

Developing 3rd Grade Children's 3D Visualization and Numeration Skills using Intuitively Accessible Models

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Realistic Mathematics Education (RME)

Mathematics is a human activity

Opportunities for students to "re-invent" mathematics through active participation in the learning process

Horizontal mathematization: Students come up with the mathematical tools to organize and solve problems located in real-life situations

Vertical mathematization: Finding shortcuts, discovering connections and applying these to new situations in increasingly more abstract ways

A diagram illustrating 3D visualization. It shows a series of small, colorful rectangular blocks arranged in a staircase pattern that ascends from the bottom left towards the top right. The blocks are connected by thin lines, suggesting a sequence or a process of building up a structure.

3D Visualization

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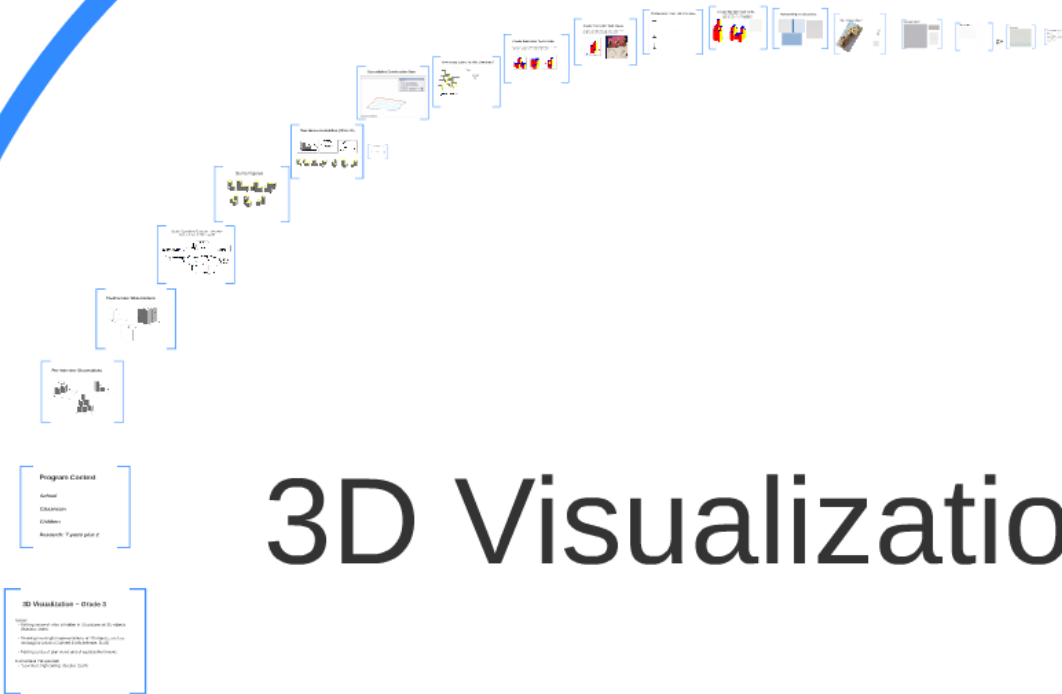
Mathematics is a human activity

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Horizontal mathematization: Students come up with the mathematical tools to organize and solve problems located in real-life situations

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3D Visualization



3D Visualization – Grade 3

Issues:

- Making sense of what is hidden in 2D pictures of 3D objects (Battista, 1999)
- Drawing meaningful representations of 3D objects, such as rectangular prisms (Outhred & Mitchelmore, 2002)
- Making sense of plan views and of top/side/front views

Instructional Perspective:

- “Low floor; high ceiling” (Boaler, 2014)

Program Context

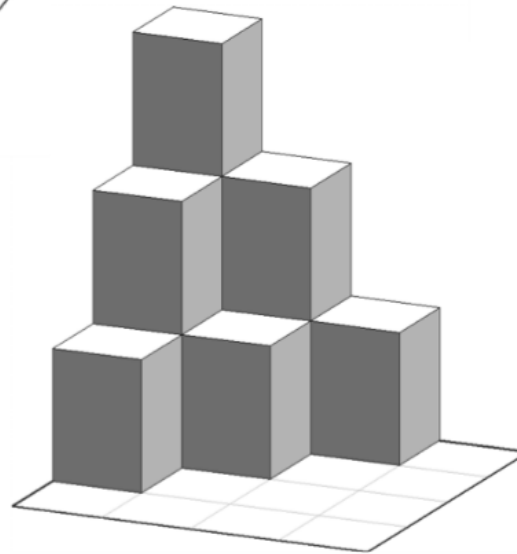
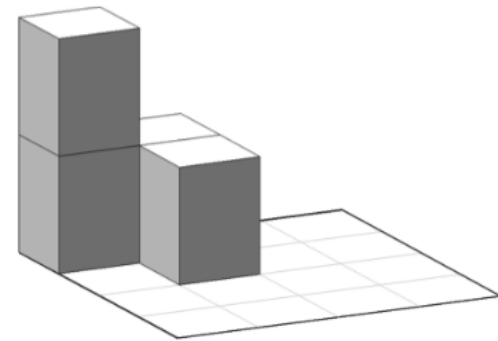
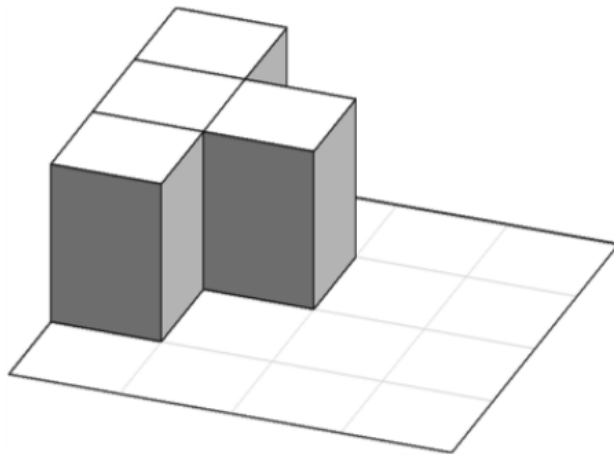
School

Classroom

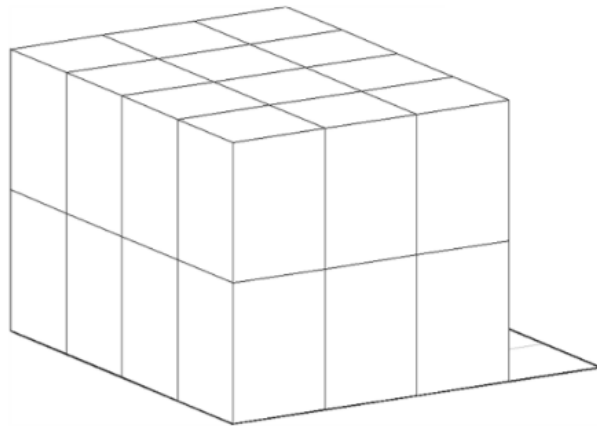
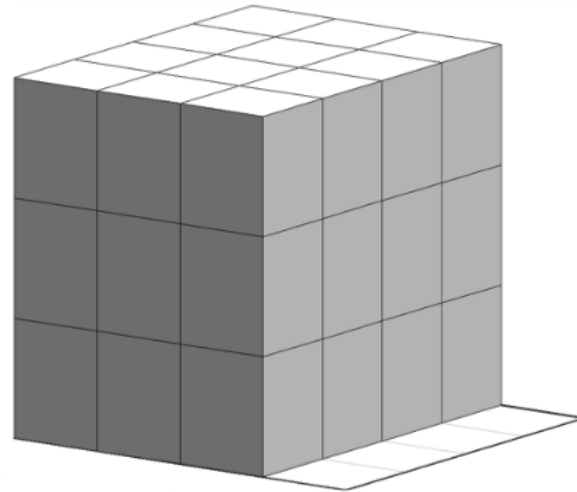
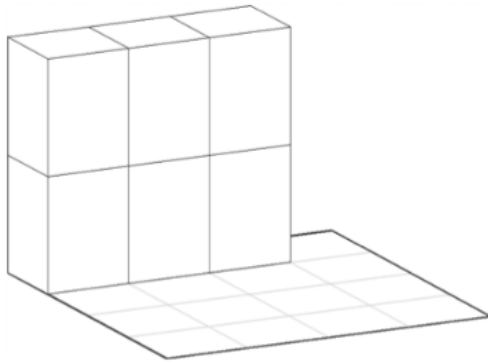
Children

Research: 7 years plus 2

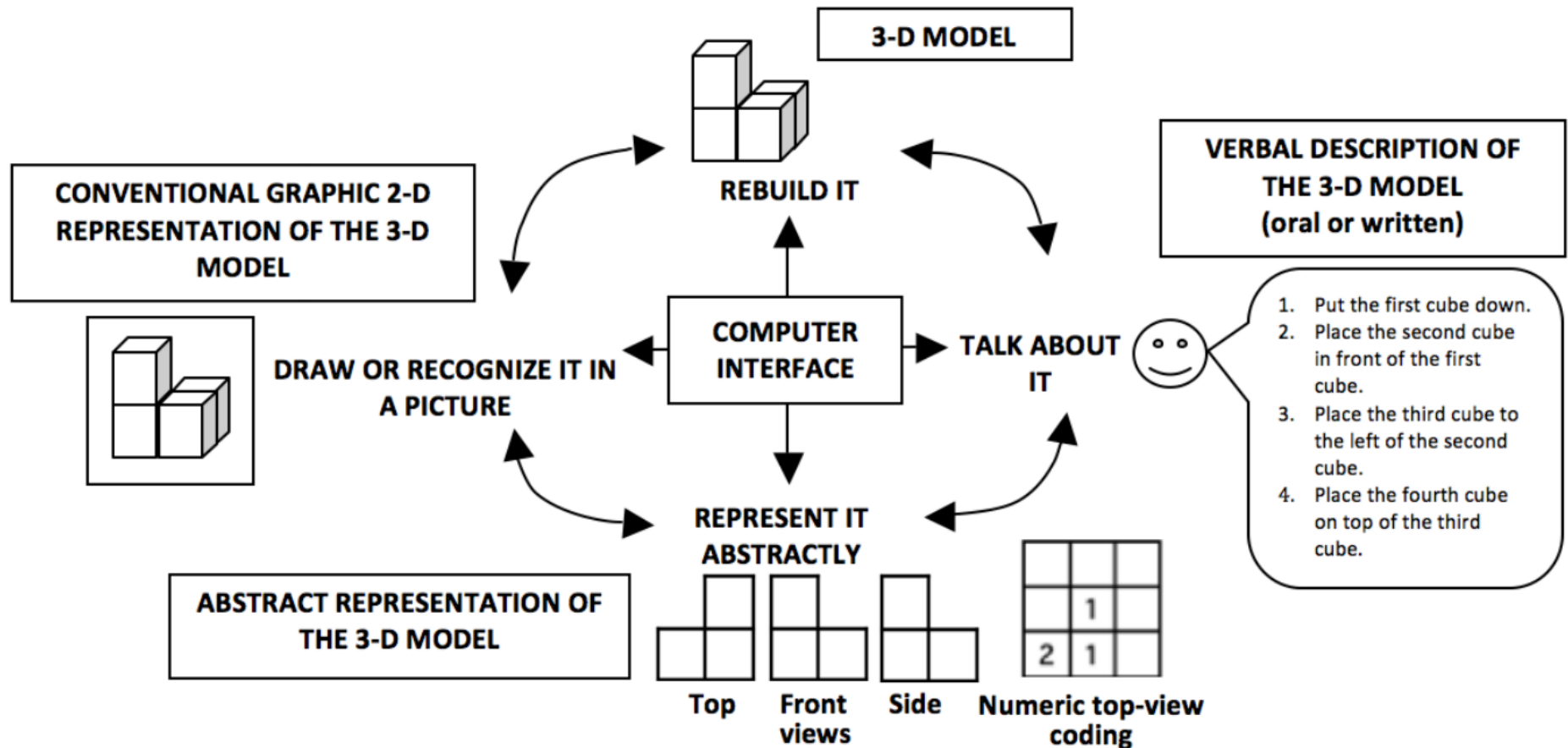
Pre-Interview Observations



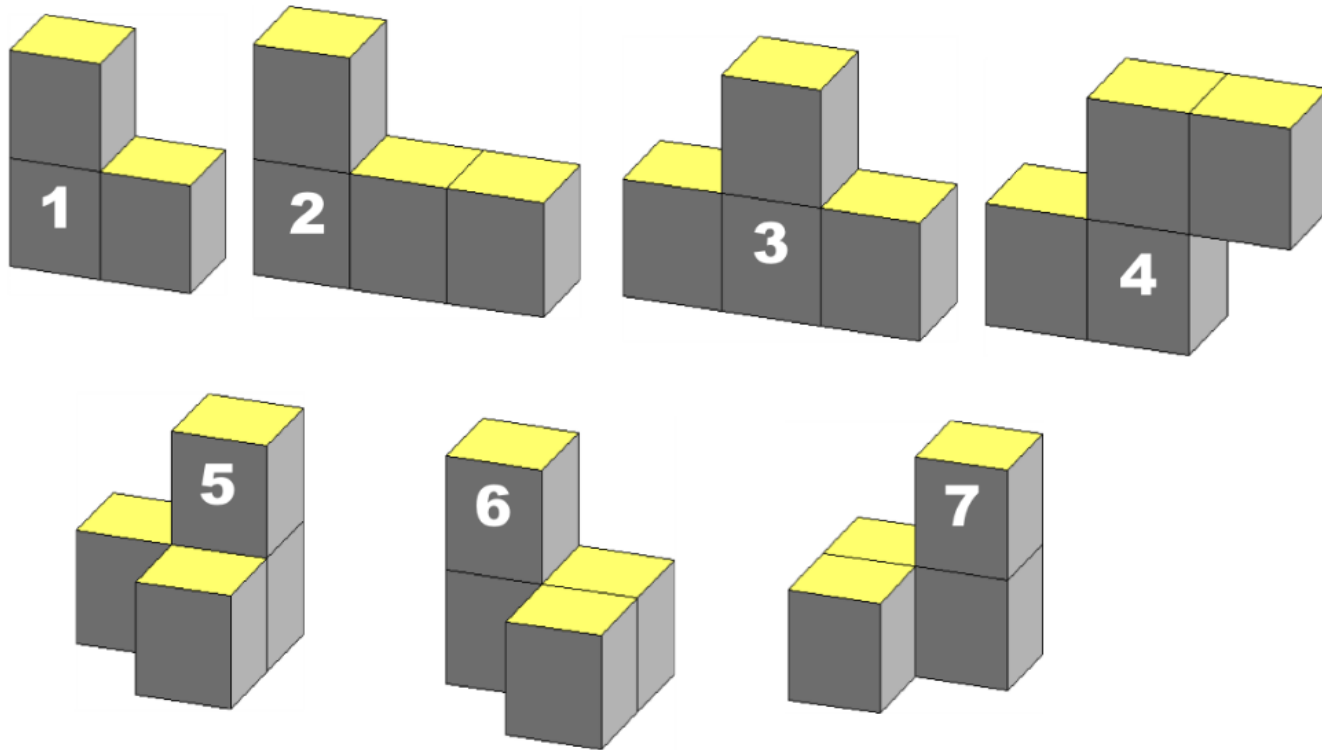
Pre-Interview Observations



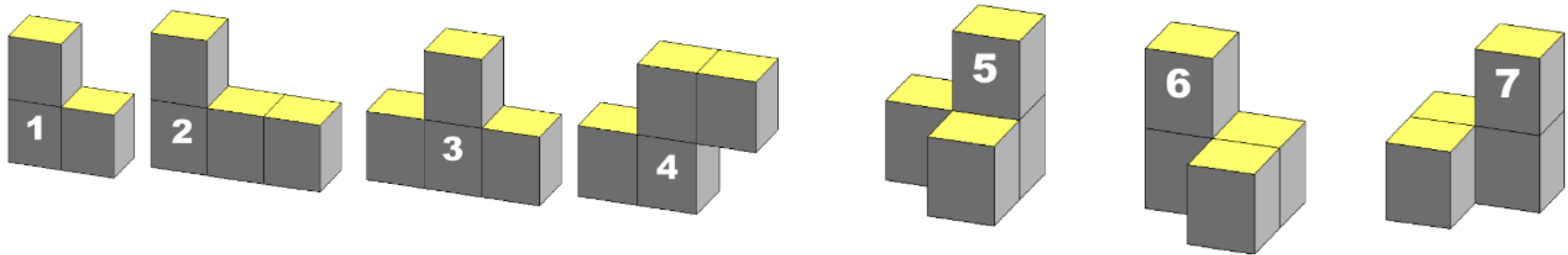
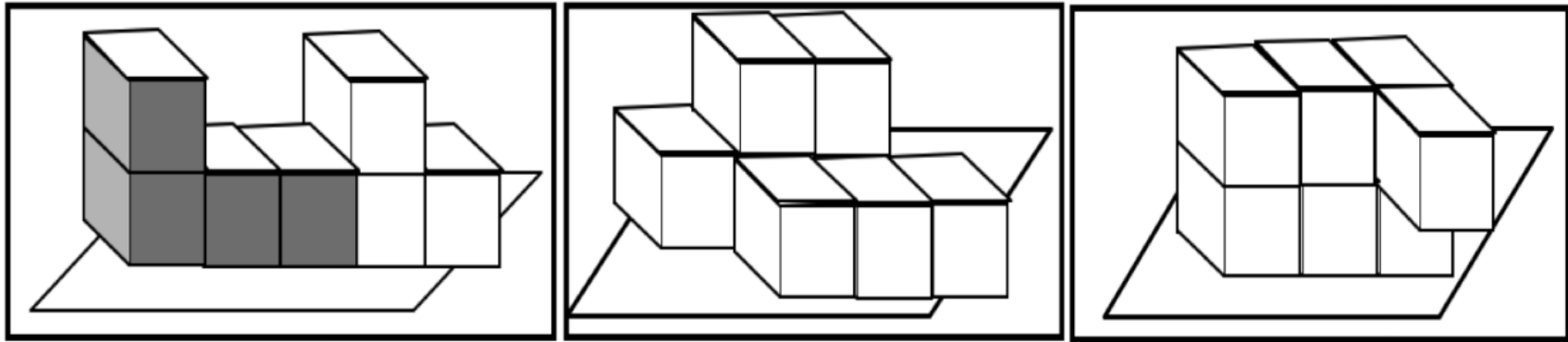
Spatial Operational Capacity Framework (Sack, & Van Niekerk, 2009)



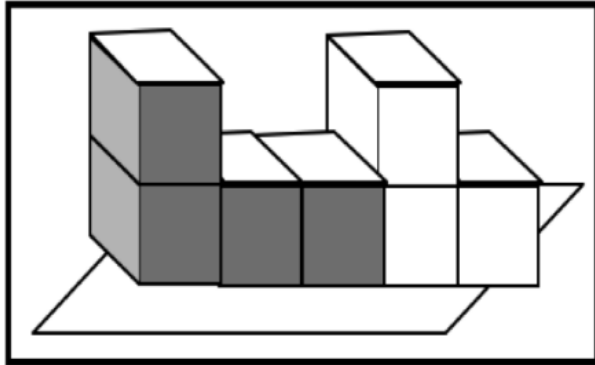
Soma Figures



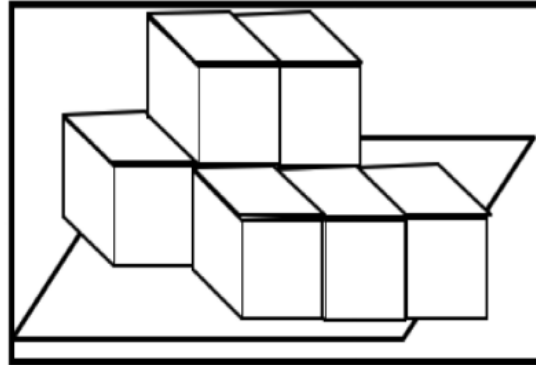
Two-Soma Assemblies (2D to 3D)



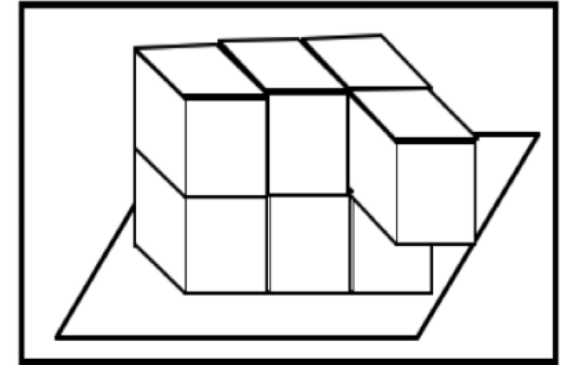
Two-Soma Assemblies (2D to 3D)



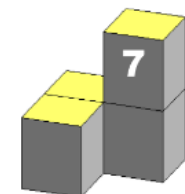
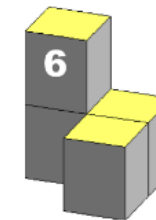
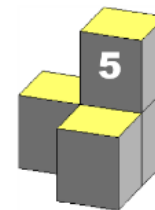
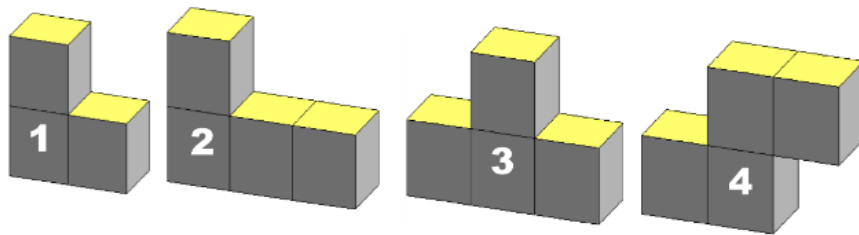
Somas #2; #1



Somas #3; #4 OR
Somas #5; #7



Somas #1; #2 OR
Somas #2; #3 OR
Somas #5; #1 OR
Somas #6; #1 OR ...



Geocadabra Construction Box

Construction box

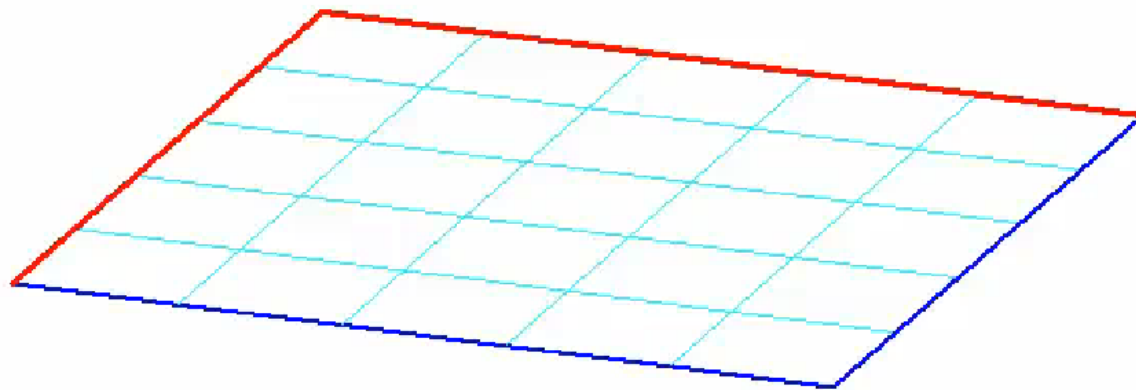
Top View

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

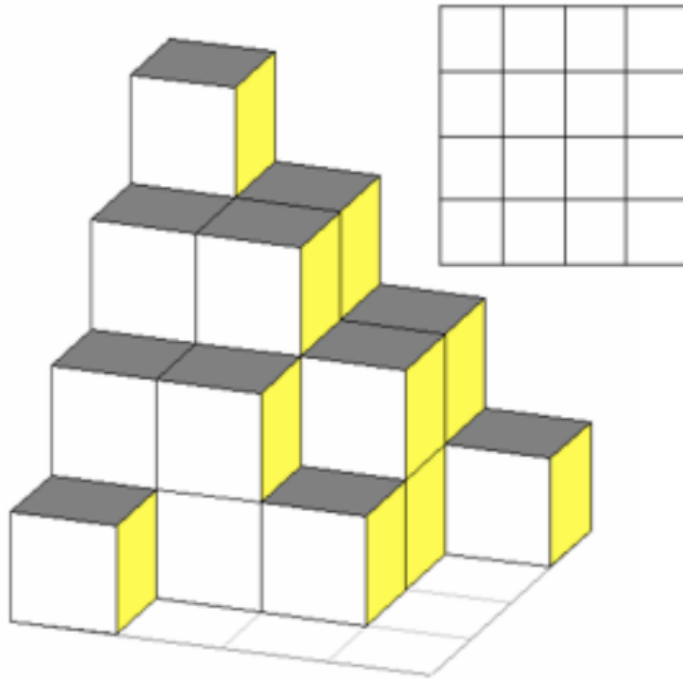
- Show Spatial Model
- Show Buttons Top View
- Fit on Screen
- Show also Vertical Grid Lines

Control View Line Stop

Control Perspective vvv



How many cubes in this structure?



Videos:
GEMSG (Mar 10 2009)
KIMMGS4 (Nov 8 2010)
M_SPDGS (Nov 11 2010) start at 2'00"

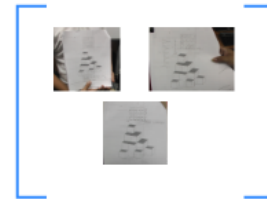


Figure 4. Task 1f

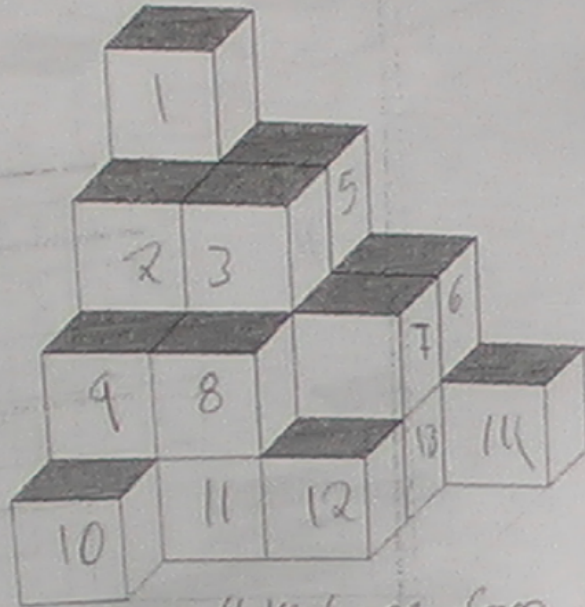
Videos:

GB3MS2 (Mar 10 2009)

KM3MES4 (Nov 8 2010)

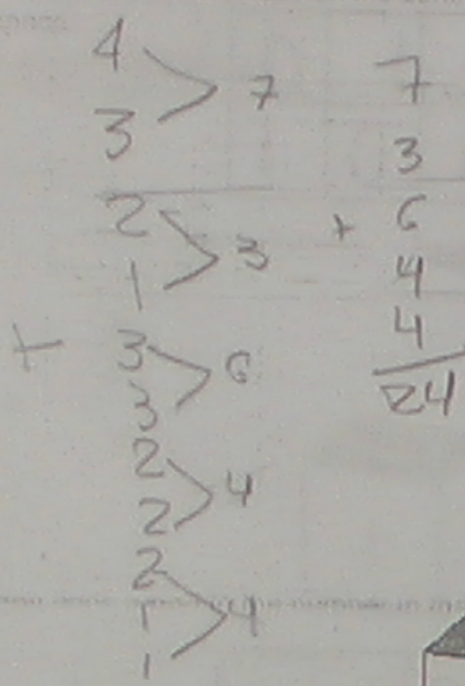
M_3FSE5 (Nov 11 2010) start at 2'00"

4	3	2	1
3	3	2	0
2	2	1	0
1	0	0	0



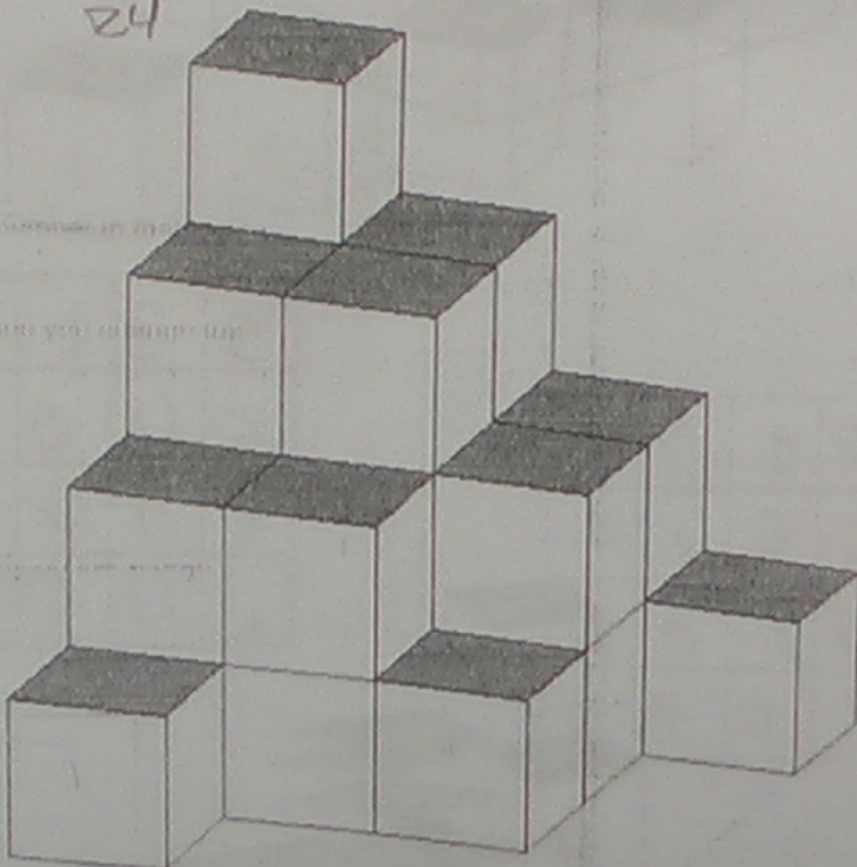
25 blocks because from the outside its 14, and in the back are more

9.



4	3	2	1
3	3	2	0
2	2	1	0
1	0	0	0

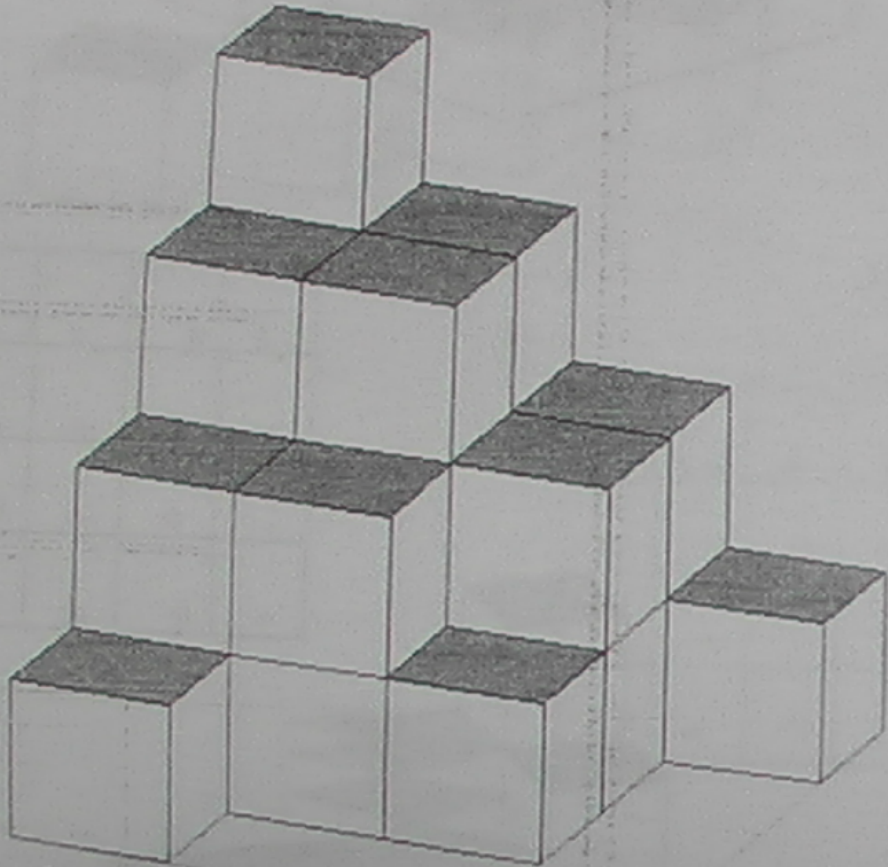
24



g.

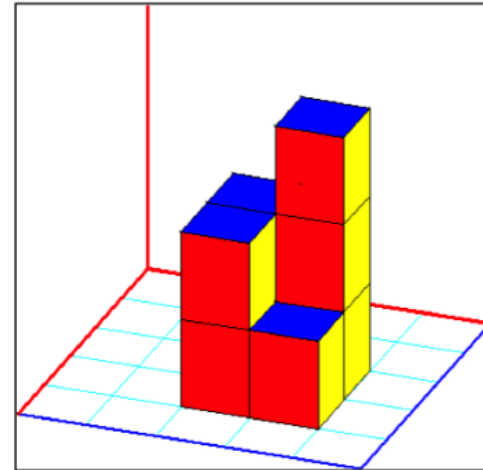
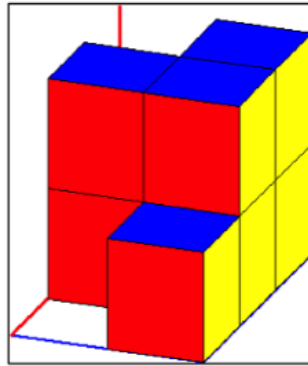
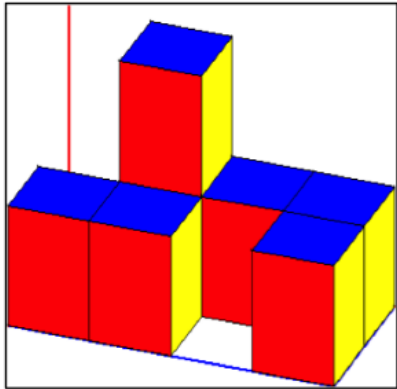
	4	3	2	1	=10
+	3	3	2	0	=8
+	2	2	1	0	=5
+	1	0	0	0	=1
					=24

24 blocks in total
 24 blocks in total figures



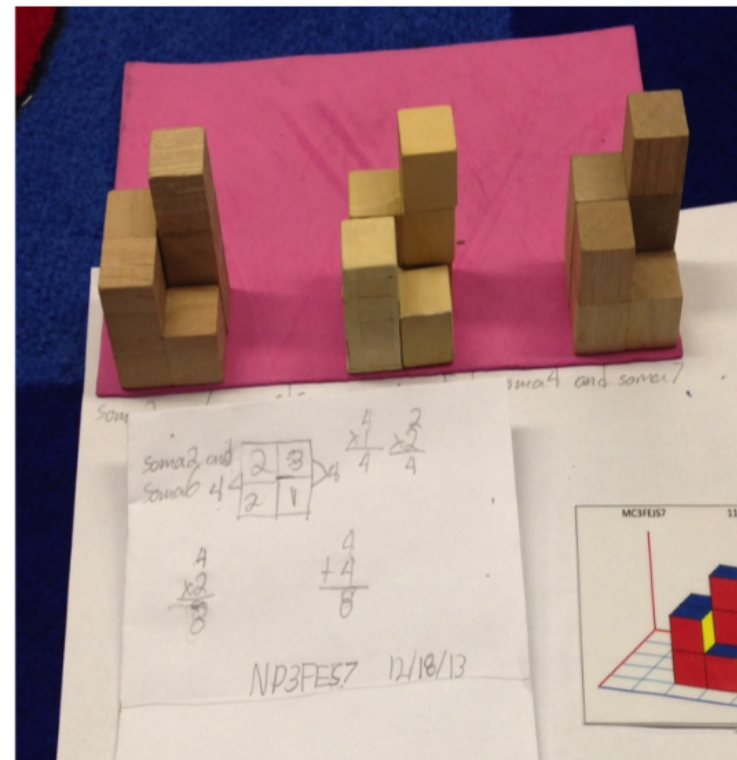
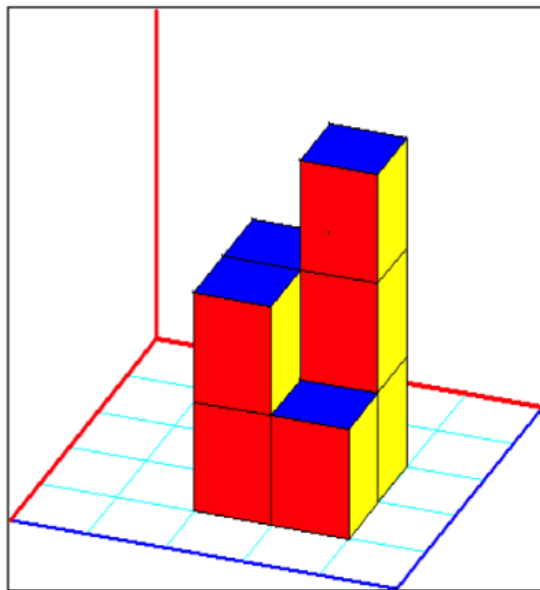
Create Your Own Task Cards

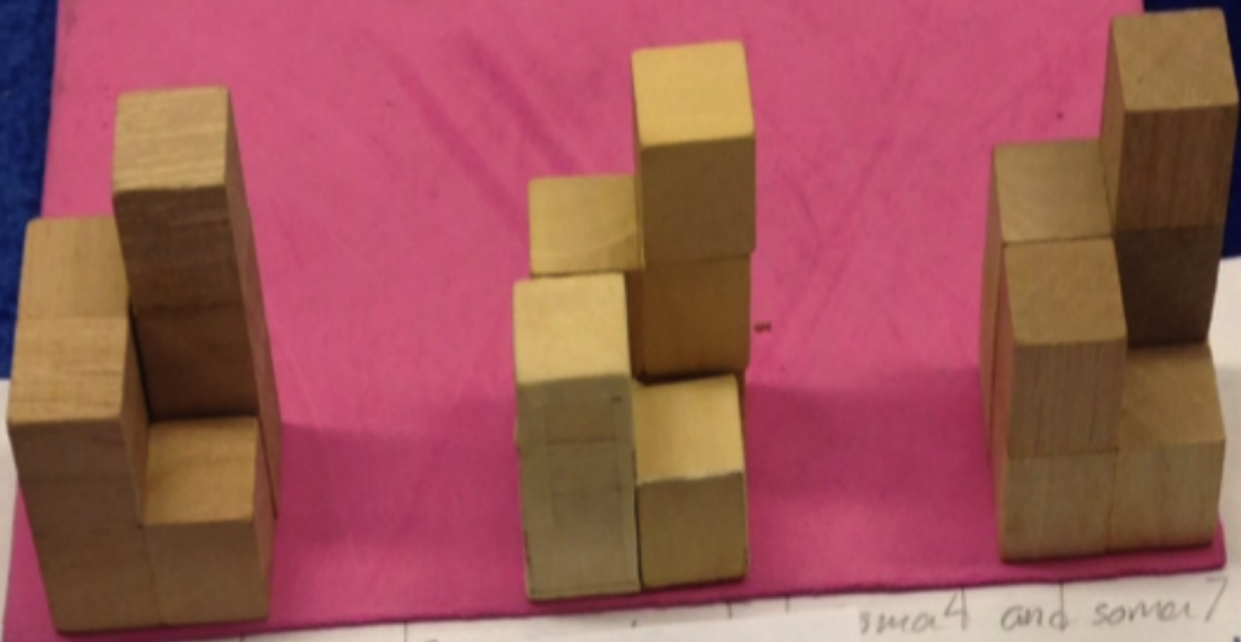
Two-Soma assemblies (Challenge your friends to see if they can figure out which two Soma figures you used)



Create Your Own Task Cards

Two-Soma assemblies (Challenge your friends to see if they can figure out which two Soma figures you used)





Soma

Soma 2 and
Soma 6 4 4

2	3
2	1

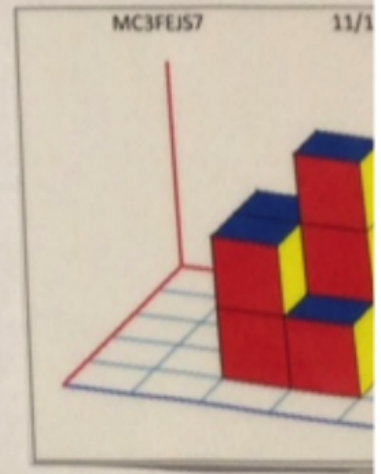
$$\begin{array}{r} 4 \\ \times 1 \\ \hline 4 \end{array} \quad \begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 4 \\ \times 2 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 4 \\ + 4 \\ \hline 8 \end{array}$$

soma 4 and soma 7

NP3FE57 12/18/13



Making sense of top, side front views

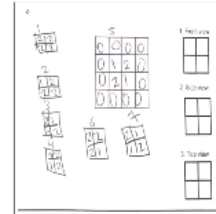
1. Front view



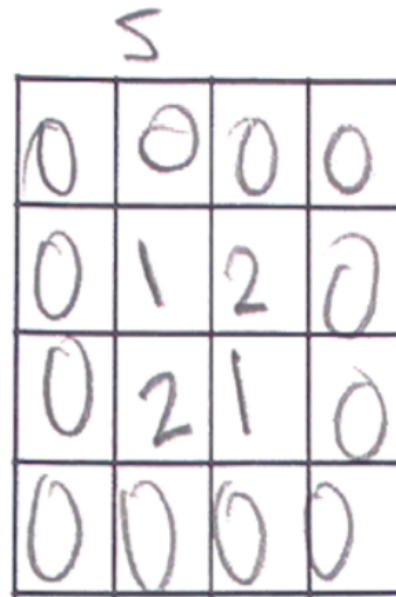
2. Side view



3. Top view



0.



1. Front view



2. Side view

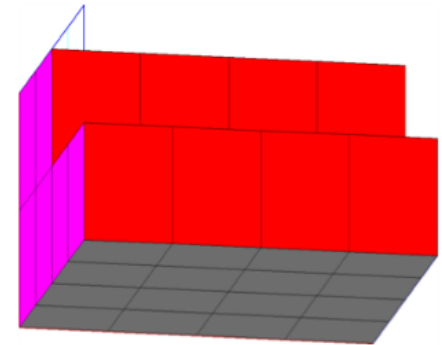
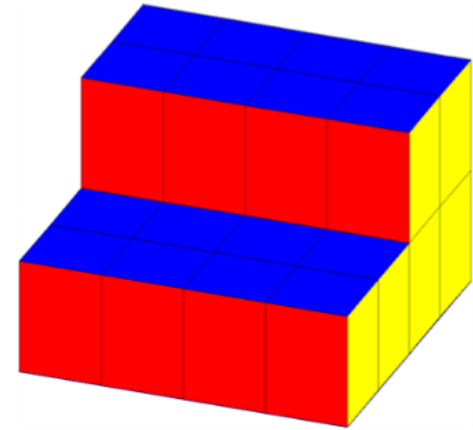
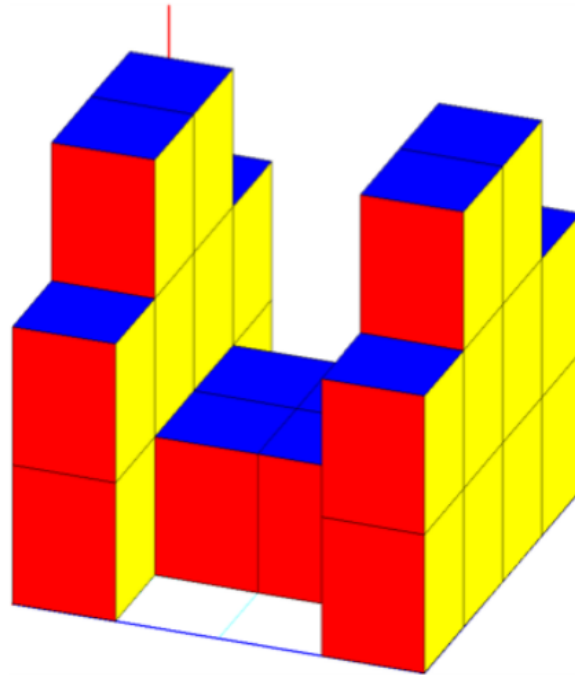
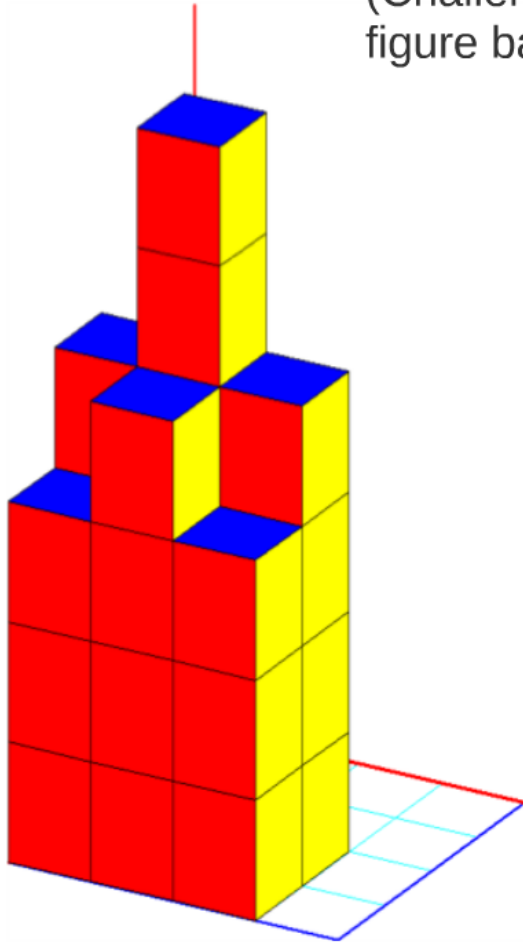


3. Top view

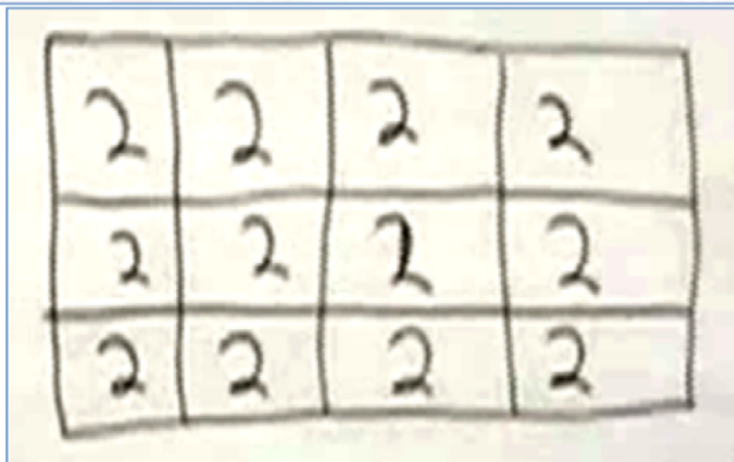
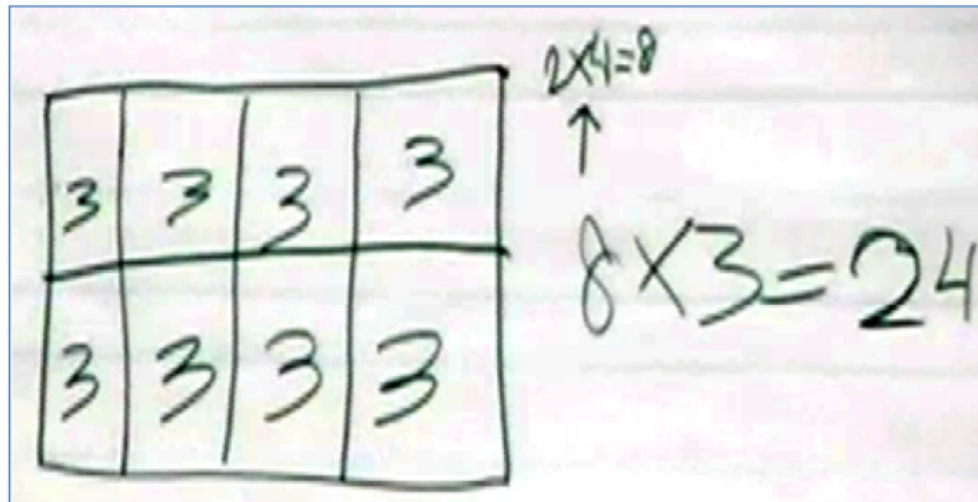


Create Your Own Task Cards

Create a symmetrical figure with 24 cubes
(Challenge your friends to see if they can build your figure based only on reading your numeric plan view)



Representing 24-cube prisms



$$\begin{array}{r} 27 \\ \times 4 \\ \hline 108 \end{array}$$

1

$$\begin{array}{r} \overset{1}{\text{un cubo}} \\ 27 \\ + 27 \\ \hline 54 \end{array}$$

1

$$\begin{array}{r} 54 \text{ doble} \\ + 54 \\ \hline 108 \end{array}$$

total de cubos

2

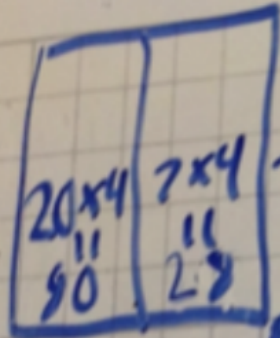
$$2 \times 27 = 54$$

$$54 + 54 = 108$$

$$3 \times 54 = 81$$

$$81 + 27 = 108$$

$$\begin{array}{r} 80 \\ + 28 \\ \hline 108 \end{array}$$

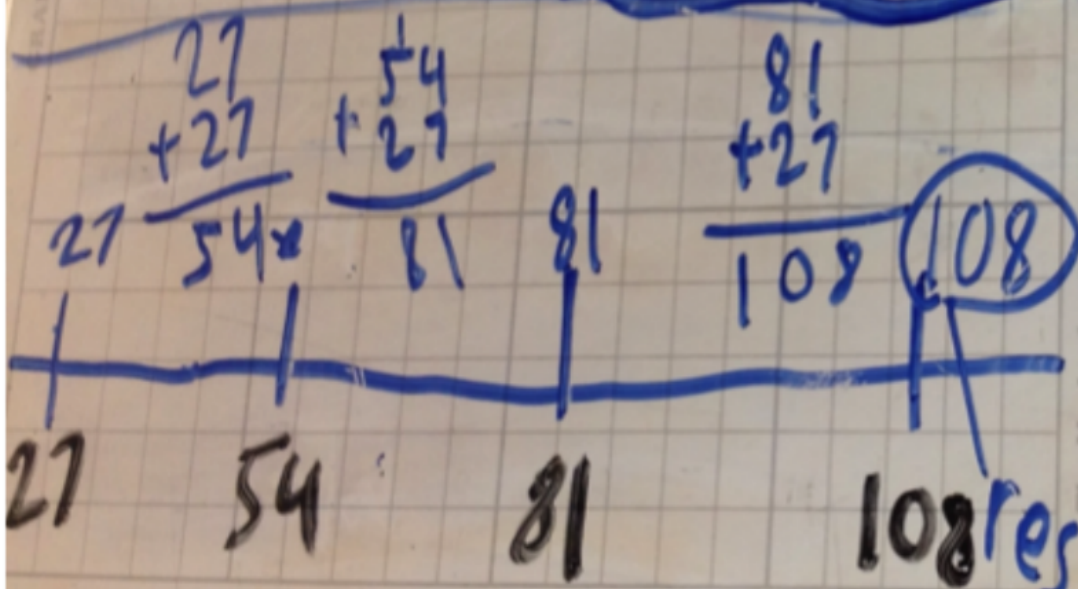


area 27

ase es 4 ves

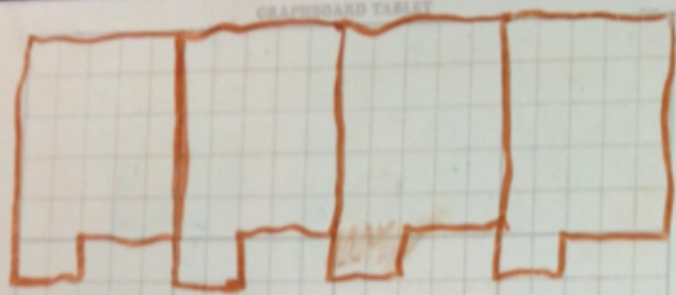
cuts res p u g m

108

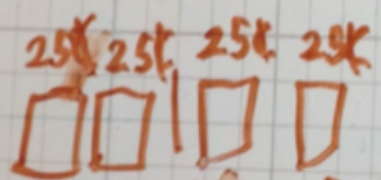


108 res

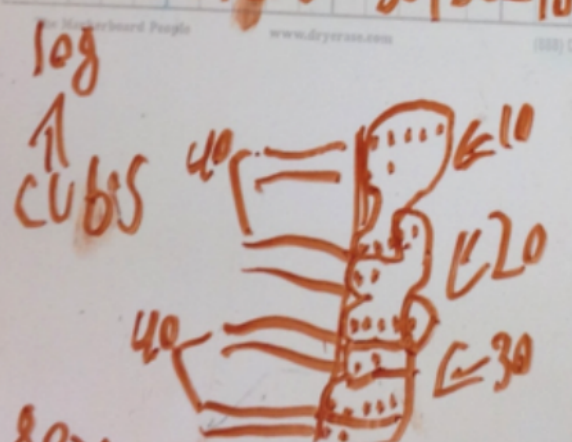
$$\begin{array}{r} \times 27 \\ \quad 4 \\ \hline 280 \\ + 80 \\ \hline 108 \end{array}$$



$$\begin{array}{r}
 27 \\
 \times 4 \\
 \hline
 108
 \end{array}
 \quad
 \begin{array}{r}
 25 \\
 \times 4 \\
 \hline
 100
 \end{array}
 + \frac{2}{8} = 100\frac{2}{8}$$

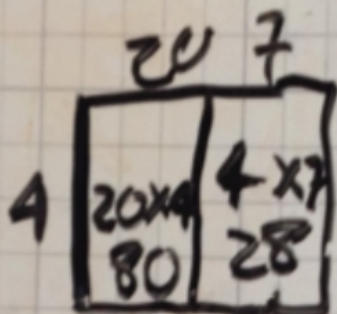


$$25 + 25 = 50 + 50 = 100 + \frac{2}{8} =$$



$$\begin{array}{r}
 80 \\
 + 20 \\
 \hline
 100
 \end{array}
 + \frac{2}{8} = 100\frac{2}{8}
 \quad
 10 + 10 + 10 = 30$$

$$27 \times 4 = 108$$

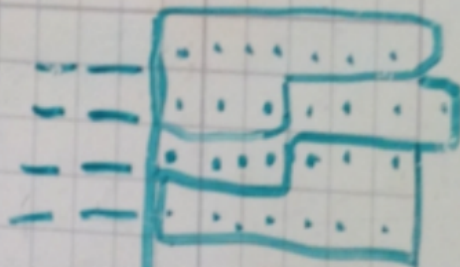


$$\begin{array}{r} 80 \\ + 28 \\ \hline 108 \end{array} \text{ Cubs}$$

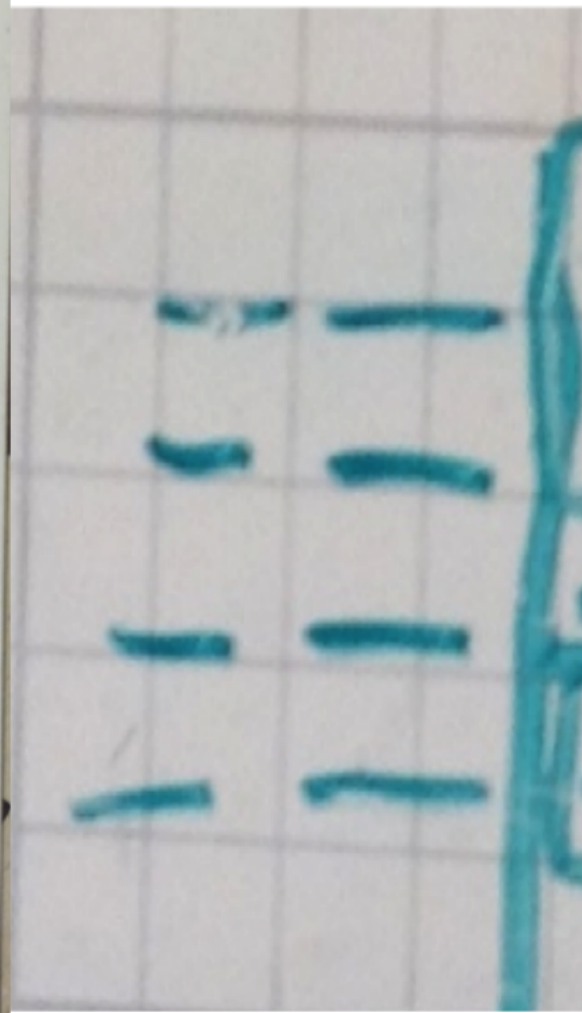
$$\begin{array}{r} 1 \\ 27 \\ + 27 \\ \hline 54 \end{array}$$

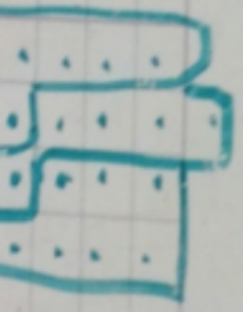
$$\begin{array}{r} + 27 \\ \hline 81 \end{array}$$

$$\begin{array}{r} + 27 \\ \hline 108 \end{array} \text{ Cubs}$$



$$\begin{array}{l} 20 \times 4 = 80 \\ 4 \times 7 = 28 \\ \hline 108 \end{array}$$





$$7 = 28$$



How many cubes?



$$\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$$
$$\begin{array}{r} 27 \\ \times 9 \\ \hline 180 \\ \hline \end{array}$$
$$9 \overline{) 243} = 27$$
$$\begin{array}{r} 180 \\ + 63 \\ \hline 243 \end{array}$$

$$\begin{array}{r} 9 \\ \times 27 \\ \hline 63 \\ + 180 \\ \hline 243 \end{array}$$
$$\begin{array}{r} 27 \\ \times 9 \\ \hline 180 \\ \hline \end{array}$$
$$\begin{array}{r} 27 \\ \times 9 \\ \hline 243 \end{array}$$
$$\begin{array}{r} 243 \\ \times 3 \\ \hline 729 \end{array}$$

$$\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} \times 207 \\ \hline 9 \overline{) 207} \\ \underline{180} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

$$9 \overline{) 207} \quad 9 \times 9 = 81 \quad 9 \times 7 = 63 \quad = 63$$

$$\begin{array}{r} 180 \\ + 63 \\ \hline 243 \end{array}$$

20

$$\begin{array}{r} 9 \\ \times 27 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \times 9 \\ \hline \end{array}$$

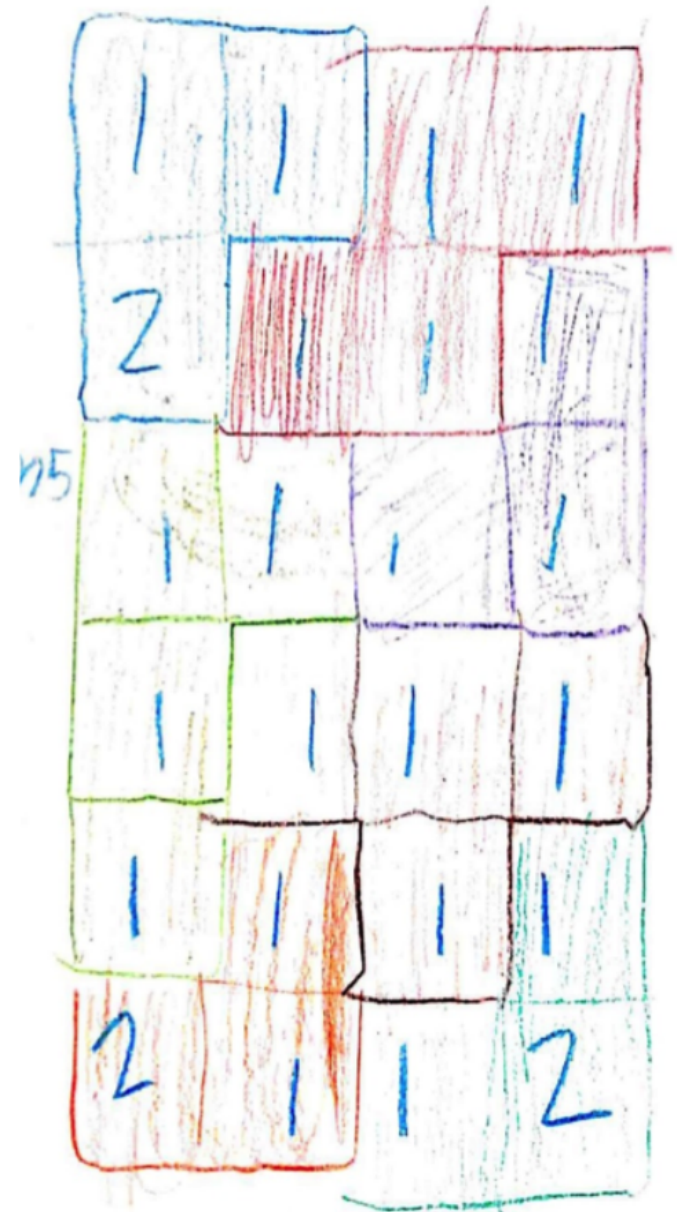
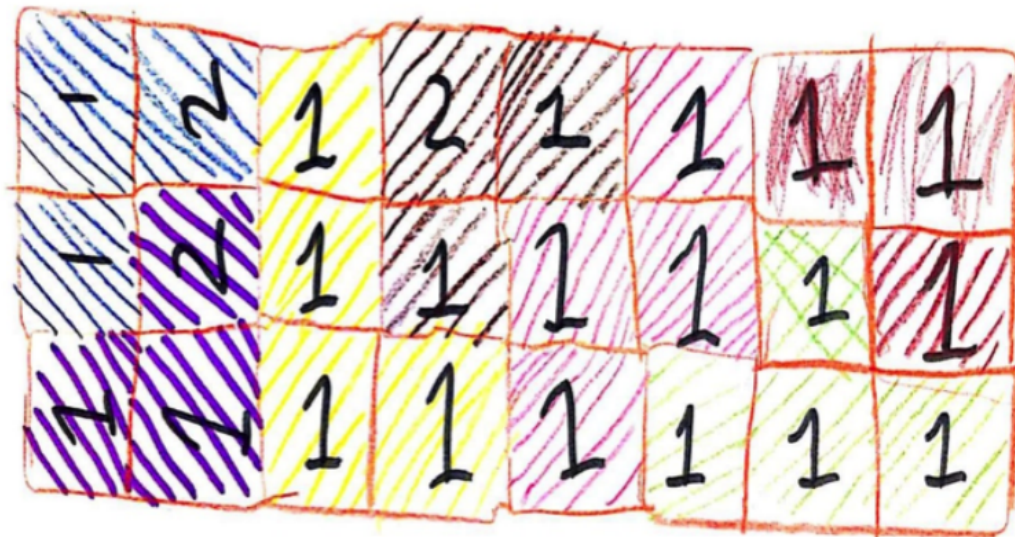
$$\begin{array}{r} 63 \\ + 180 \\ \hline 243 \end{array}$$

$$\begin{array}{r} \downarrow \\ 243 \end{array}$$

$$\begin{array}{r} \downarrow \\ 243 \end{array}$$

$$\begin{array}{r} 243 \\ \times 3 \\ \hline 729 \end{array}$$

Cake design . . .



Permutations

1	1	1	1	2	1	1	1
1	1	1	1	1	2	1	1
1	1	1	1	1	1	2	1

#1	#5	#6	#7	
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D
15	A	B	C	D
16	A	B	C	D
17	A	B	C	D
18	A	B	C	D
19	A	B	C	D
20	A	B	C	D
21	A	B	C	D
22	A	B	C	D
23	A	B	C	D
24	A	B	C	D
25	A	B	C	D
26	A	B	C	D
27	A	B	C	D
28	A	B	C	D
29	A	B	C	D
30	A	B	C	D

$4! = 24$
 $3! = 6$
 $2! = 2$
 $1! = 1$
 $4! \times 3! \times 2! \times 1! = 24 \times 6 \times 2 \times 1 = 288$

SGF3E

B	5	5	6	6	7	7
3	7	6	7	5	5	6
E	6	7	5	7	6	5

1, 6, 7, 5

7th grade work

Factorial

$4! = 4 \times 3 \times 2 \times 1 = 24 = 4!$

Factorial

Notes

- Bad what I do
- Show what I learned
- Show

1	7	6	5	1
1	6	7	5	1
1	6	5	7	1
1	5	6	7	1
1	5	7	6	1
6	7	5	1	1
6	5	7	1	1
6	5	1	7	1
6	5	1	1	7
5	7	6	1	1
5	6	7	1	1
5	6	1	7	1
5	7	1	6	1
5	7	1	1	6

SG F 3 E

113 B

5	5	6	7	7
7	6	7	5	6
6	7	5	7	6

1, 6, 7, 5,

7th grade work

DLF4E

Factors

Factorial

7-6-5-1
7-6-1-5
7-5-6-1
7-5-1-6
7-1-6-5
7-1-5-6

$4 \times 3 \times 2 \times 1 = 24 = 4!$

1-7-5-6
1-7-6-5
1-6-7-5
1-6-5-7
1-5-6-7
1-5-7-6

6-7-5-1
6-7-1-5
6-1-5-7
6-1-7-5
6-5-1-7
6-5-7-1

5-1-7-6
5-1-6-7
5-6-7-1
5-6-1-7
5-7-1-6
5-7-6-1

Notes

- Say what I did
- Say what I learned
- Stay concentrated

	#1	#5	#6	#7
1	A	B	C	D
2	A	B	D	C
3	A	C	B	D
4	A	C	D	B
5	A	D	B	C
6	A	D	C	B
7	B	A	C	D
8	B	A	D	C
9	A	C	A	D
10	B	C	D	A
11	B	D	A	C
12	B	D	C	A
13	C	A	B	D
14	C	A	D	B
15	C	B	A	D
16	C	B	D	A
17	C	D	A	B
18	C	D	B	A
19	D	A	B	C
20	D	A	C	B
21	D	C	A	B
22	D	B	C	A
23	D	C	D	B
24	D	C	B	A

Another way: $3 \times 2 = 6$
 $6 \times 6 \times 4 = 24$
 spots solutions

Cake design



3D Visualization – Grades 3-16

Issues:

- Making sense of what is hidden in 2D pictures of 3D objects (Battista, 1999)
- Drawing meaningful representations of 3D objects, such as rectangular prisms (Outhred & Mitchelmore, 2002)
- Making sense of plan views and of top/side/front views

Instructional Perspective:

- “Low floor; high ceiling” (Boaler, 2014)
- Common Core Geometry . . . ?