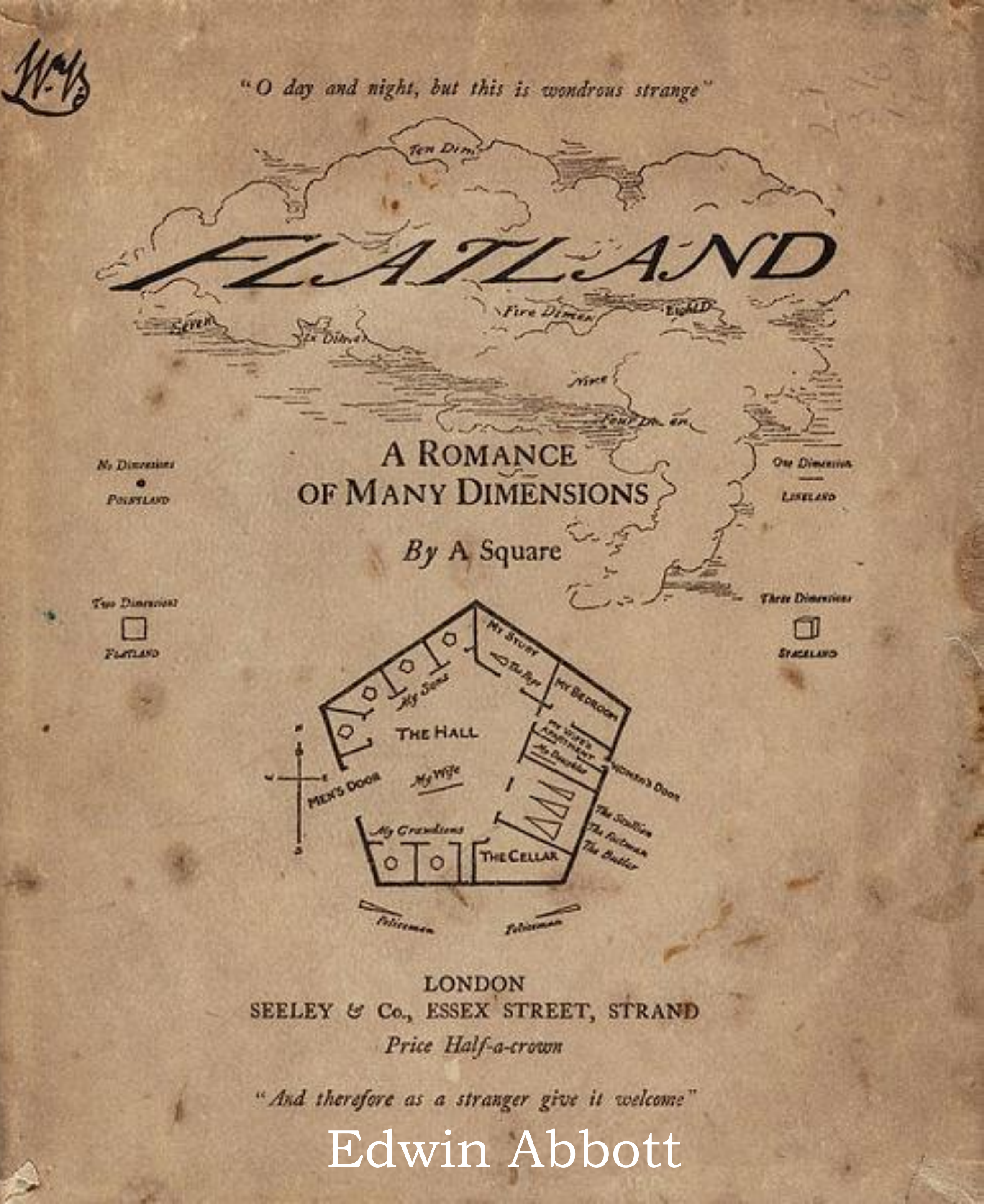
Overview

- A summary of the guiding principles of RME
- A blind spot in RME?
- An online game to introduce early algebra in primary school
- A new project aimed at making the primary school mathematics curriculum more mathematical

Netherlands Initiative for Education Research

Development of domain-specific higher-order skills

- ✗ Proposal submitted in 2012
- ✓ Proposal submitted in 2014



Beyond

FLATLAND

in primary school mathematics education

Beyond

FLATLAND

in primary school mathematics education

dynamic data modeling

probability

early algebra

Netherlands Initiative for Education Research

Utrecht University

IPN

Senior staff

UU	IPN
Marja van den Heuvel	Aiso Heinze
Paul Leseman	Anke Lindmeier
Jan Boom	
Michiel Doorman	
Michiel Veldhuis	

PhD students

Carolien Duijzer

Roos Blankespoor

Mara Otten

Realistic Mathematics Education

- activity principle
- reality principle
- level principle
  * various levels of understanding
  * progressive schematization
  * models as bridges
- intertwinement principle
- interactivity principle
- guidance principle

Theoretically enhanced by

- **Embodiment theory**
- **Representational re-description theory**
- **Variation theory**

- Our sensori-motor system has an important role in developing conceptual understanding
- The same neural substrate used in imagining is used in understanding

(Gallese & Lakoff, 2005)

- Embodiment theory

- Representational re-description theory

- Variation theory

- Embodiment theory

The RR theory describes the development of representations, which can bring students to higher levels of thinking.
The initial implicit, embodied knowledge, is in a next step re-described in verbal or other types of symbolic representations and, as such, becomes available for explicit verbal-symbolic reasoning and explicit hypothesis-led experimentation.

(Karmiloff-Smith, 1992)

- Representational re-description theory

- Variation theory

- Embodiment theory

- Representational re-description theory

- **Variation theory**

A necessary condition for learning is the possibility to experience variation and distinguish between what changes and what remains invariant.

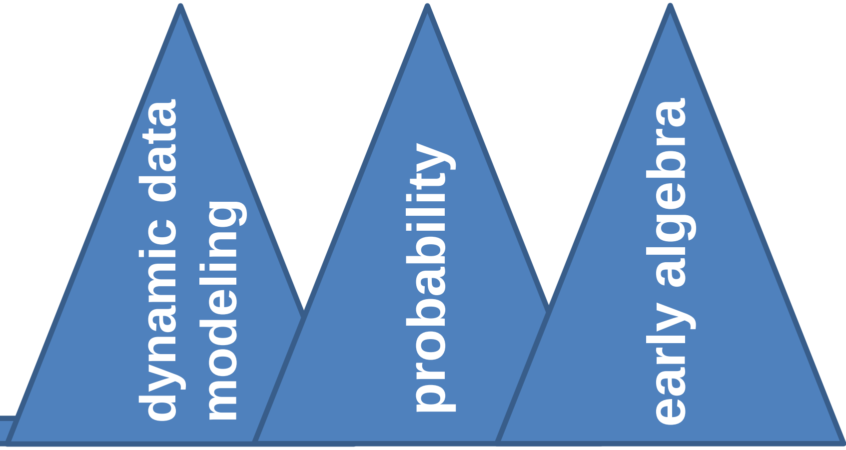
(Marton & Booth, 1997; Marton & Pang, 2013)

Being able to discover structure and to identify patterns is considered the essence of mathematics

(Watson & Mason, 2006)

Therefore, variation theory is considered a powerful design principle for mathematics education

(e.g. Sun, 2011; Li, Peng & Song, 2011)



Aim Flatland project
Investigating *whether* and *in what ways* these content domains do have potential to foster **H**igher-**O**rder **T**hinking skills in primary school students

Research questions

1. Which mathematical HOT skills emerge in primary school students in solving problems on dynamic data modeling / probability / early algebra?

2. To what degree can theory-based learning facilitators (variation in tasks, opportunities for embodiment, and hints for representational re-description) contribute to the (further) development of these HOT skills?

3. What constitutes a teaching sequence for developing HOT skills in primary school mathematics education?

Year 1 (Sept 2015-Aug 2016): Pilot phase (design of tasks and try-out)

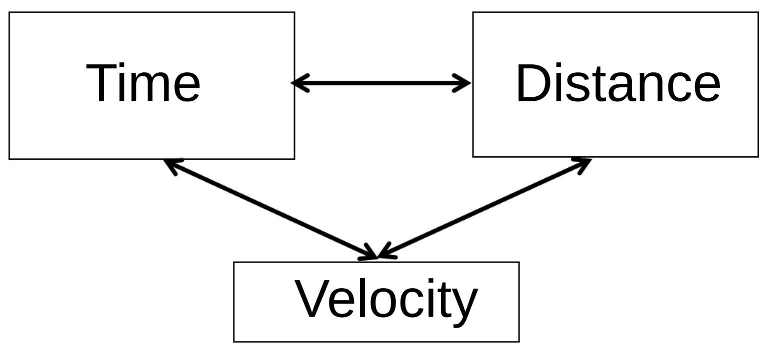
Year 2 (Sept 2016-Aug 2017): Main experiments

- *Staged comparison design*
- *Micro-genetic and macro-genetic analyses*

Condition	classes/ students	Oct-Nov ' 16		Jan-Feb '17		Apr-May '17		Jun '17
	1/ 25	M1	LESSON 1-6 m1-6	M2		M3		M4
A/B	1/ 25	M1		M2	LESSON 1-6 m1-6	M3		M4
	1/ 25	M1		M2		M3	LESSON 1-6 m1-6	M4

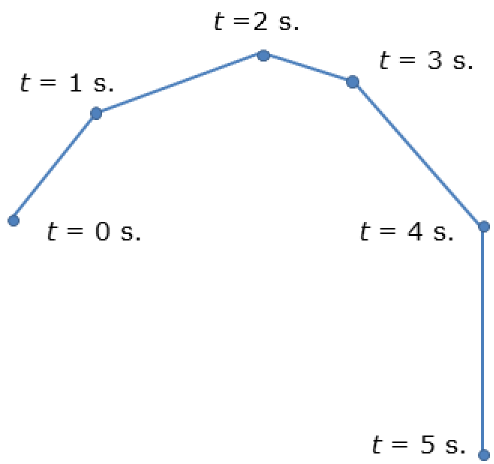
Part-project 1: HOT in dynamic data modeling

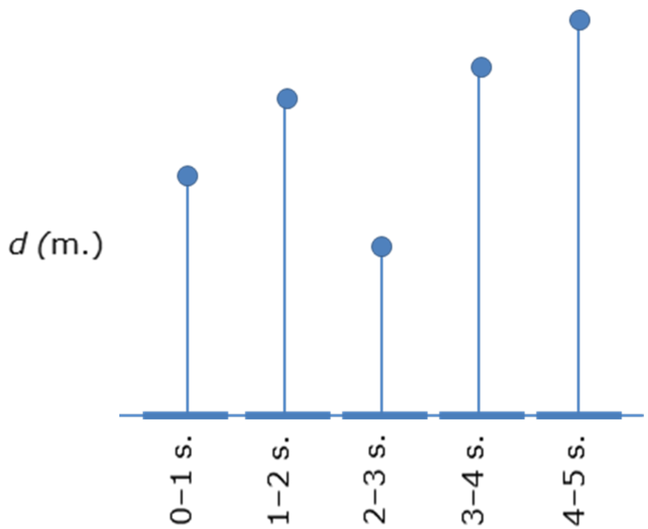
The HOT skills aimed at in this project include representing dynamic data related to motion, reflecting on these representations, refining them and using them for reasoning, hypothesizing and testing predictions about

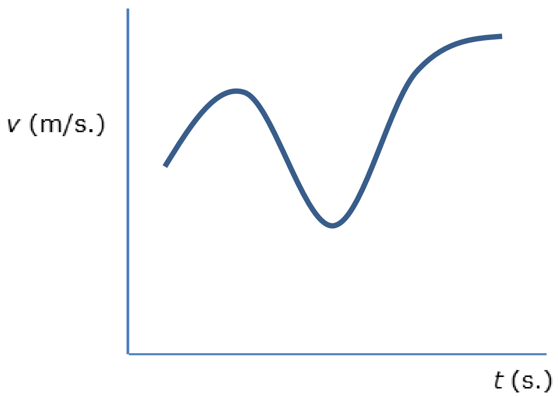


- DiSessa et al. (1991)
- Nemirovsky et al. (1998)
- Radford (2009)
- Van Galen et al. (2012)

Changing speed of a moving object can be described at different levels of understanding:







by tracing the traveled distance in a geographical map

by an interval graph

by a conventional time-velocity graph

Learning facilitators

General: Task variation

Variation in
- *motion* (with pauses, with sudden stop)
- *visual representation*
- *perspective* (equal time segments → equal distance segments)

Condition A: → Hints for representational re-description

Making the implicit knowledge about the relation between time-distance-speed more explicit

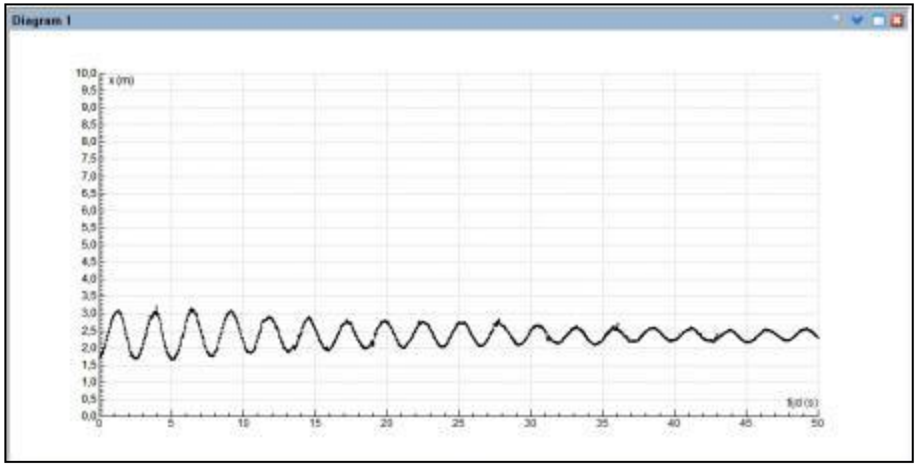
Condition B: → A + Opportunities for embodiment

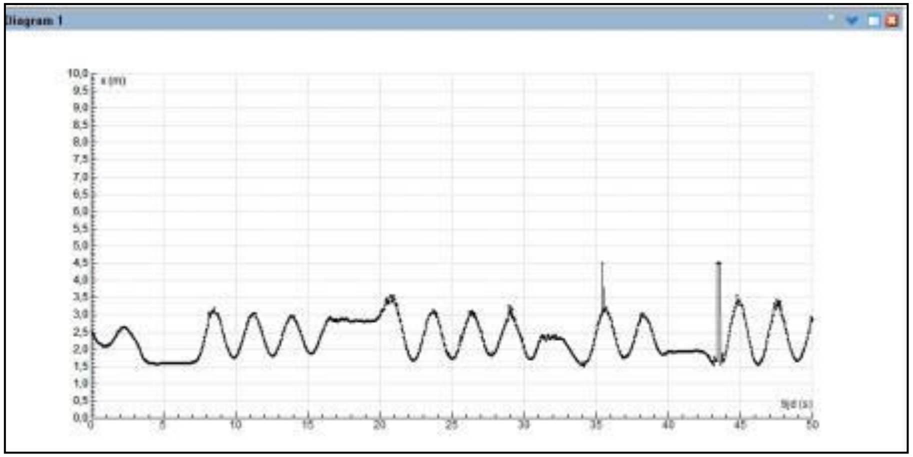
Using student-operated sensors that generate graphical representations

Design of tasks

(1) Exploring swing movements

- The rucksack

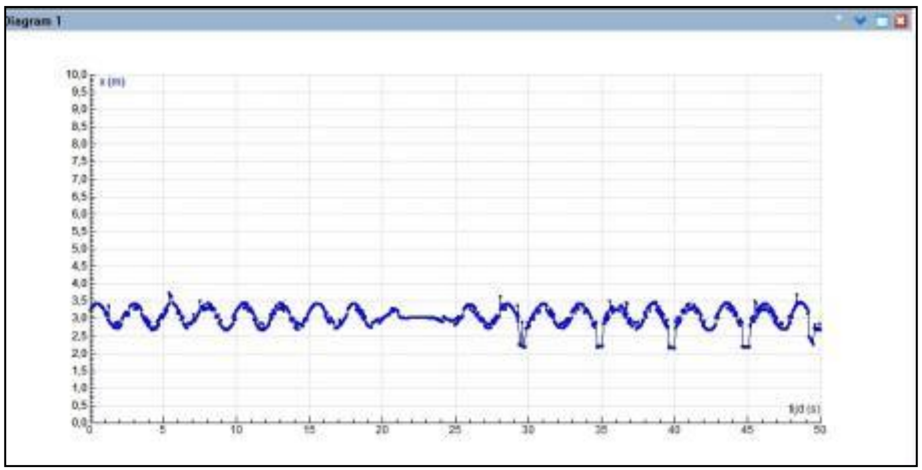




Design of tasks

(1) Exploring swing movements

- The rucksack
- Carolien in the swing

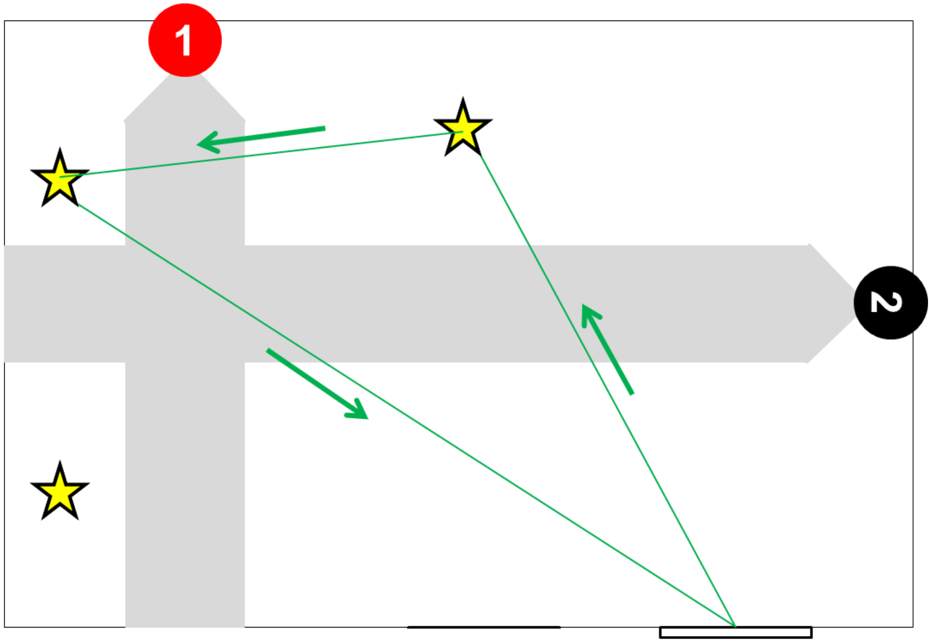


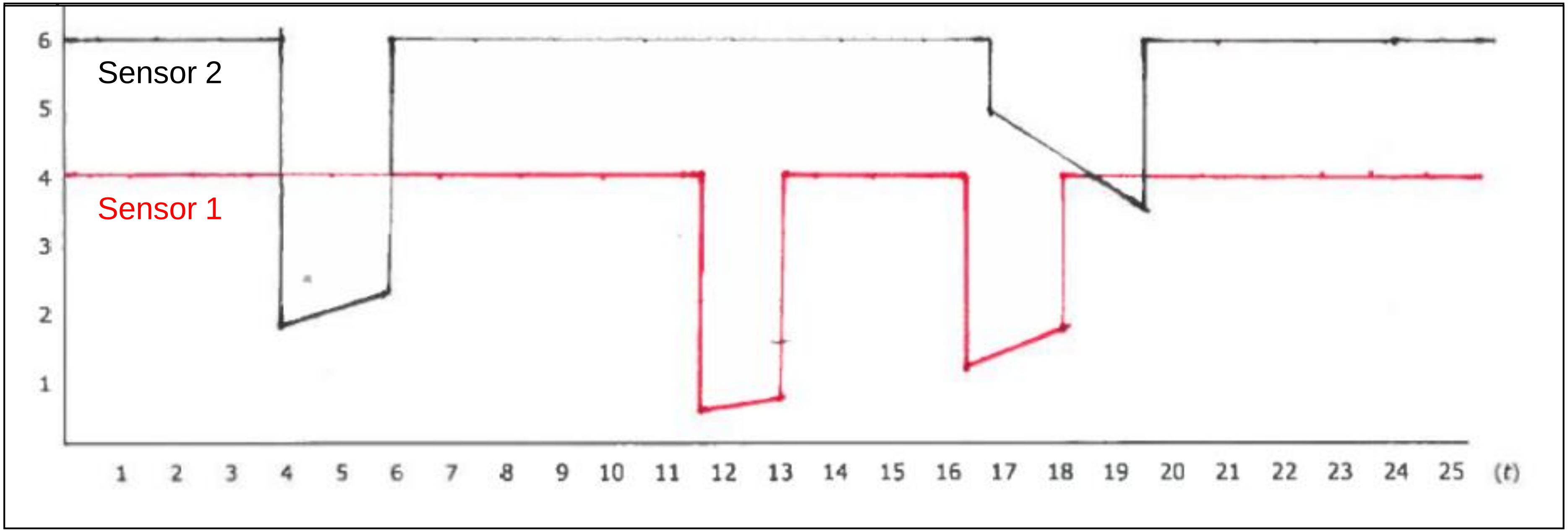
What happened in the second phase?

Design of tasks

(1) Exploring swing movements
- The rucksack
- Carolien in the swing

(2) Tracing the intruders' movement





Part-project 2: HOT in dealing with probability in primary school

The HOT skills aimed at in this project include having a qualitative elementary understanding of the probability of events, using **sample space** as a basis for predicting outcomes of probabilistic events, and being able to reflect on and explain these predictions

> “[T]he first and essential step in solving any probability problem is to work out all the possible events and sequences of events that could happen [...]
> and working out the sample space is not just a necessary part of the calculation of the probabilities of particular event, but also an essential element in understanding the nature of probability.”
>
> *(Bryant & Nunes, 2012, p. 3)*

Flip the two coins.
What is the chance you will have a tail?

sample space

Flip the two coins.
What is the chance you will have a tail?

sample space

Learning facilitators

General: Task variation

Variation in
- *context* (coins, dice, spinners) with same sample space
- *sample space* within same context

Condition A: → Opportunities for embodiment
+ Hints for representational re-description

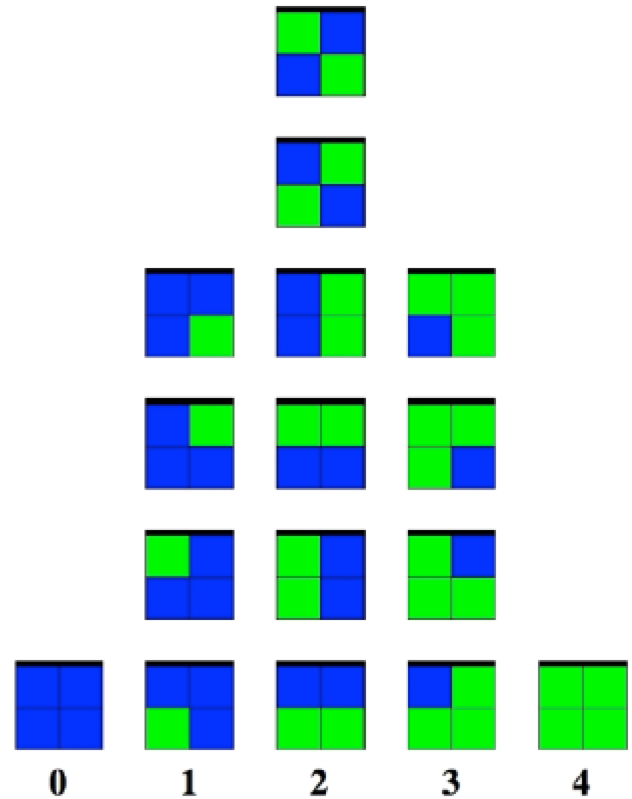
Giving opportunities for carrying out probabilistic events physically and giving hints to focus students' attention to all possible outcomes

Condition B: → A + Perceptual approach

A marble-scooper random generator is used that is considered to function as an epistemic resource (Abrahamson, 2014)

Abrahamson, D. (2014). Rethinking probability education: Perceptual judgment as epistemic resource. In E.J. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking: presenting plural perspectives* (pp. 239-260). New York: Springer.

four-marbles-scooper



16 possible outcomes
5 categories of outcomes

Part-project 3: HOT in solving early algebra problems

The HOT skills aimed at in this project include comparing combinations of quantities, revealing the structure of these combinations in order to create new equivalent or non-equivalent combinations of quantities, using **isolating and/or substituting strategies** to identify the values of unknowns, and developing context-connected representations, eventually evolving into more abstract notations

- Non-symbolic or pre-symbolic approach to algebraic thinking in the primary grades (Kieran, 2004)
- Algebraic thinking is searching for generalizations (Caspi & Sfard, 2012)
- Since the ability to generalize requires distinguishing between what changes and what remains invariant in particular instances, experiencing variation is considered significant for the learning of algebra (Al-Murani, 2006).

Learning facilitators

General: Task variation

Variation in

- *the combination of quantities* which students have to compare
- *tasks affordances* (prompts for an isolation and/or a substitution strategy)

Condition A: → Opportunities for ICT-based embodiment
+ Hints for representational re-description

Giving opportunities to experience embodiment through dynamic interactive applets

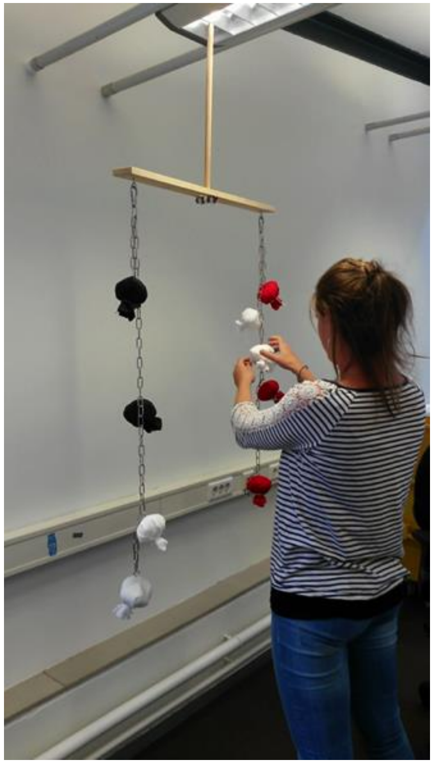
physical embodiment

Condition B: → Opportunities for "surrogate" embodiment
+ Hints for representational re-description

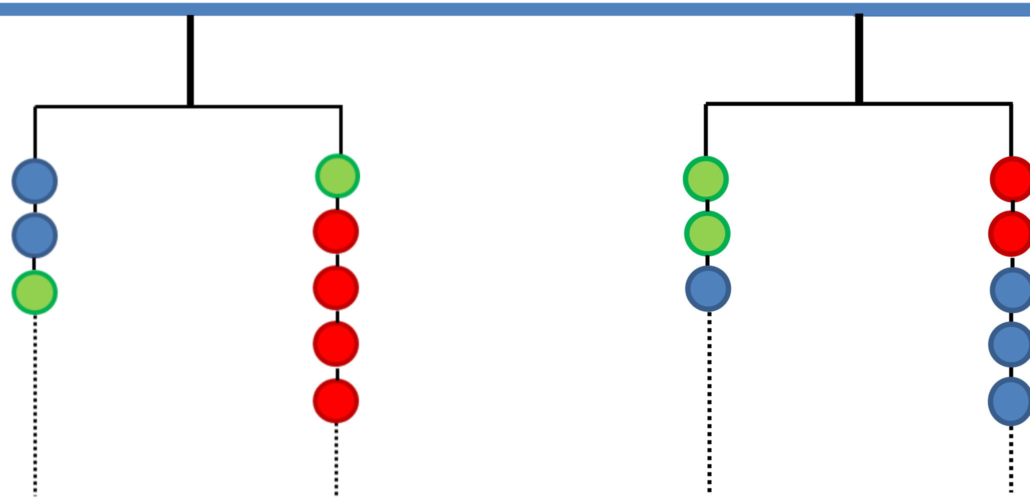
Providing students with "learning movies" in which they can observe problem solving of knowledgeable others

Design of tasks

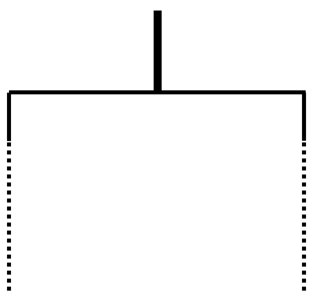
- Keep the hanging mobile in balance

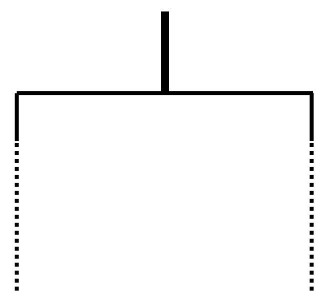


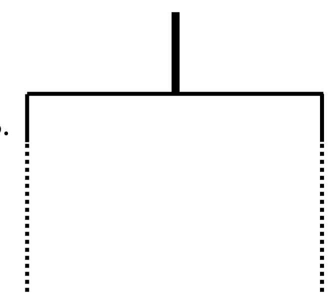
Can you find out how this thing works?



Take a look at the above pictures
and draw the correct amount of balls in the pictures below

1.


2.


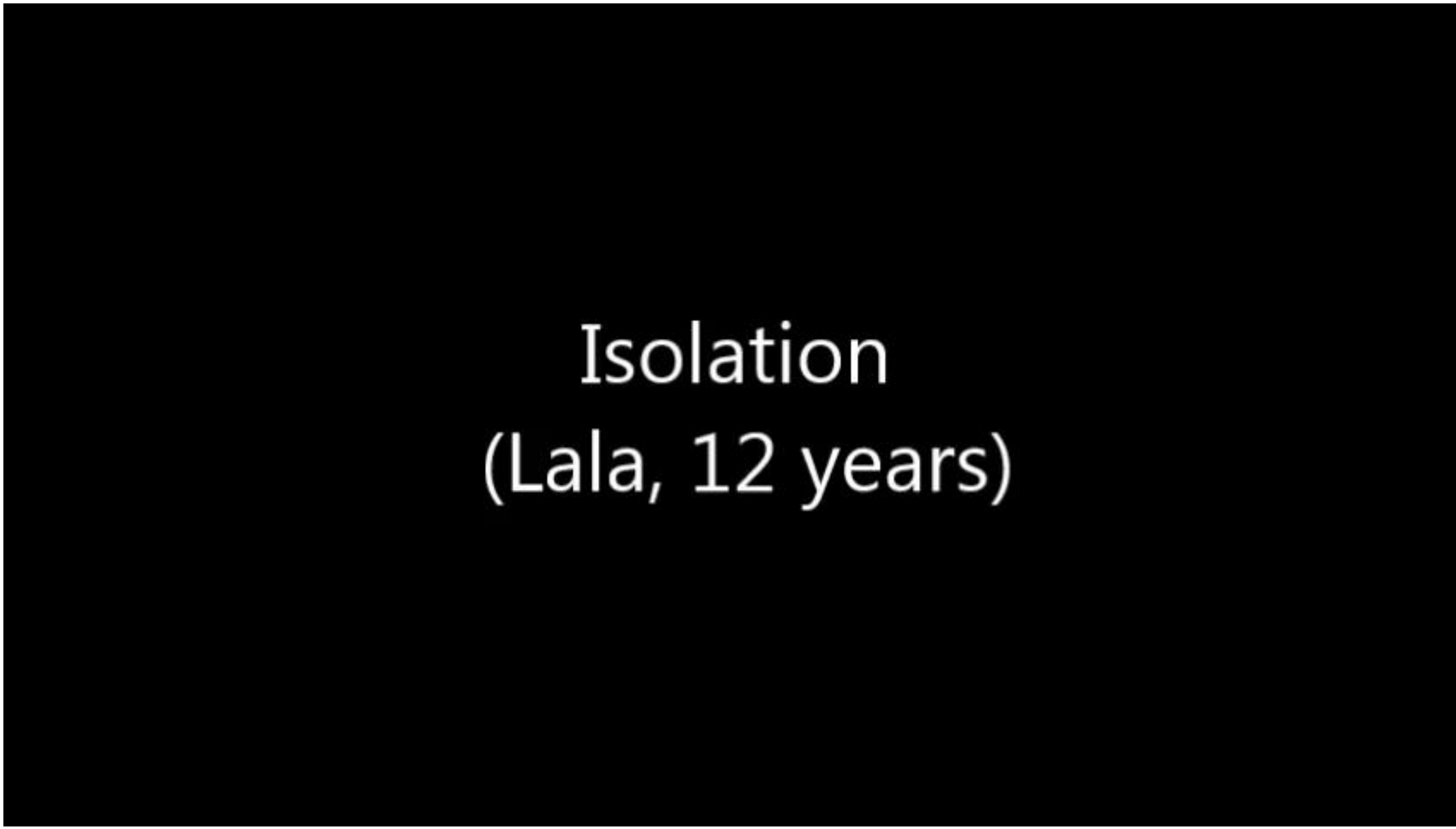
3.


How many ? How many ? How many ? How many ? How many ? How many ?

Now do the same thing with as little balls as possible

Example of student work:

1. solving by isolation
2. solving by isolation and substitution



Will be continued…

m.vandenheuvel@fi.uu.nl
m.vandenheuvel-panhuizen@uu.nl