Framing a Geometry Trajectory on RME Principles for Methods and Content Courses for Undergraduate Preservice Teachers

Jacqueline Sack
Associate Professor

sackj@uhd.edu

Judith Quander
Assistant Professor

quanderr@uhd.edu

University of Houston DOWNTOWN

Traditional Geometry Sequence (in the US)

- Points, Lines, Planes
- Deductive Reasoning
- Perpendicular and Parallel Lines
- Congruent Triangles
- Triangles
- Quadrilaterals
- Transformations
- Similarity
- Circles
- Area
- Surface Area and Volume

Realistic Mathematics Education (RME)

2D Visualization

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Isometric Transformations (Chapter 9)

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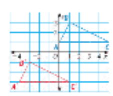
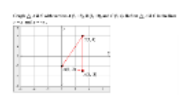
Isometric Transformations (Chapter 9)

1. Translations
2. Matrices
3. Reflections
4. Rotations
5. Composition of Transformations
6. Symmetry
7. Dilations

Translations



Reflections



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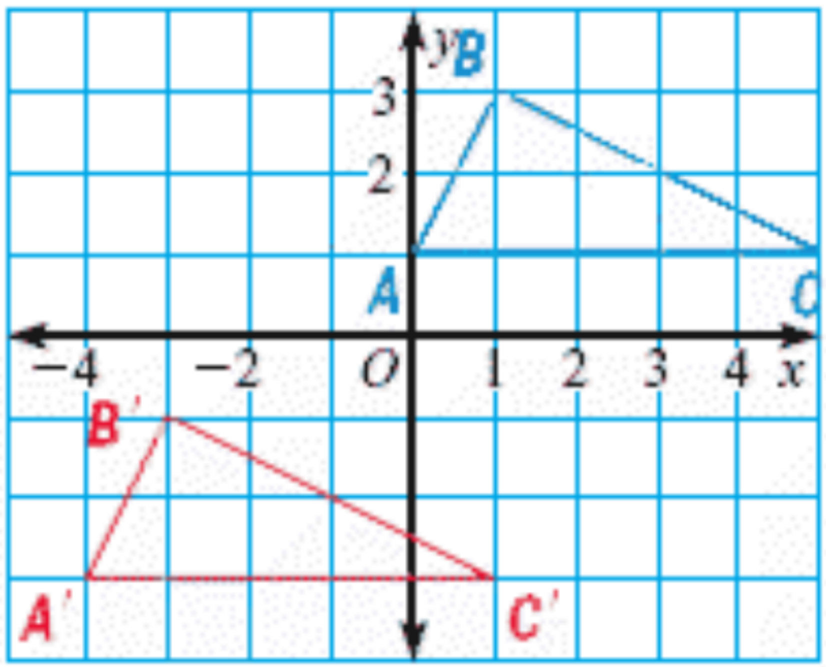
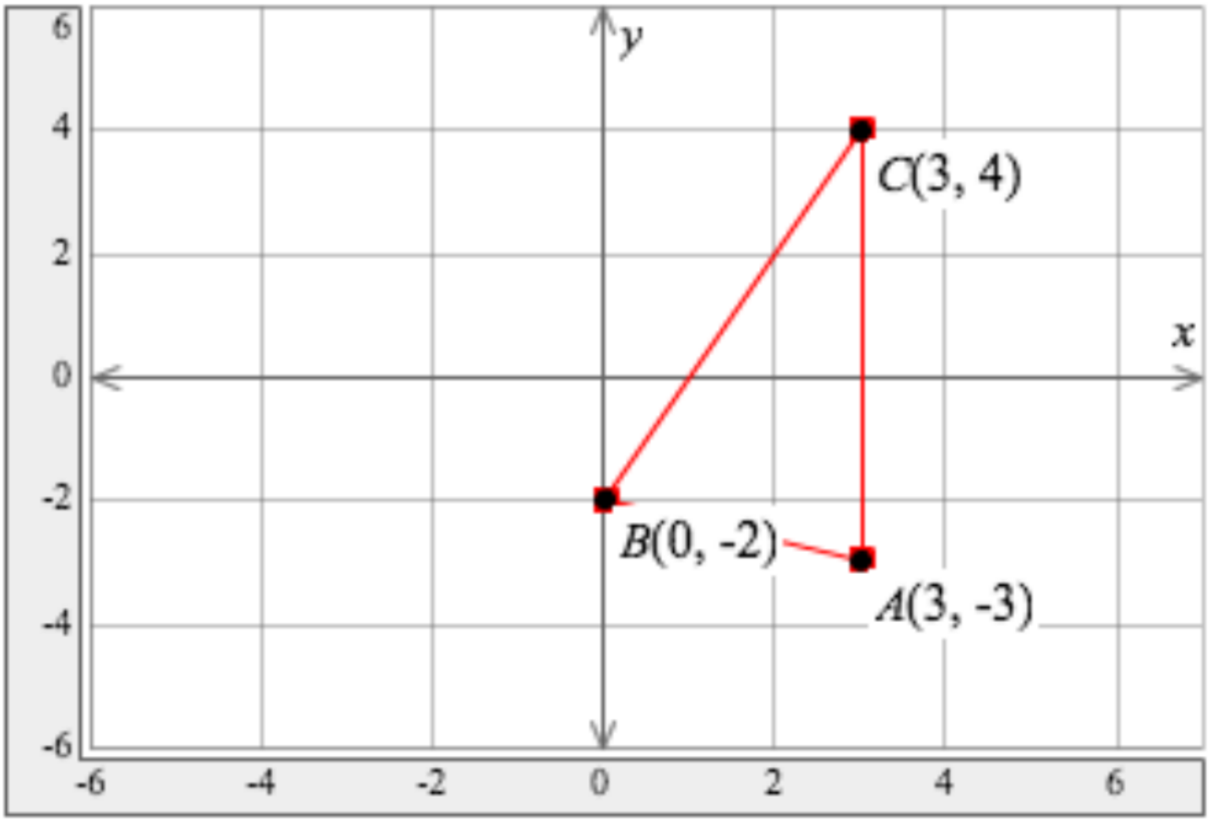


Reflections

Graph △ ABC with vertices A (3, −3), B (0, −2), and C (3, 4). Reflect △ ABC in the lines $y = x$ and $y = -x$.



Rotations

$\triangle ABC$ has vertices $A(-7, 0)$, $B(-6, 2)$, and $C(-2, 3)$. Rotate $\triangle ABC$ 90° about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?

- A. $A'(-7, 0)$, $B'(-6, -2)$, and $C'(-2, -3)$
- B. $A'(0, 7)$, $B'(-2, 6)$, and $C'(-3, 2)$
- C. $A'(0, -7)$, $B'(-2, -6)$, and $C'(-3, -2)$
- D. $A'(0, -7)$, $B'(-2, -6)$, and $C'(3, -2)$

Quadrilateral $FGHJ$ has vertices $F(3, 4)$, $G(3, 8)$, $H(5, 8)$, and $J(6, 4)$. Rotate quadrilateral $FGHJ$ 90° clockwise. What are the coordinates of the vertices of the image, $F'G'H'J'$?

- A. $F'(3, -4)$, $G'(3, -8)$, $H'(5, -8)$, $J'(6, -4)$
- B. $F'(-3, 4)$, $G'(-3, 8)$, $H'(-5, 8)$, $J'(-6, 4)$
- C. $F'(4, 3)$, $G'(8, 3)$, $H'(8, 5)$, $J'(4, 6)$
- D. $F'(4, -3)$, $G'(8, -3)$, $H'(8, -5)$, $J'(4, -6)$

Quadrilateral *FGHJ* has vertices *F*(3, 4), *G*(3, 8), *H*(5, 8), and *J*(6, 4). Rotate quadrilateral *FGHJ* 90° clockwise. What are the coordinates of the vertices of the image, *F′G′H′J′*?

- A. *F′*(3, −4), *G′*(3, −8), *H′*(5, −8), *J′*(6, −4)
- B. *F′*(−3, 4), *G′*(−3, 8), *H′*(−5, 8), *J′*(−6, 4)
- C. *F′*(4, 3), *G′*(8, 3), *H′*(8, 5), *J′*(4, 6)
- D. *F′*(4, −3), *G′*(8, −3), *H′*(8, −5), *J′*(4, −6)

1. Translations
2. Matrices
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Realistic Mathematics Education (RME)

Mathematics is a human activity

Opportunities for students to “re-invent” mathematics through active participation in the learning process

Horizontal mathematization: Students come up with the mathematical tools to organize and solve problems located in real-life situations

Vertical mathematization: Finding shortcuts, discovering connections and applying these to new situations in increasingly more abstract ways

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2D Visualization

2D Visualization: The Van Hiele framework

Levels of Understanding	*Instructional phases (cf 5 E's)*
Visual	Information (Engage)
Descriptive (Analytical)	Guided orientation (Explore)
Relational	Explicitation (Explain / Evaluate)
Deductive	Free Orientation (Elaborate / Evaluate)
	Integration (Evaluate / Engage...)

Early Attempts

2D Visualization: Revisiting Transformations

2D Visualization: Revisiting Transformations

Revisiting Transformations

Revisiting Transformations

Revisiting Transformations

Revisiting Transformations

What makes a triangle?

Student Work

2D Visualization:
The Van Hiele framework

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Visual
Descriptive (Analytical)
Relational
Deductive

Instructional phases (cf 5 E's)

Information (Engage)
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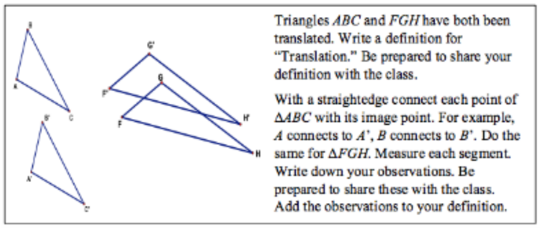
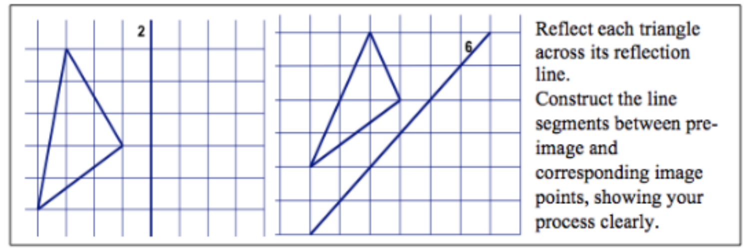
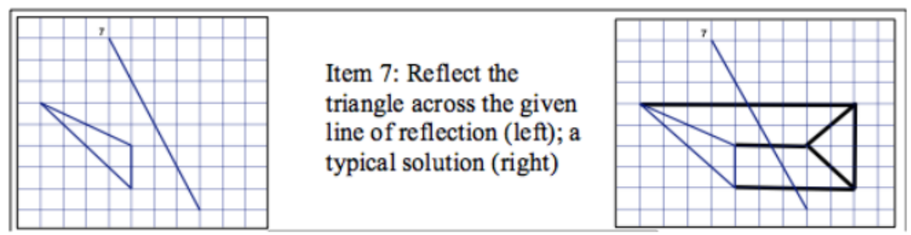
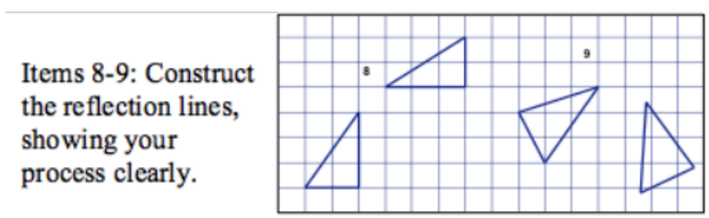
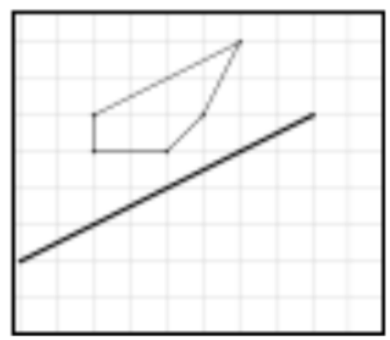
Early Attempts

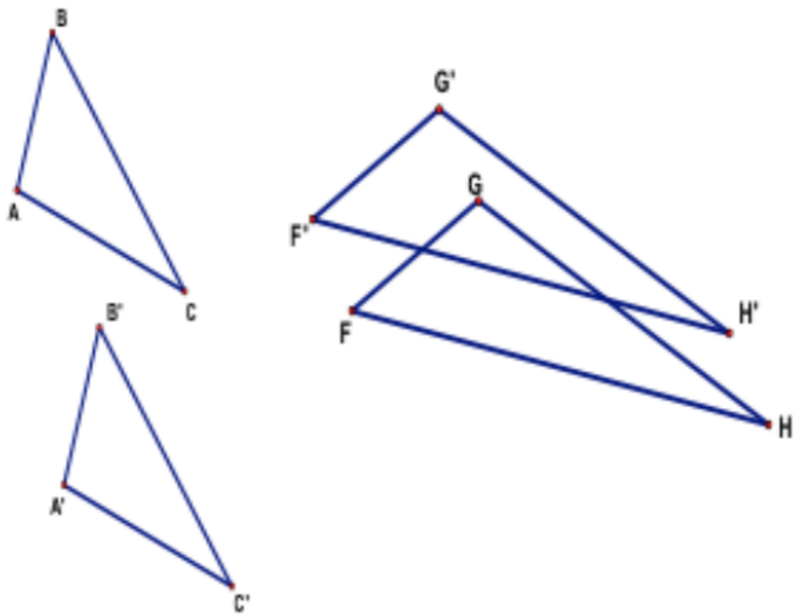






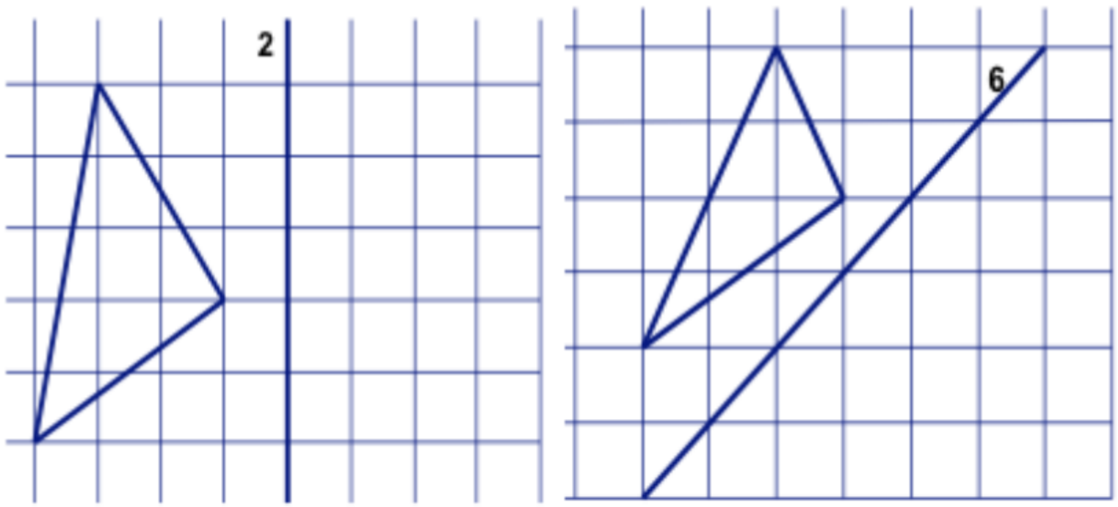




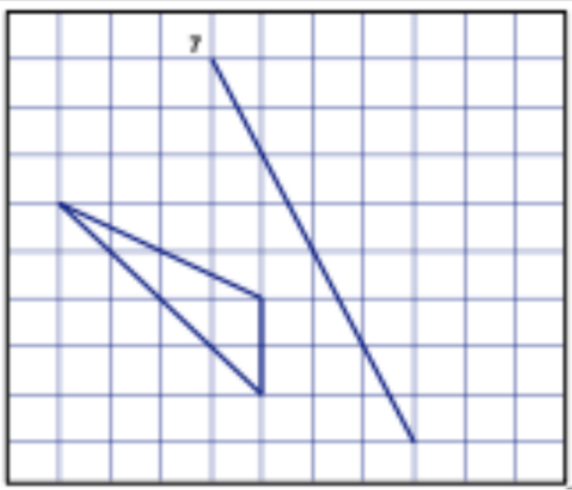


Triangles *ABC* and *FGH* have both been translated. Write a definition for "Translation." Be prepared to share your definition with the class.

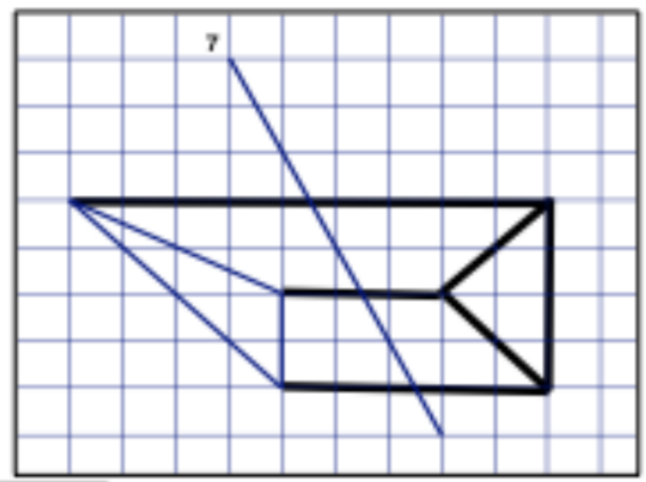
With a straightedge connect each point of ΔABC with its image point. For example, *A* connects to *A*', *B* connects to *B*'. Do the same for ΔFGH. Measure each segment. Write down your observations. Be prepared to share these with the class. Add the observations to your definition.

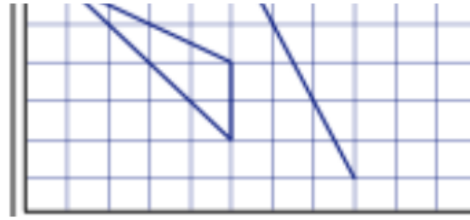


Reflect each triangle across its reflection line.
Construct the line segments between pre-image and corresponding image points, showing your process clearly.



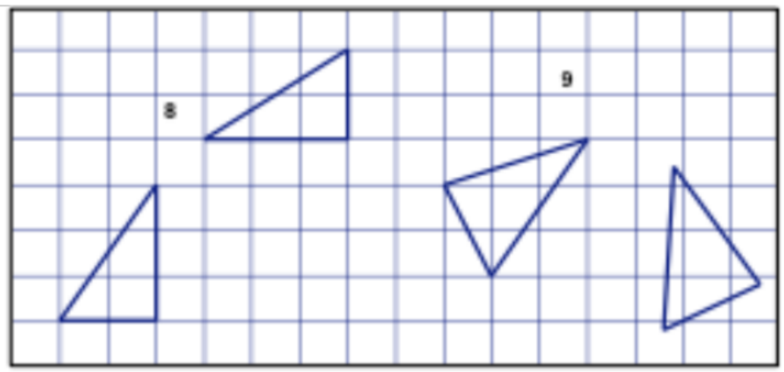
Item 7: Reflect the triangle across the given line of reflection (left); a typical solution (right)

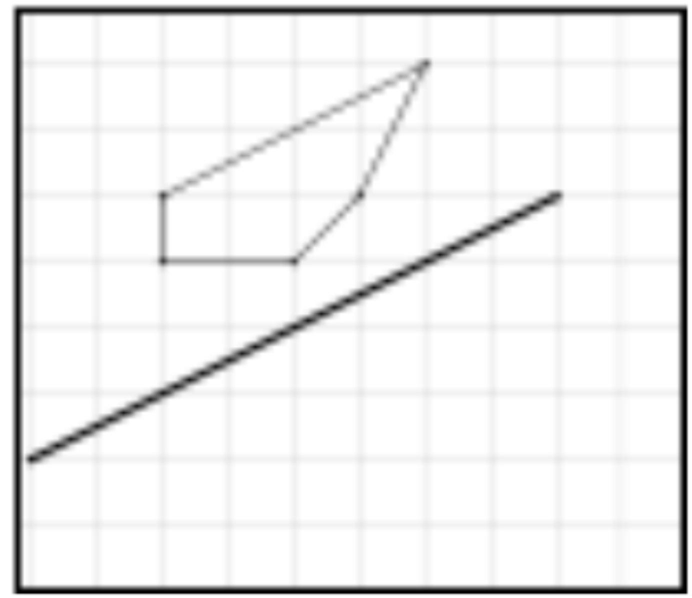




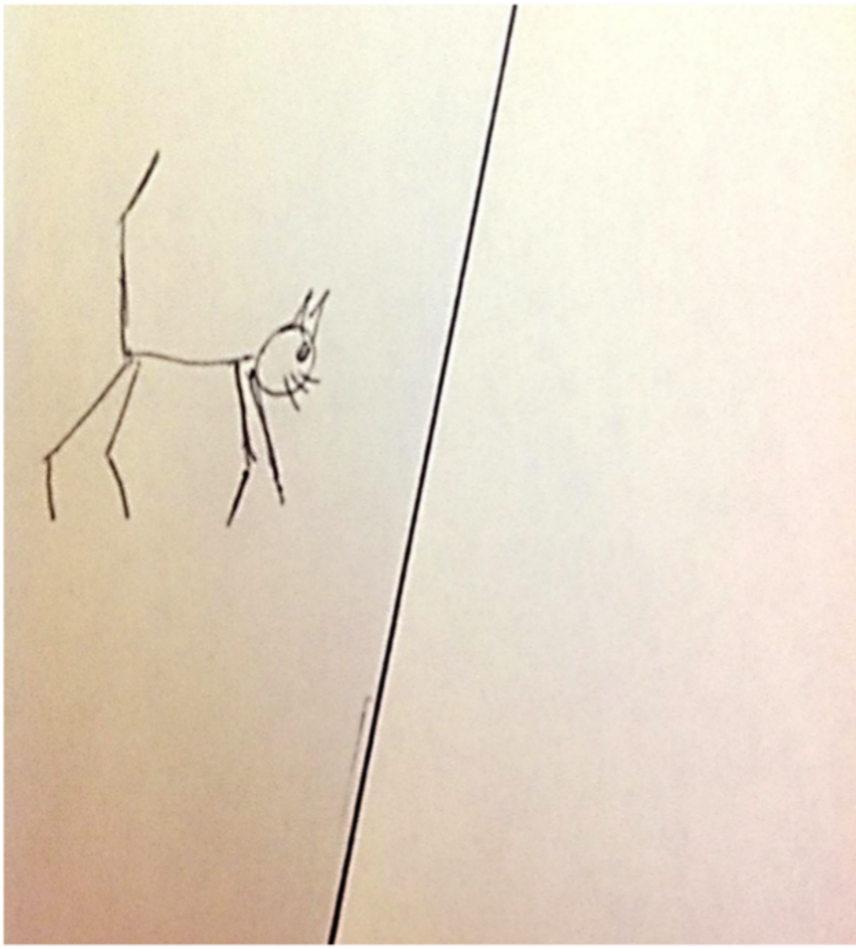
line of reflection (left); a typical solution (right)

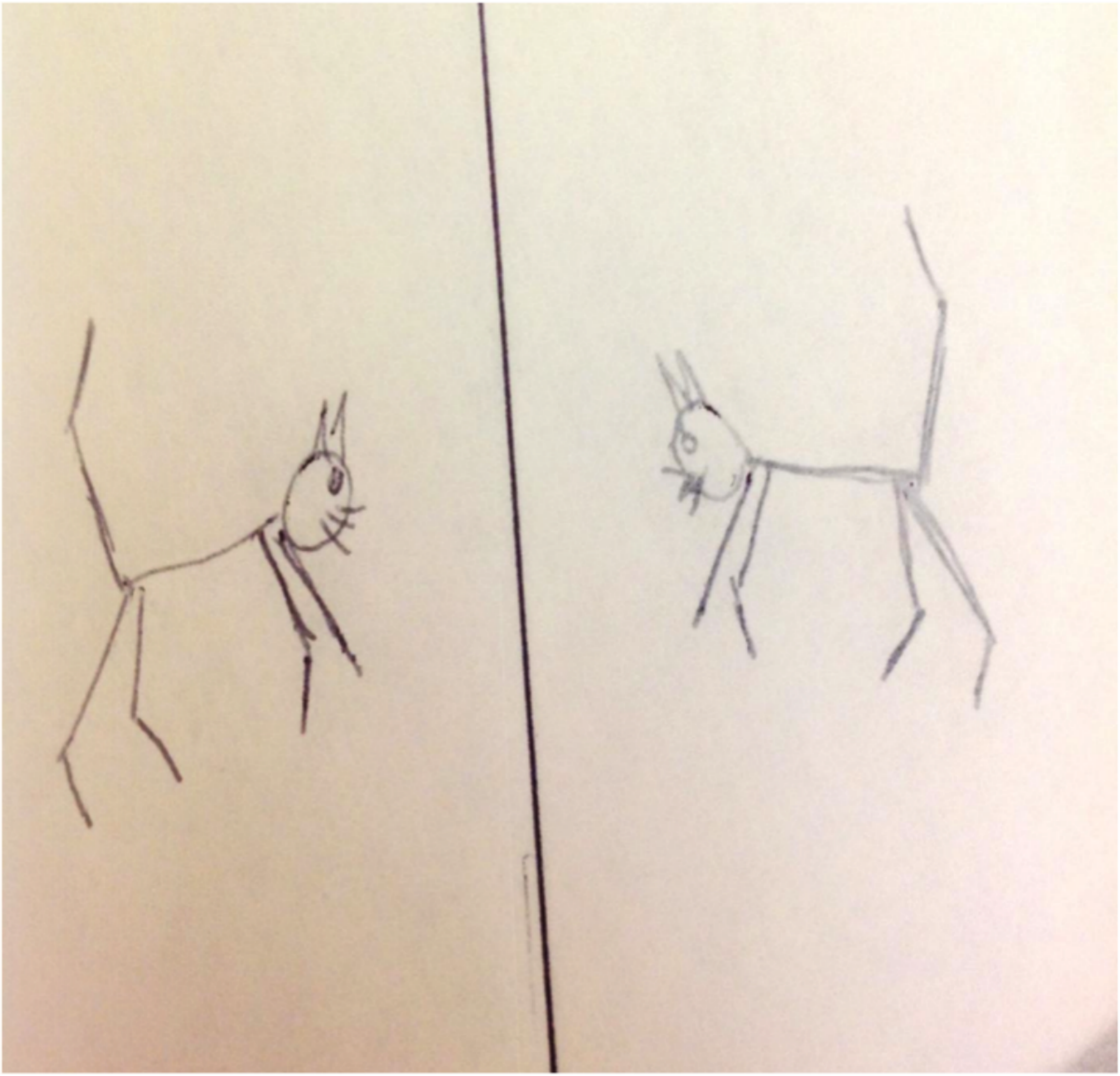
Items 8-9: Construct the reflection lines, showing your process clearly.





2D Visualization: Revisiting Transformations





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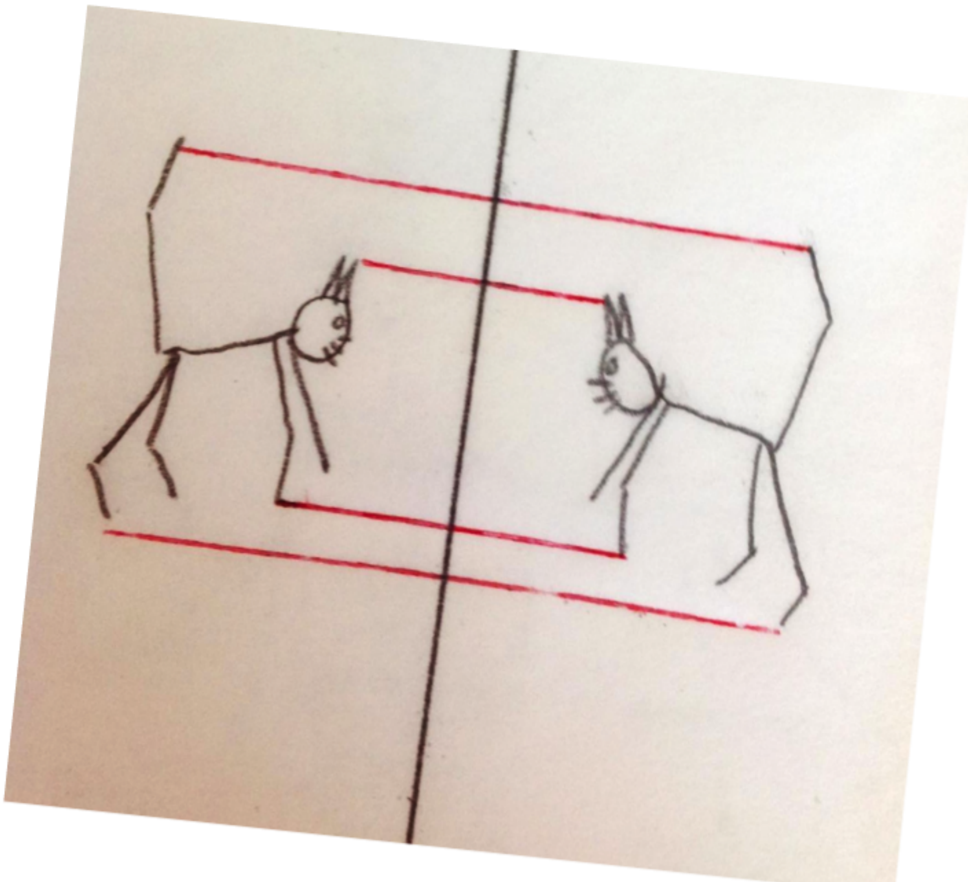
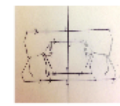
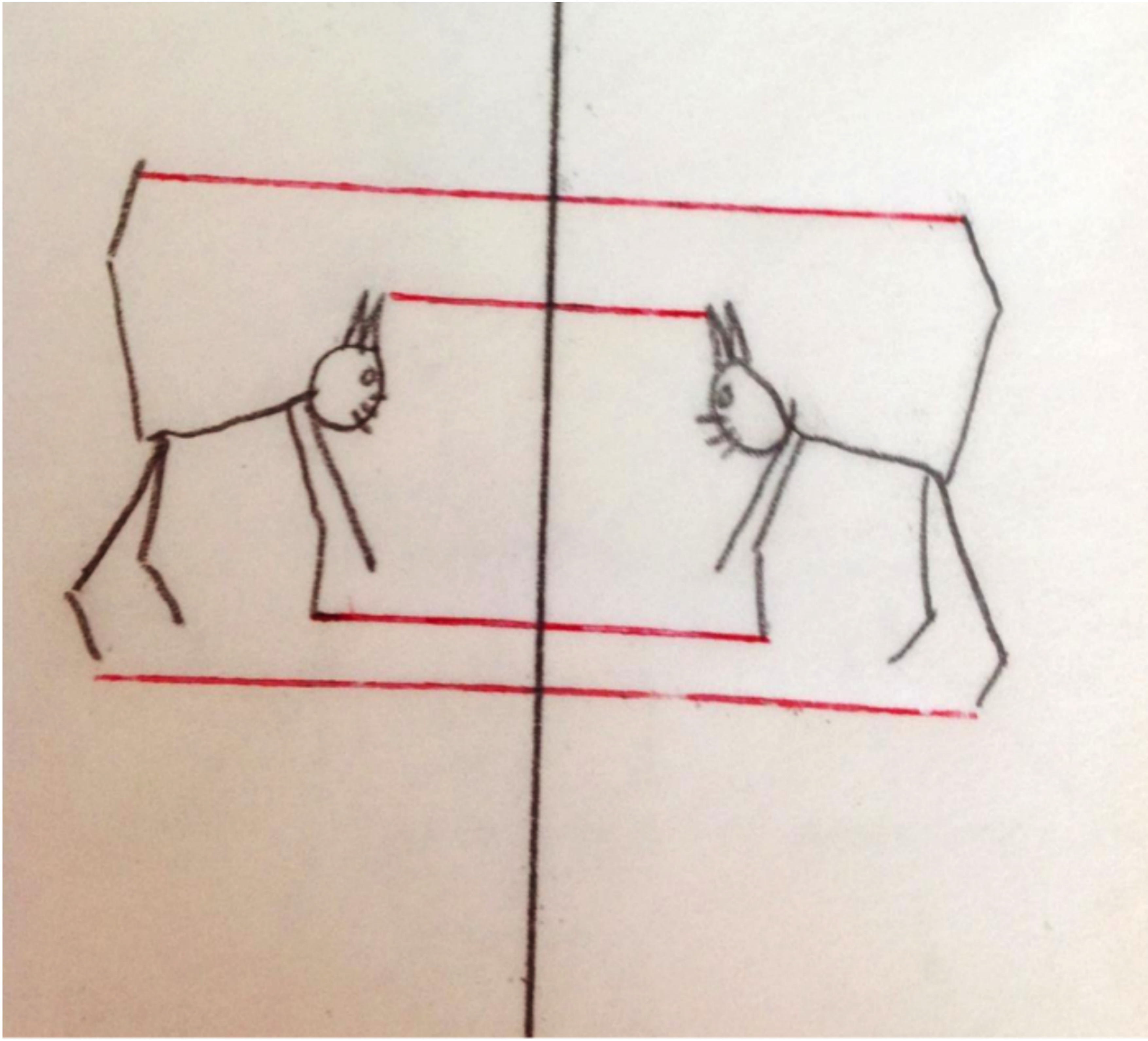
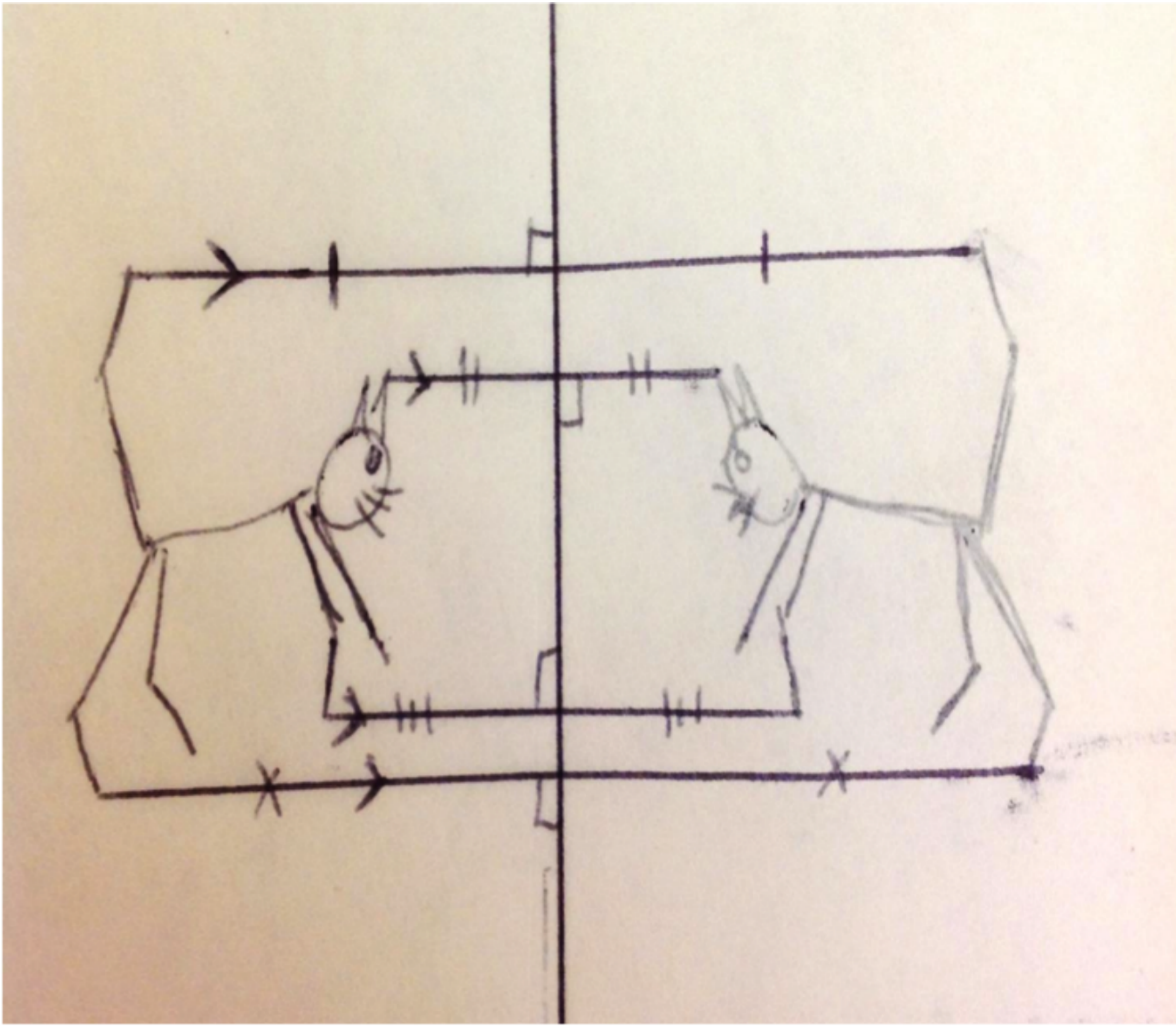
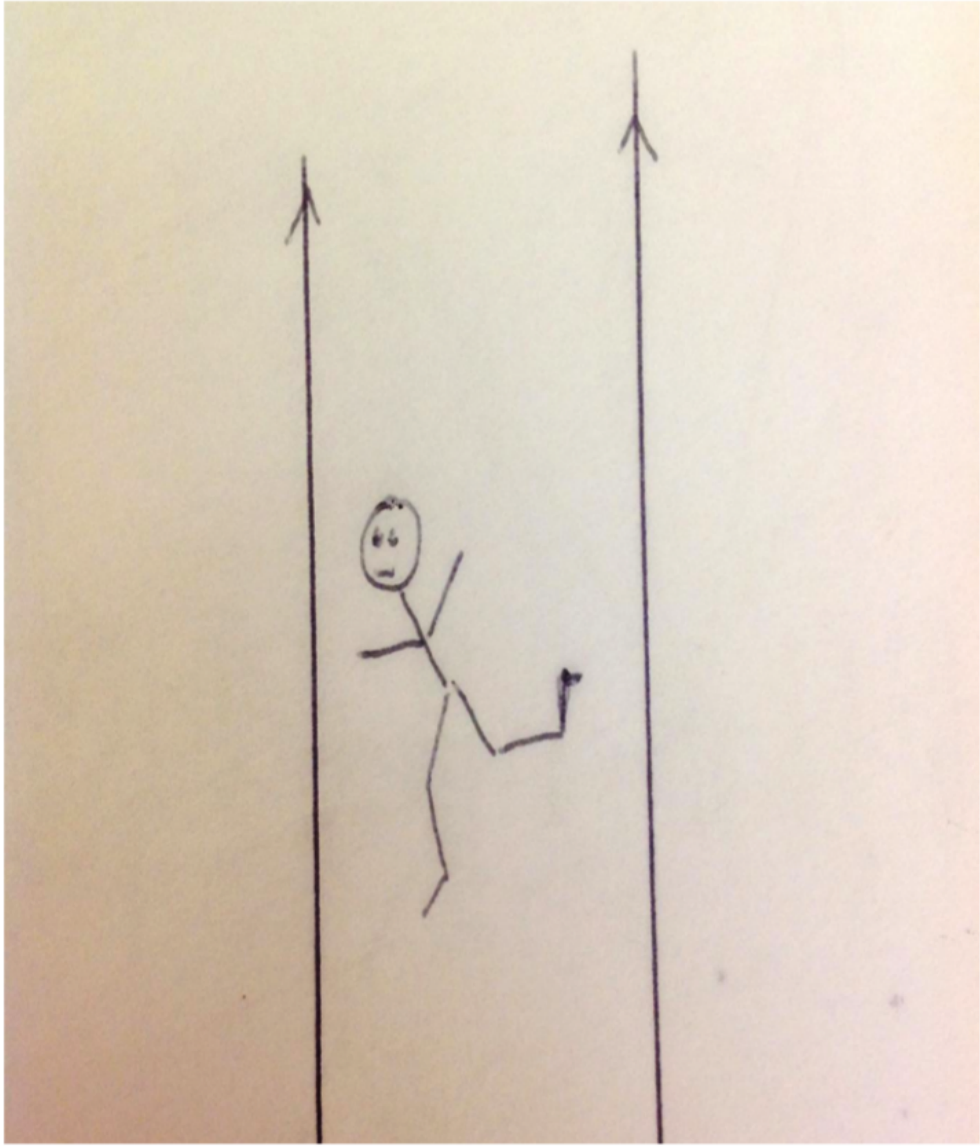


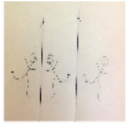


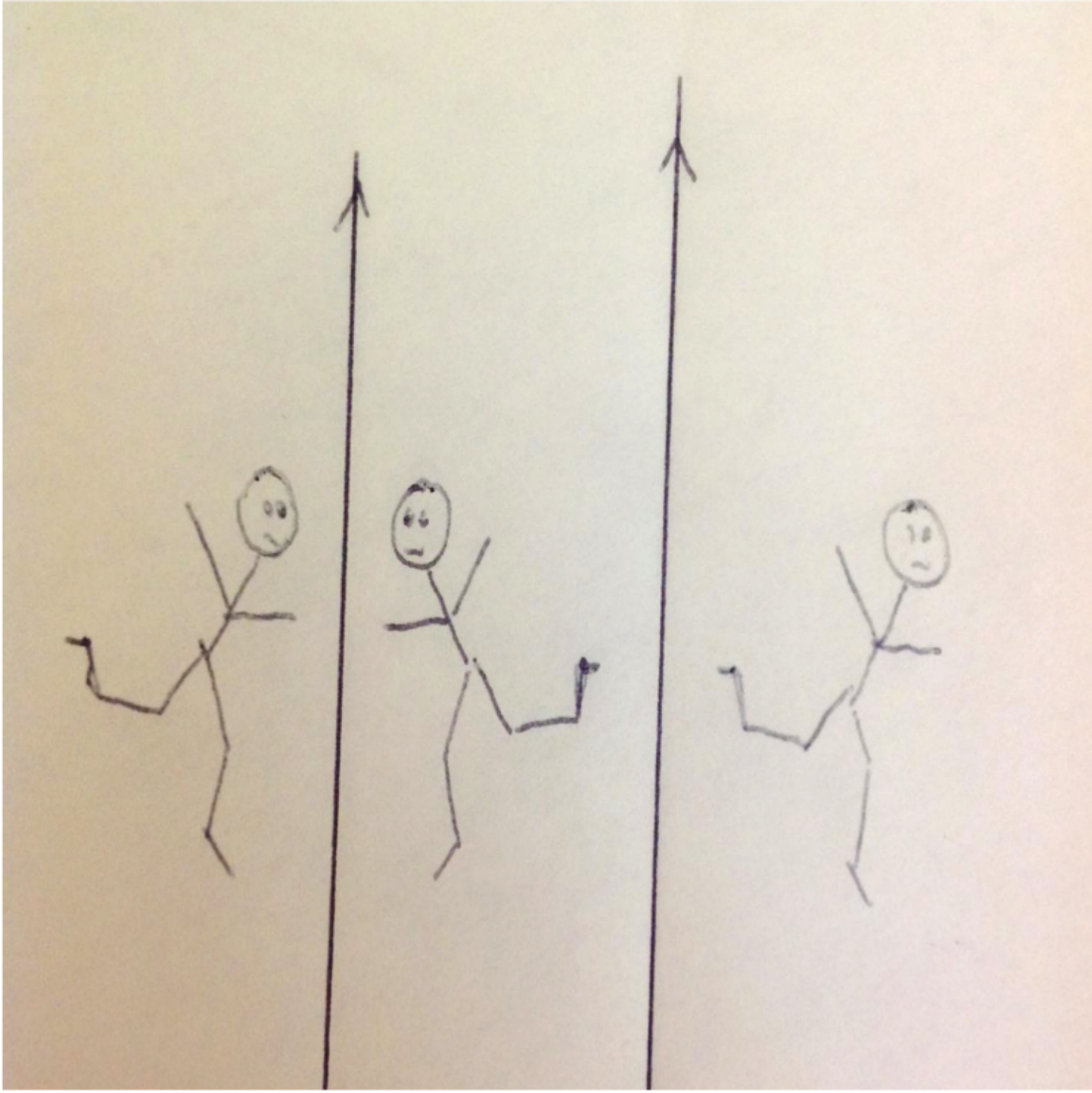




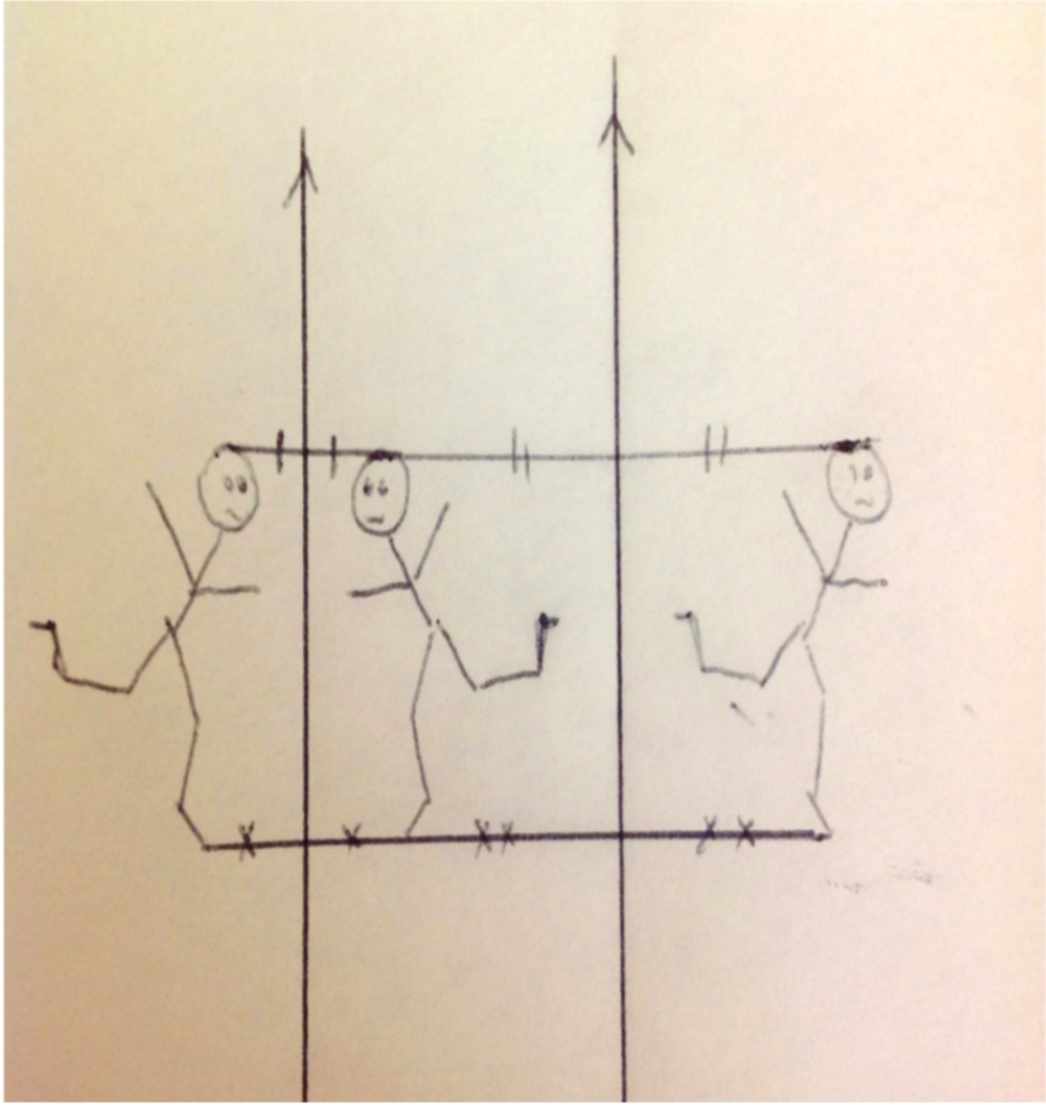
Revisiting Transformations



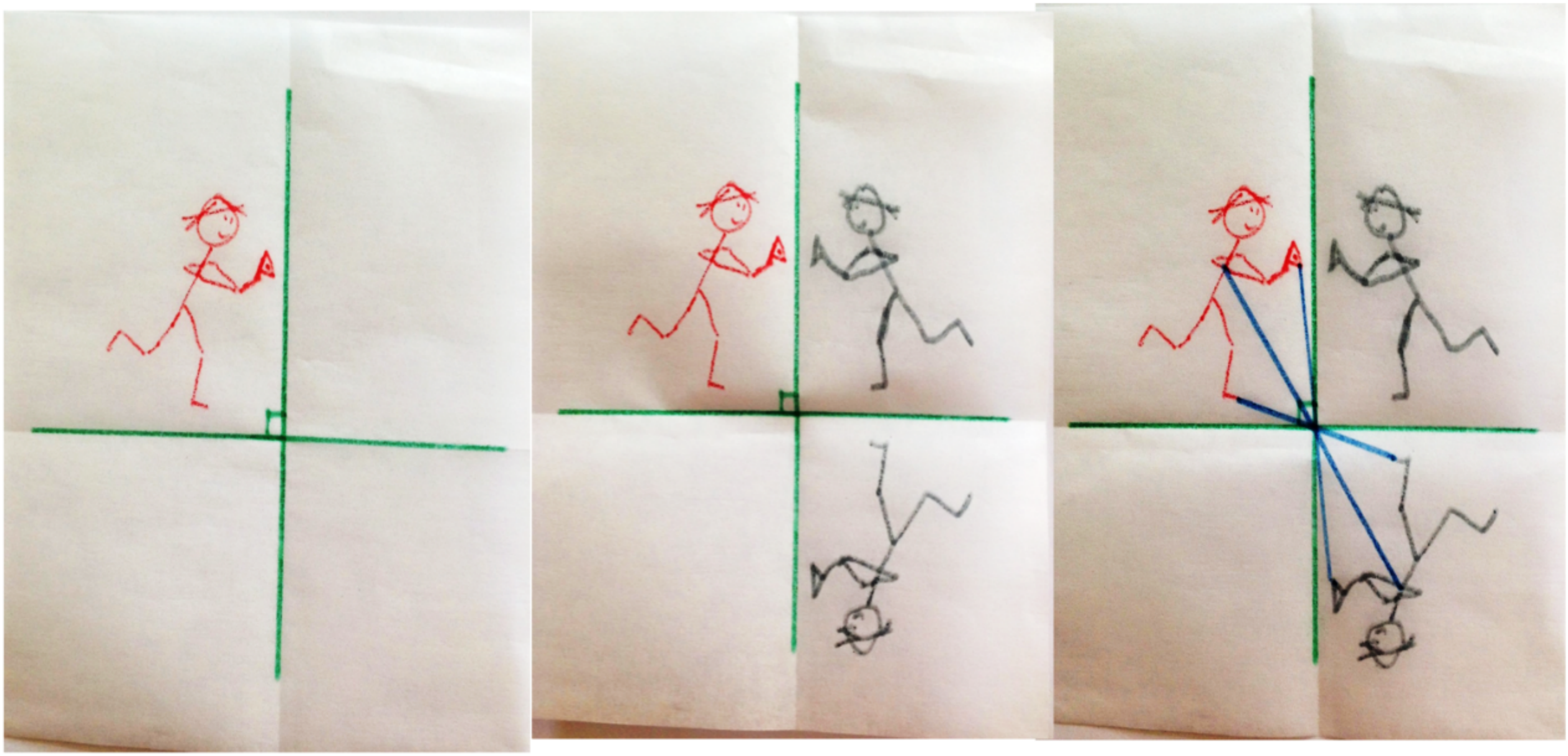




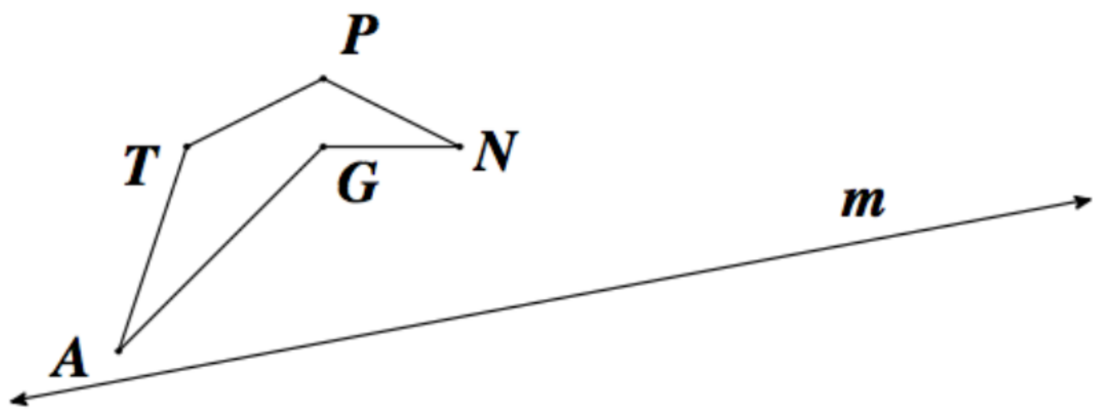
Revisiting Transformations

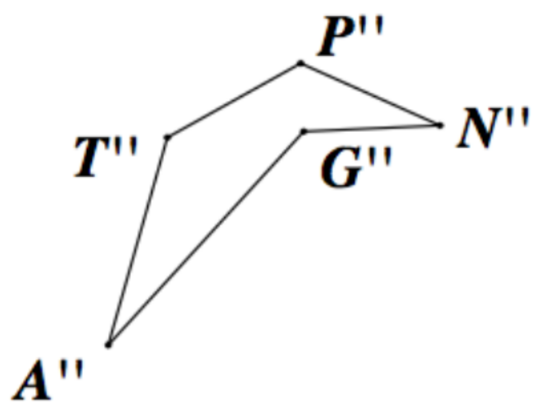


Revisiting Transformations



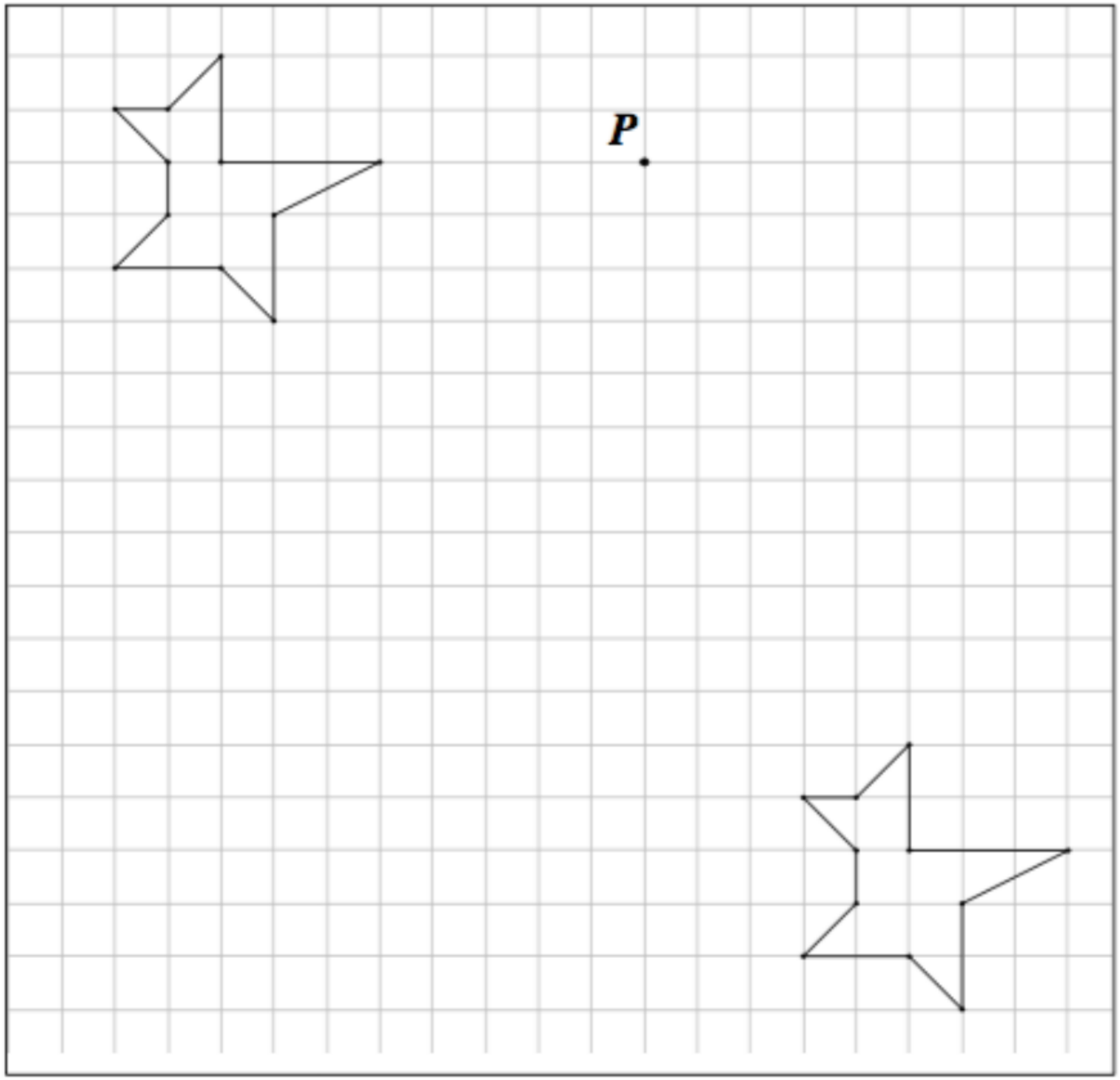
PTAGN is translated to image $P''T''A''G''N''$ by reflection first across line *m* and then across line *n* that is parallel to line *m*. Use your ruler and pencil to construct the lines needed to accurately locate line *n*.





Look carefully at the segments AA'', TT'', NN''. Write down what you notice about their distance and direction from the pre-image points?

ss
er



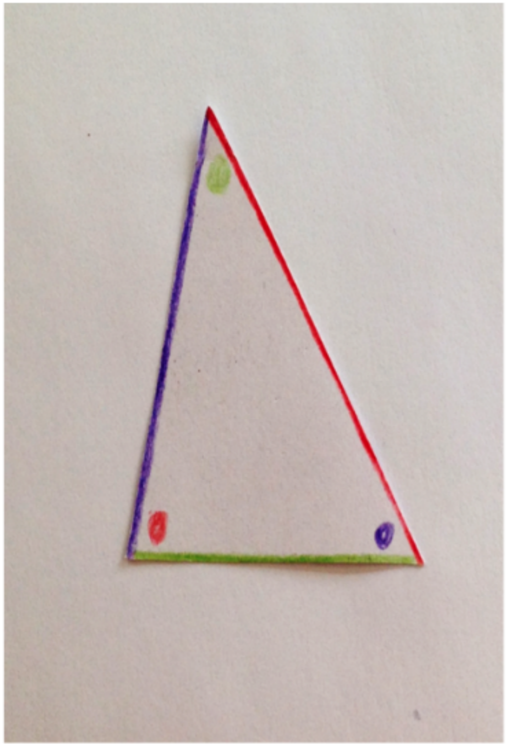
you
s?

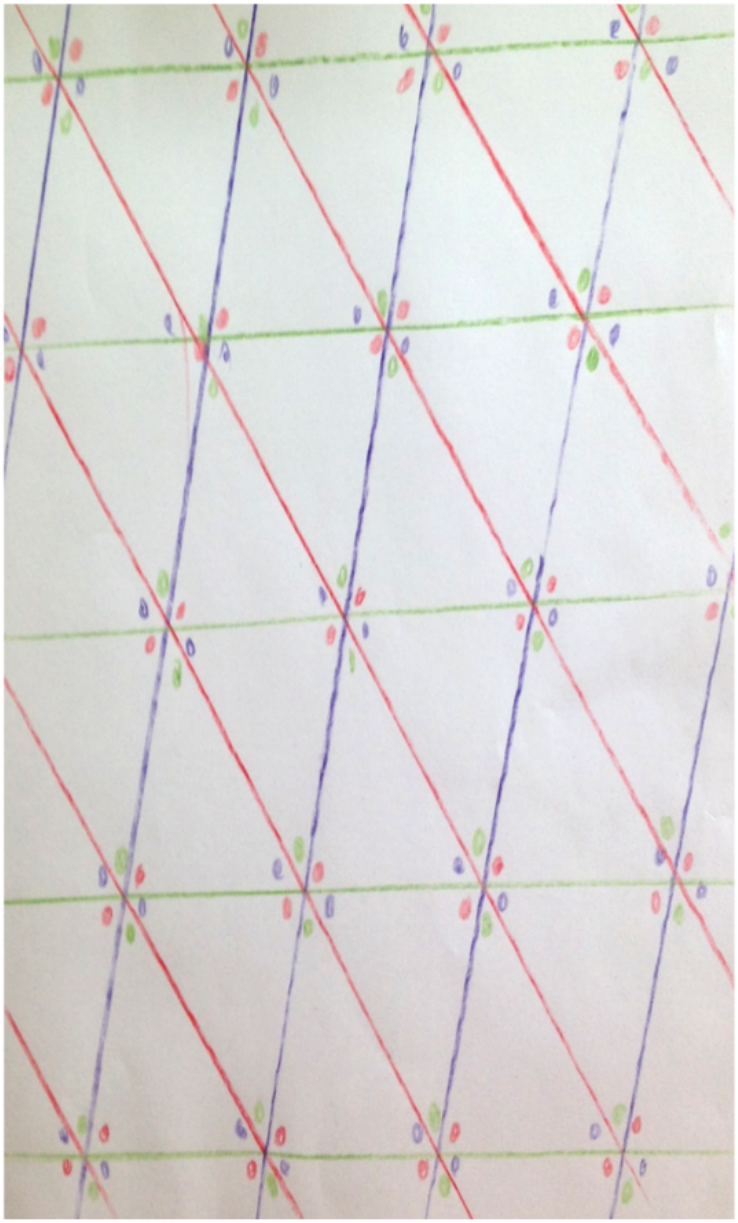
Label the figure in the top left above. The figure in the top left is translated to the position in the lower right. The first reflection line passes through point P. Using what you know about the properties of reflection that constitute a translation, use a pencil and ruler to accurately place the two reflection lines and the translation vectors for at least 4 points on the figure. Use two different color pencils for the reflection lines and for the translation vectors

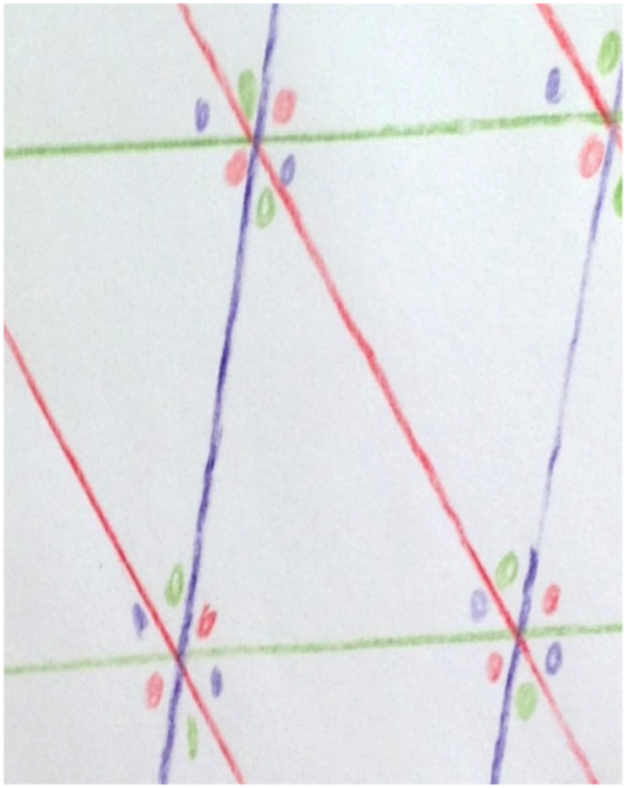
What makes a triangle?

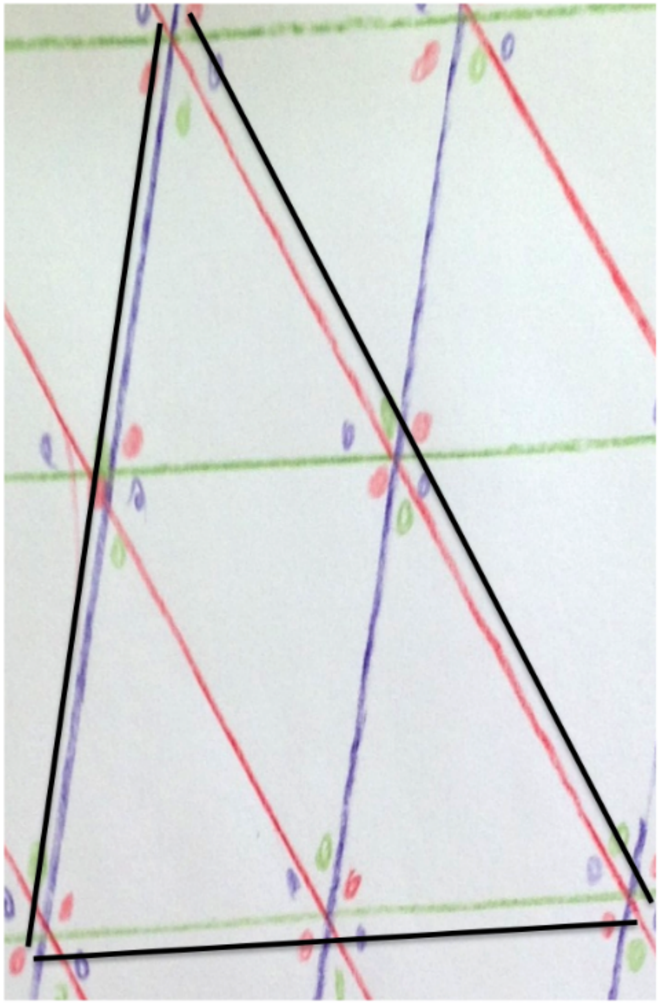
- Triangle inequalities
- Angle and segment (side) relationships
- Isosceles triangles
- Equilateral triangles

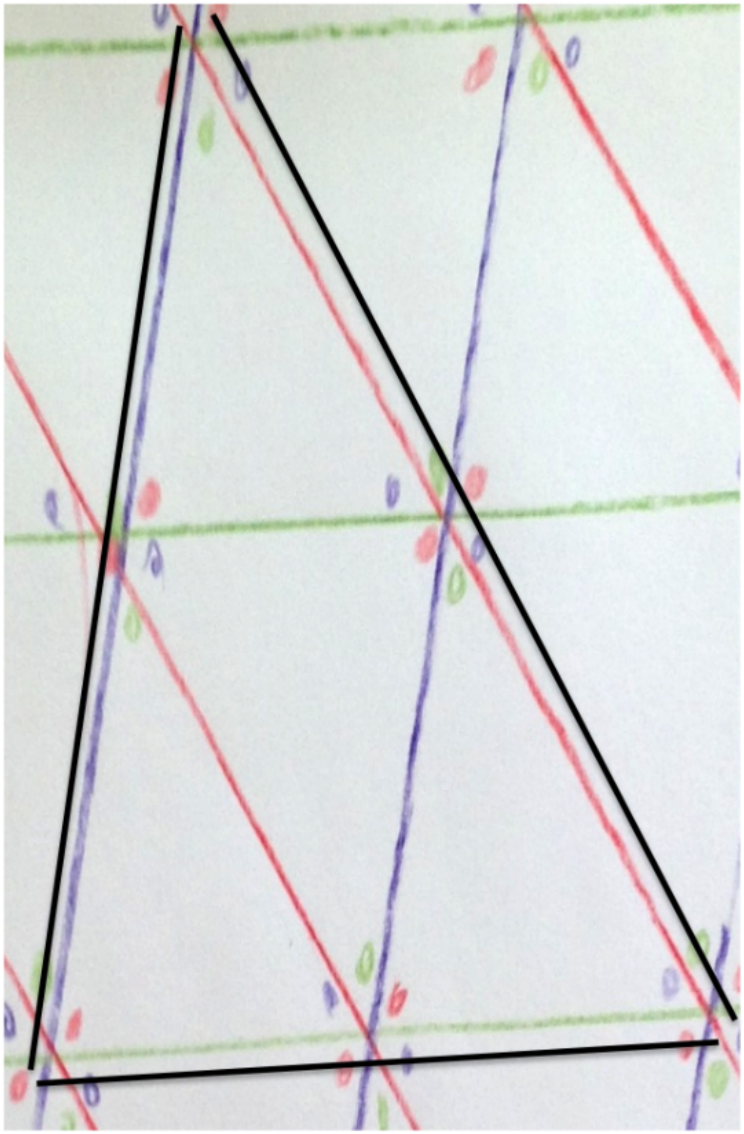
Tessellations: Rotate a triangle

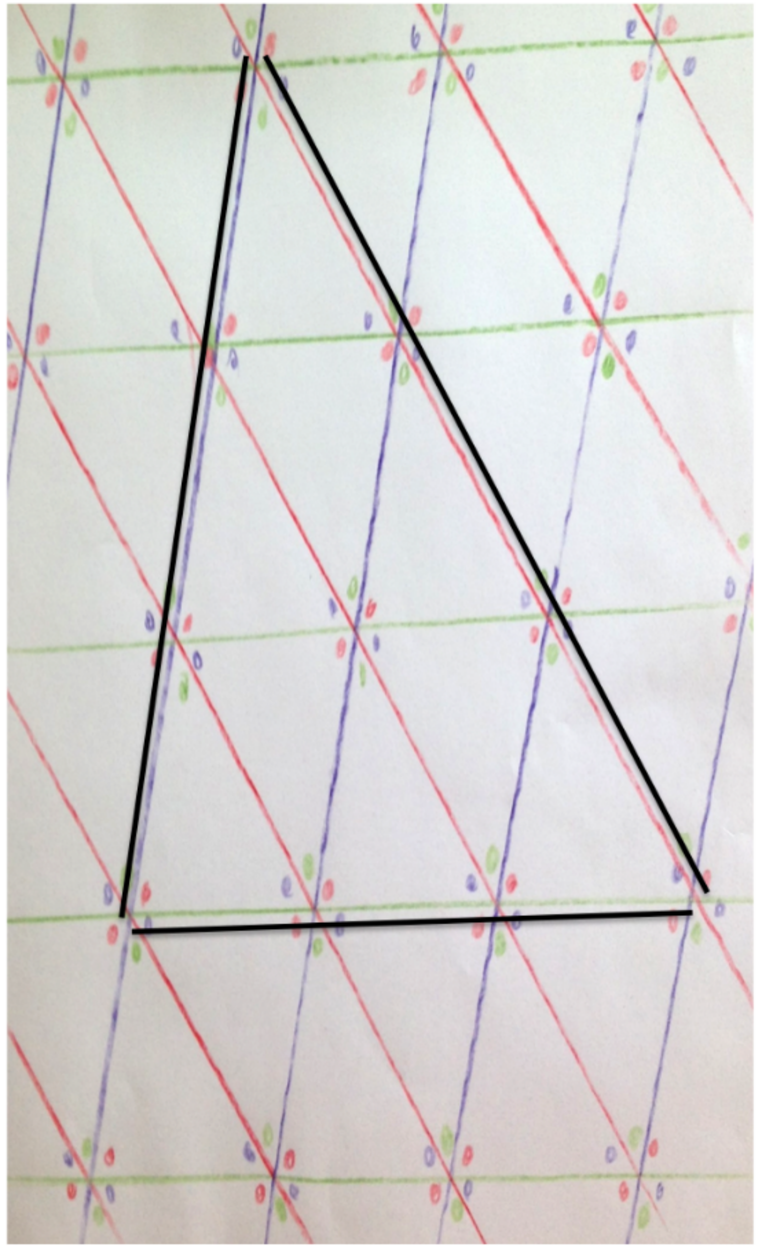




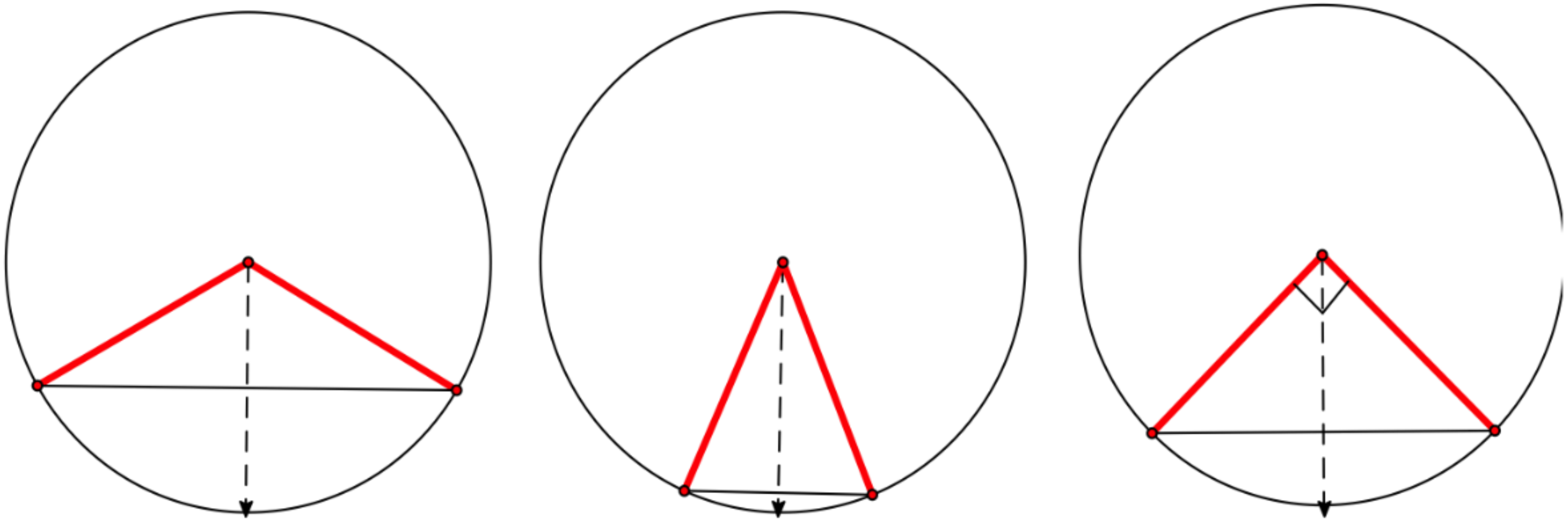








Isosceles Triangles



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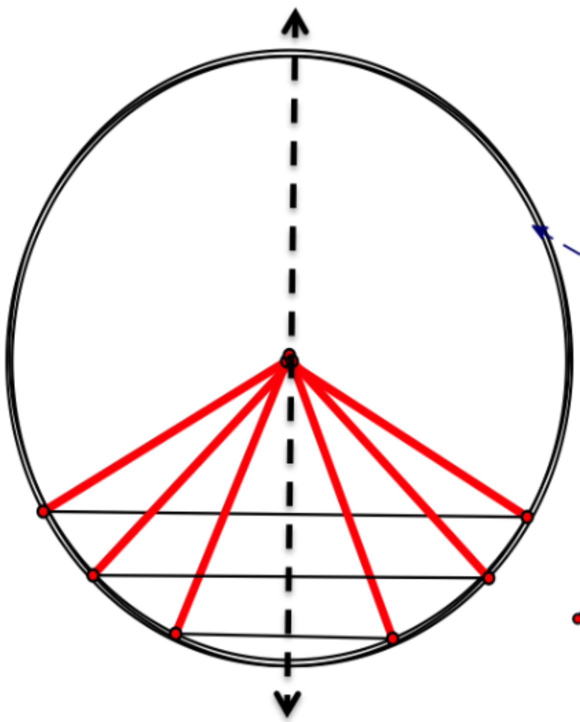
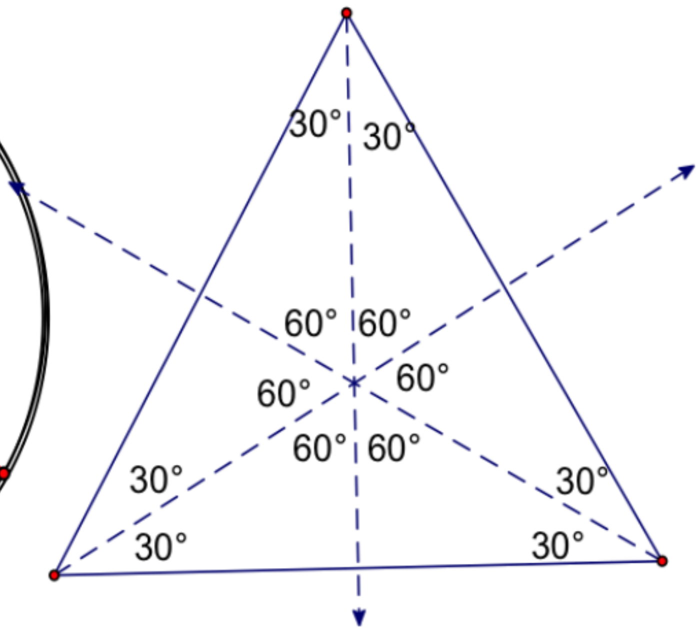
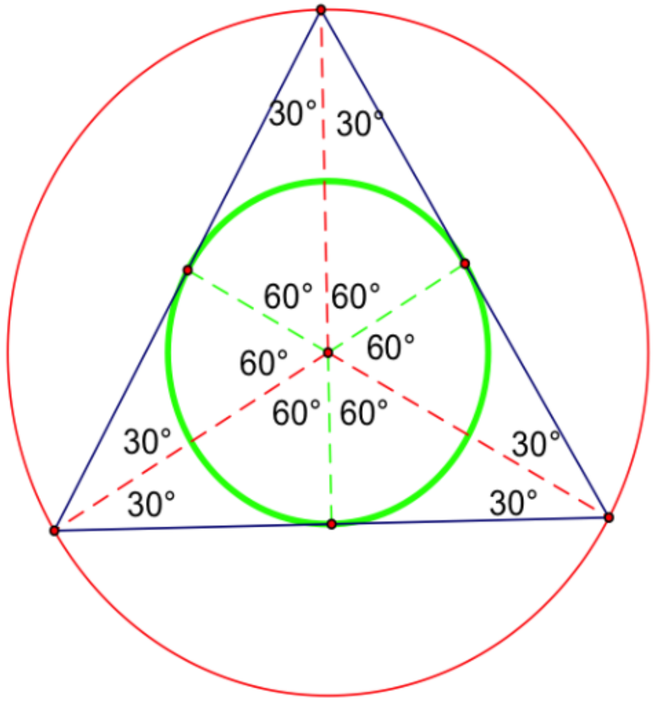
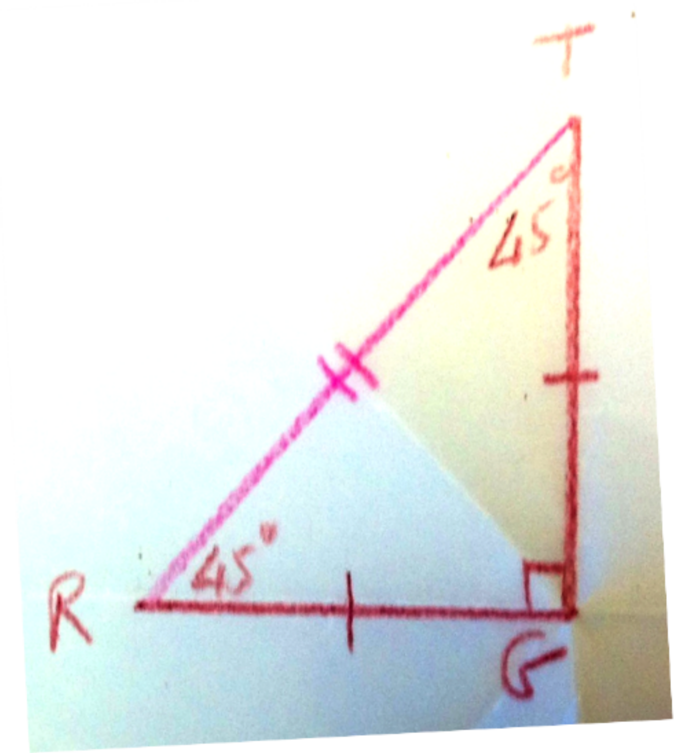
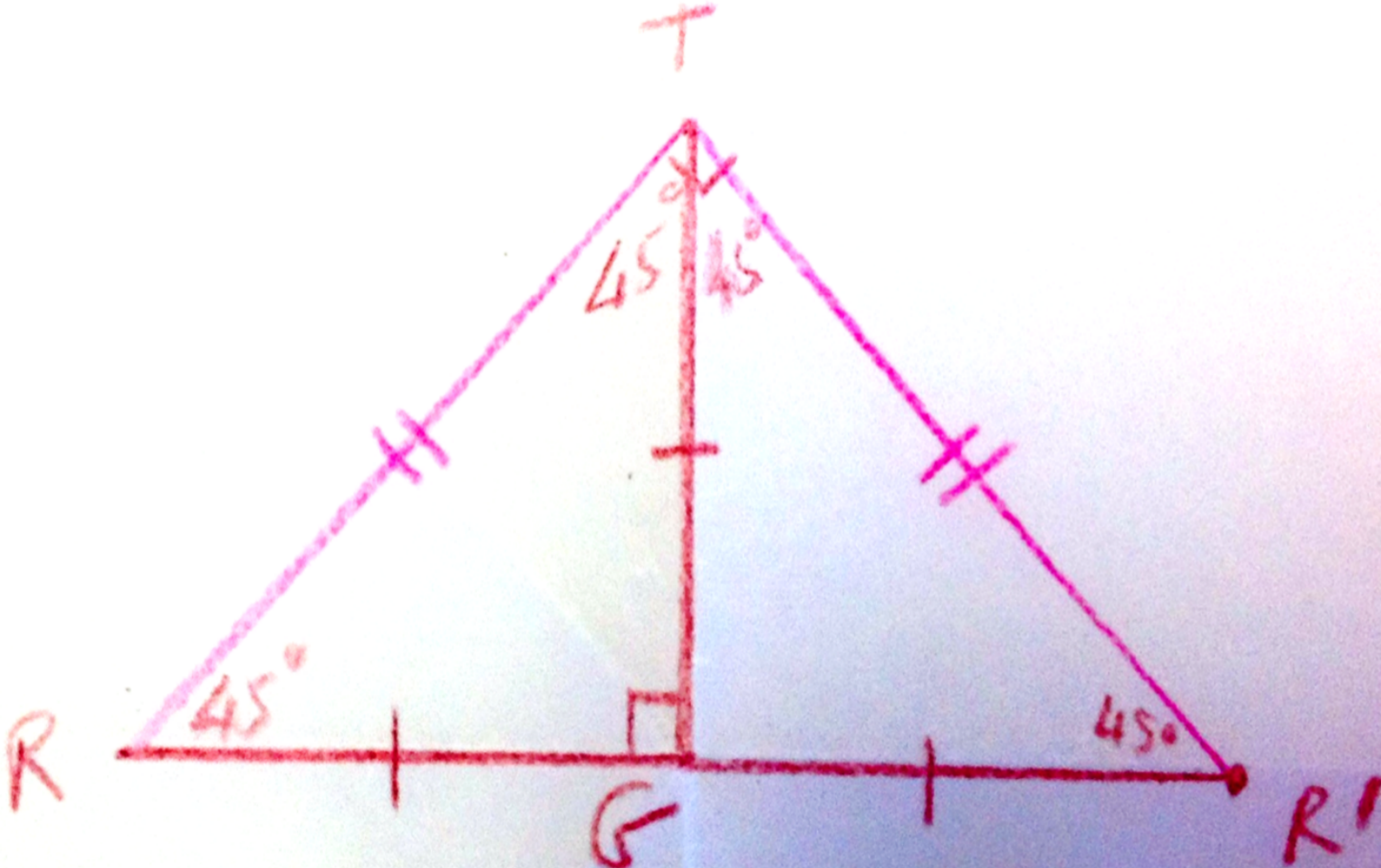


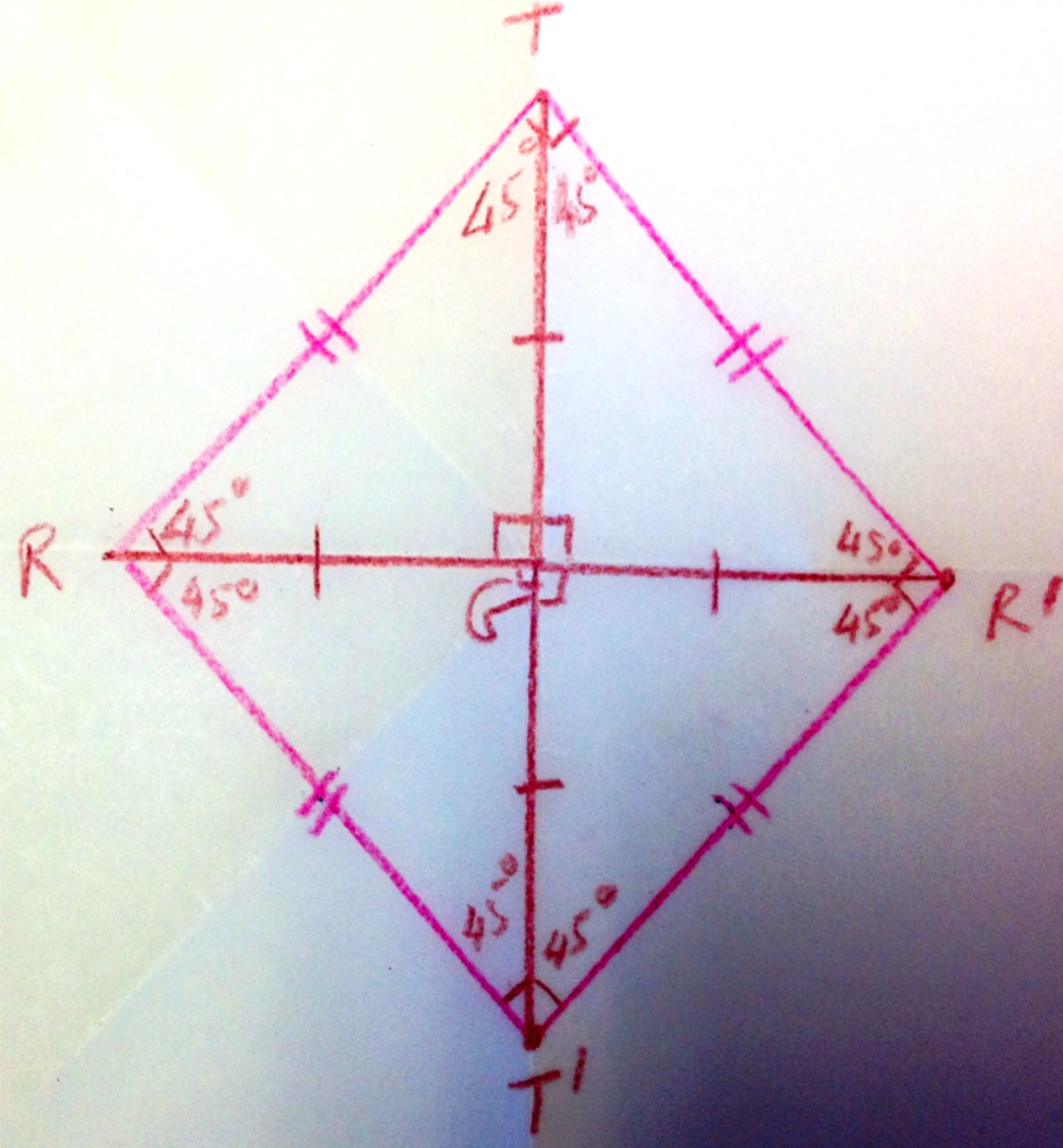




Reflect a triangle




T
45
45°
45°
45°
R
G
R'


T
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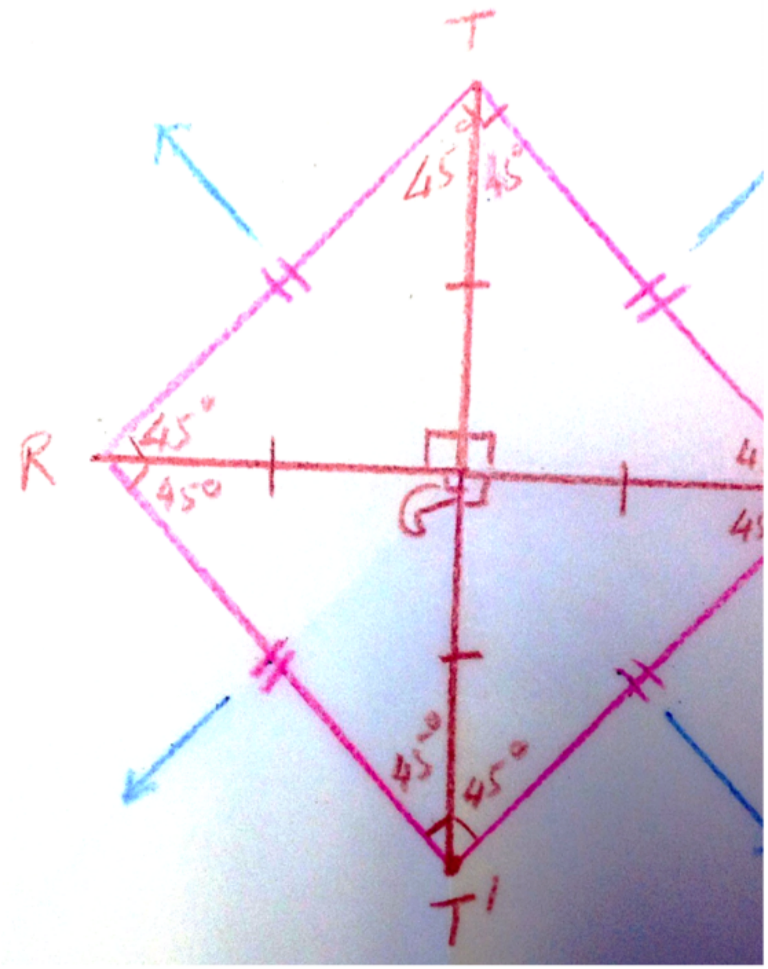
T'



SQUARE

Sides

4 congruent sides

Opp. sides are parallel ($\overline{RT} \parallel \overline{T'R'}$)

Consec. sides are $\perp$ to each other ($\overline{RT} \perp \overline{TR'} \perp \overline{R'T'}$ etc.)

Vertex ∠s

4 congruent, 90° (right) angles

Sum = 360°

Bisected by the diagonals

Diagonals

Form 4 large 45°-45°-90° △s
e.g., △RTT'

Form 4 small 45°-45°-90° △s
e.g., △RTG

Intersect at rt. ∠s (⊥ to each other)

Congruent to each other.

Bisect each other

Bisect the vertex ∠s

Symmetry

2 lines of symm. through vertices of □.

2 " " " bisect sides of □.

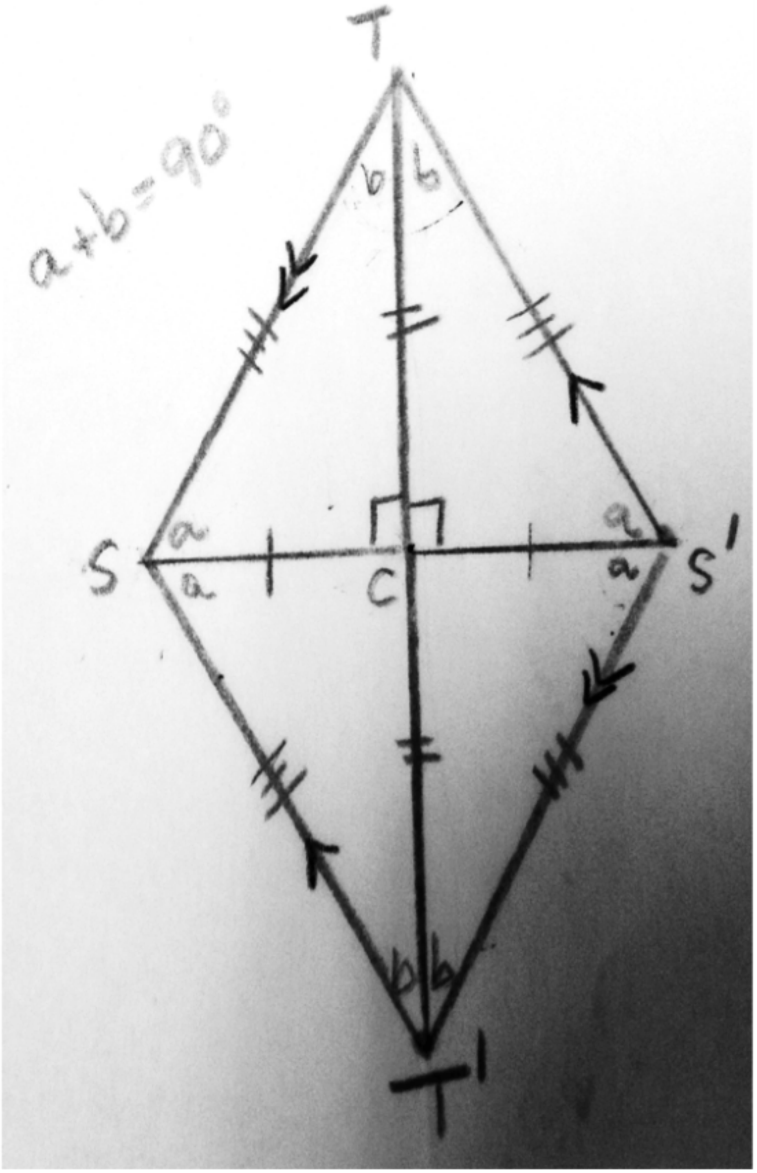
→ forming 4 smaller ≅ squares.

→ forming 4 ≅ rectangles

– 90° (4-fold) rotation symm

– 180° (2-fold) " "

Reflect a scalene right triangle...



Rhombus

Sides
- 4 ≅ sides
- Opp sides are parallel

Vertex ∠s

Opposite ∠s are ≅

Consecutive ∠s are supplementary ($2a + 2b = 180°$)

Sum of 4 ∠s is 360° ($4a + 4b = 360°$)

Bisected by the diagonals

Diagonals
- bisect vertex angles
- Form 4 scalene right Δs
- Form 2 congruent isos. acute Δs
- " " " " obtuse Δs
- Bisect each other } ⊥ bisectors of each other
- ⊥ to each other }
- 2 lines of symmetry

Symmetry

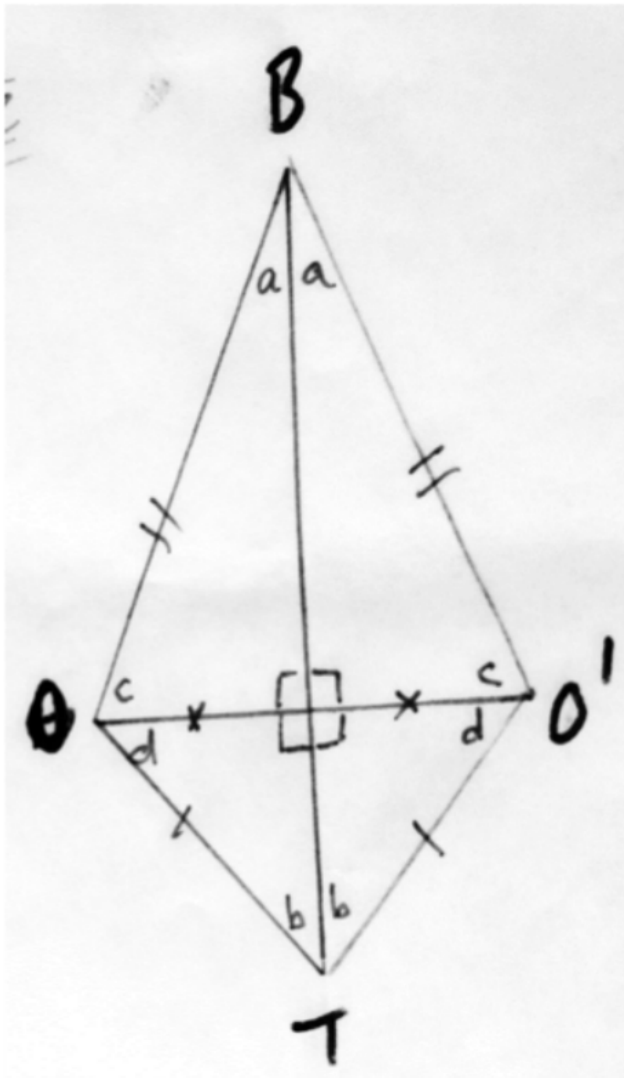
2 lines of symmetry (diagonals)

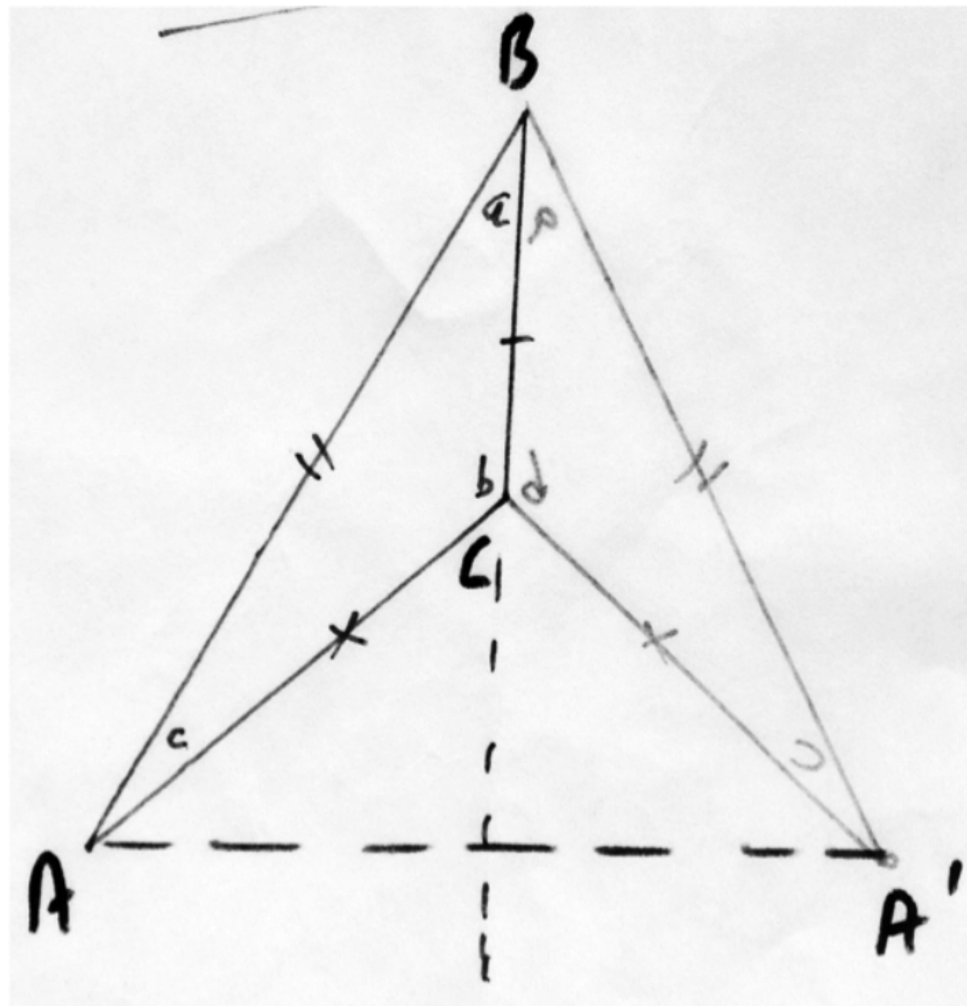
180° (2-fold) rotation symmetry

- Reflect a scalene obtuse or acute triangle

OR

- Reflect a right triangle across its longest side

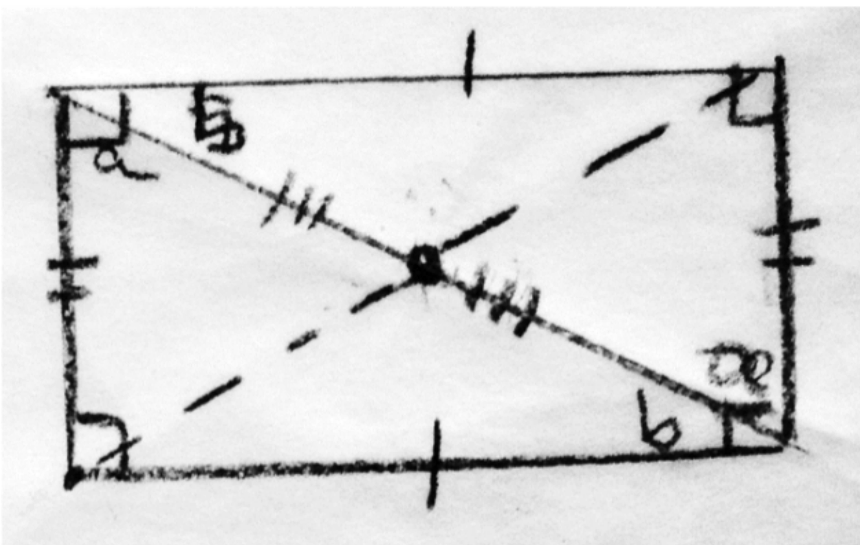


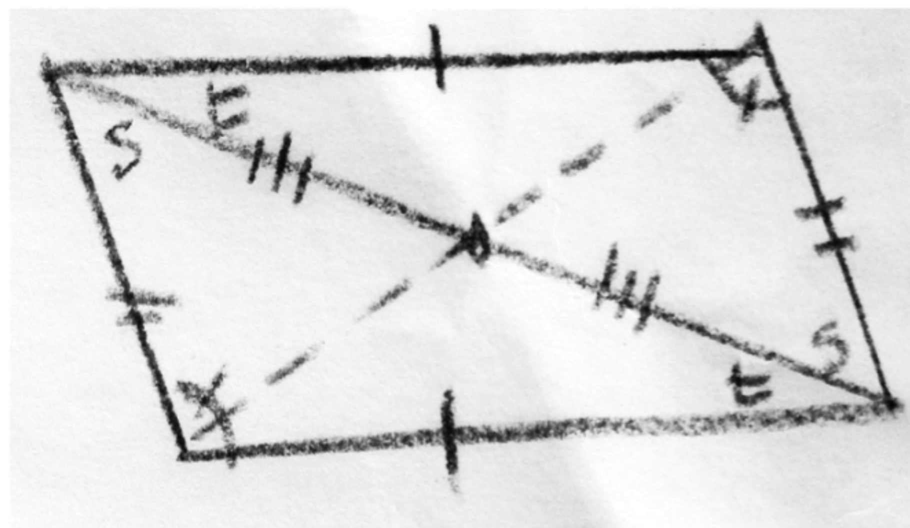


- Rotate a scalene right triangle about the midpoint of its longest side

AND

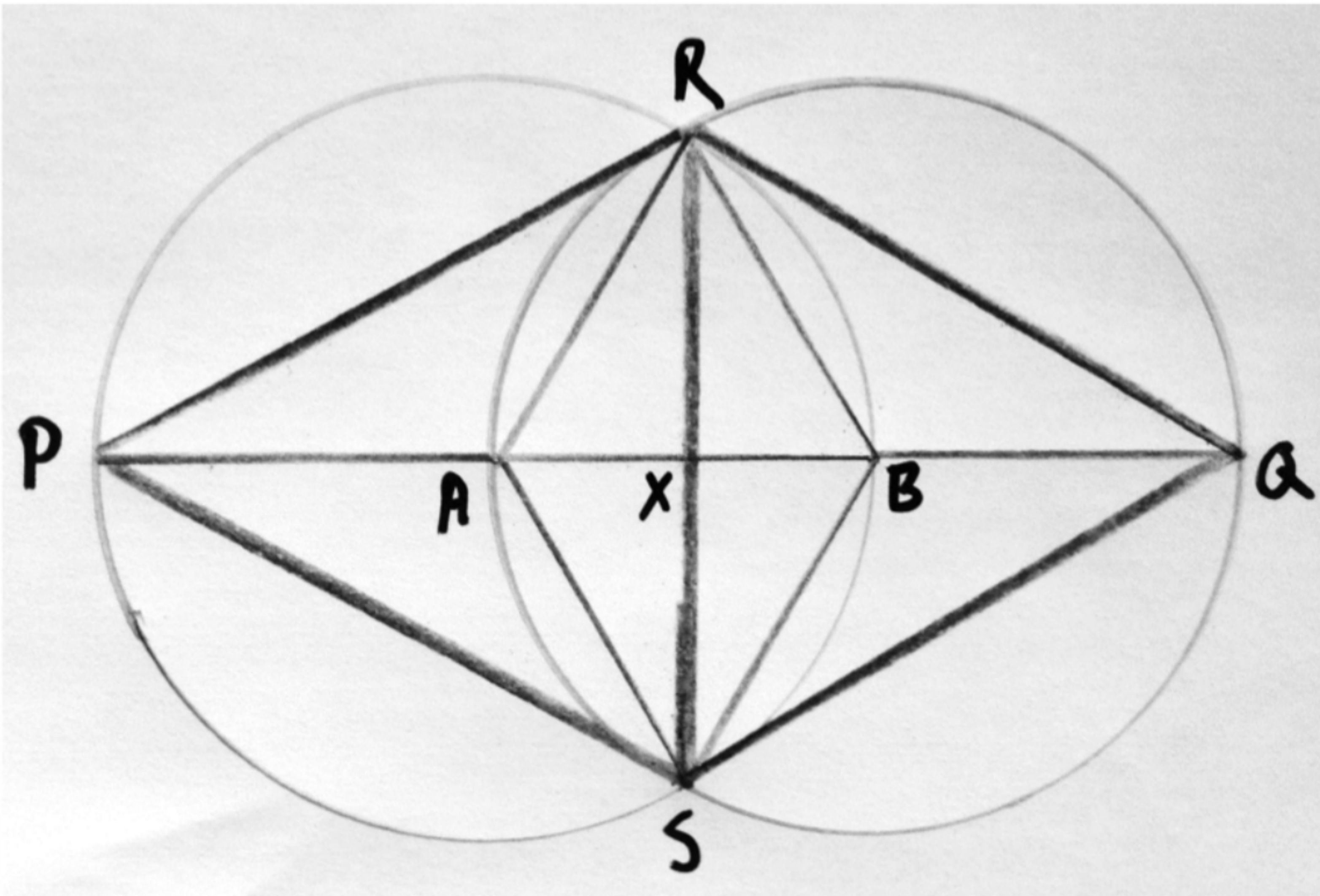
- Rotate a scalene obtuse or acute triangle about the midpoint of a side





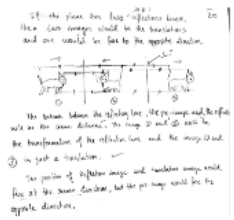
Vesica Pisces

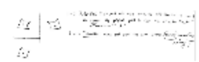
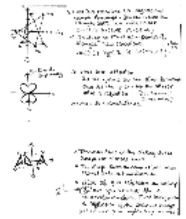
Construct two circles:
Circle A centered on circle B;
Circle B centered on circle A

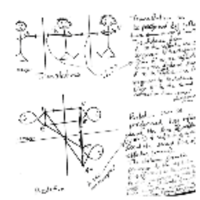


Student Work

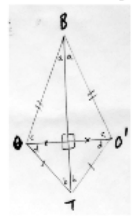
Describe how isometric rotations and translations can be performed using the properties of isometric reflection.
Be sure to describe these transformations in terms of distance and direction that the image "moves" in relation to the position of the pre-image.
Please provide pictures in your descriptions.

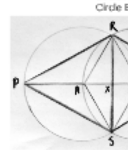



- Reflect a scalen
- Reflect a right tri



Vesica Pisces

Construct
Circle A ce
Circle B c

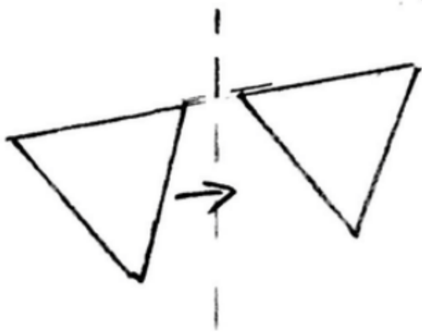


Rotation: Figure can be turned / from one of the original points. The figure can rotate any amount of degrees to the right or left but (must) keep one of the pre-image points constant.

Translation: Figure slides from one spot to anothe[r]. Points do not have to (connect) to pre-ima[ge] points. The figure can slide any direct[ion] whether it be up, down, left or rig[ht]

or skew?

Distance?



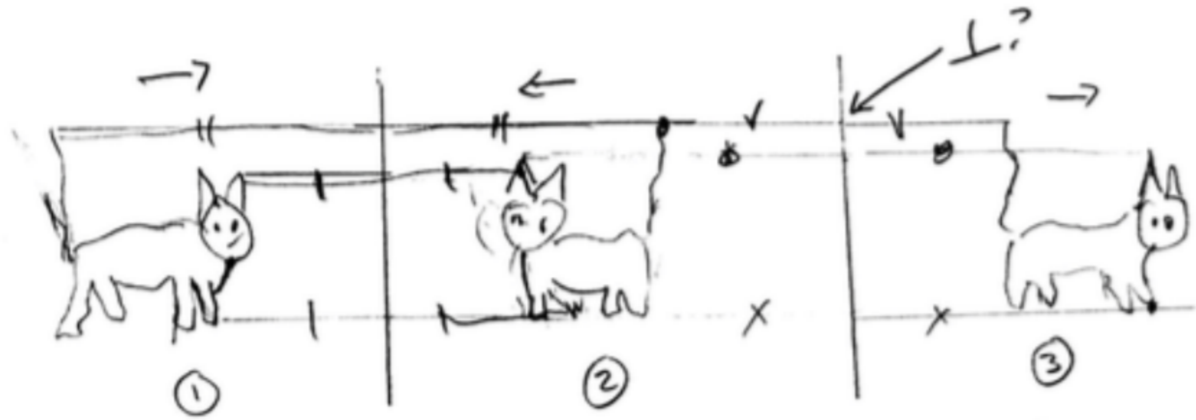
Reflection: Figure flips from one spot to another. The points may have the same ("x") or ("y") value depending on where the line of reflection is, but the (sign) will change.

Distance?
Direction?

Why?

2c

If the plane has two reflections lines, then two images would be the translations and one would be face to the opposite direction.



The Distance between the reflection line, the pre-image and the reflection will be the same distance. The image ① and ② will be the transformation of the reflection line and the image ① and ③ is just a traslation.

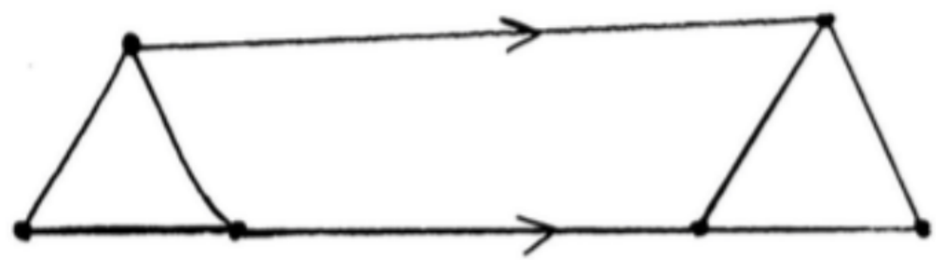
The postion of Reflection image and translation image would face at the same direction, but the pre-image would face the opposite direction.

pictures in your descriptions.

Distance? Direction? Ze

Reflection - is a mirror image of a shape

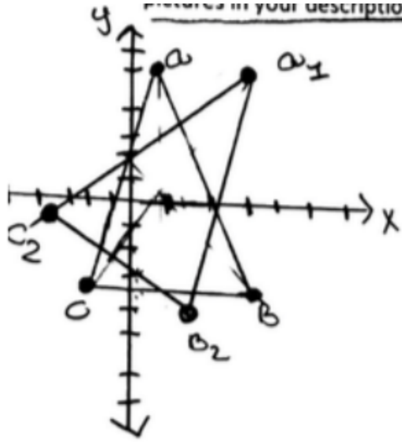
Translation - is a shape that if you take all the points of the shape and move in same direction and same distance. you can Also get a translation from 2 parrallel lines. where is "reflection" in this?



Examples would be if I was to draw a picture on patty paper then fold over and trace I would get a reflection, then if I was to fold over again and trace I would have a translat the image moved but is the same size and when you have 2 parallel lines.

Illust thi

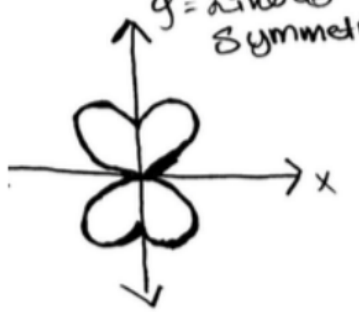
pictures in your descriptions.



* What is a rotations. Use degrees and simple Language. You can rotate the triangle 360°. We will rotate
a=(1,5) B=(3,-4) C=(-2,-4)

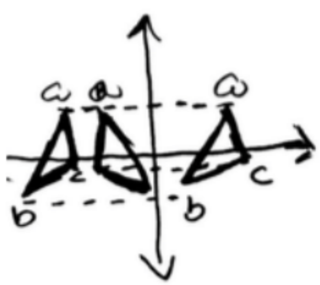
* rotating to find new points of triangle new rotations.
a1(3,5) B2=(2,-5) C=(-4,-1)

How >> ...



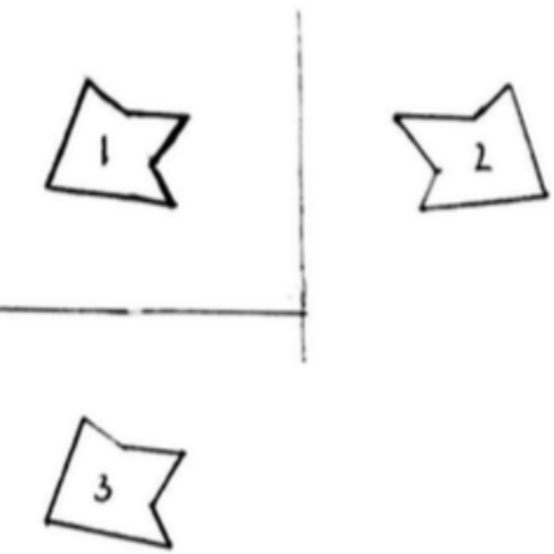
* What is a reflection.
Let the y-axis be the line of symmetry and let the y-axis me the mirror line of reflection. Distance? Direction?

* What is translations.



- Translations is the sliding of the Shape in simple turns.
- The shape looks the same it just have moved into a new Location
- also if you reflected the shape 2! → ③ times you will get ~~the~~ a transalation of the First image and a reflection of the second image.

What are you reflecting across?



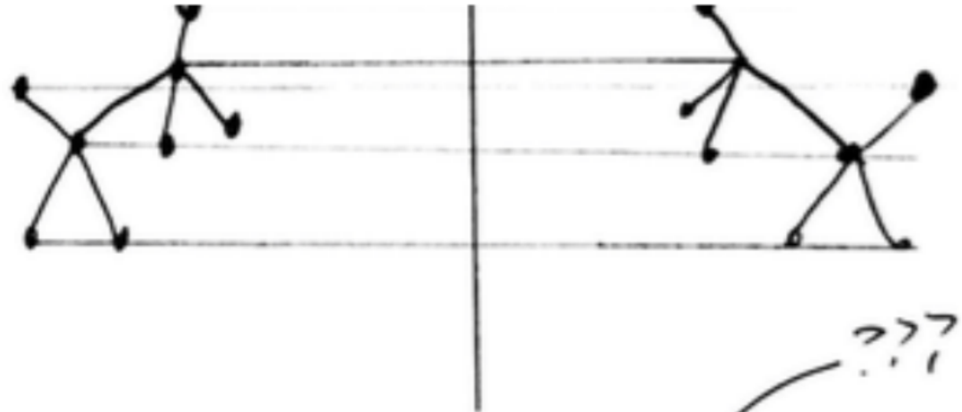
1 to 2 Reflection: Each point will move across the reflection line at two ti

the distance the original point is from the reflection line. ✓

direction?? ⊥?

1 to 3 Translation: Move each point the same distance vertically and/or horizo

why?



Distances:
∥?
⊥?

/20

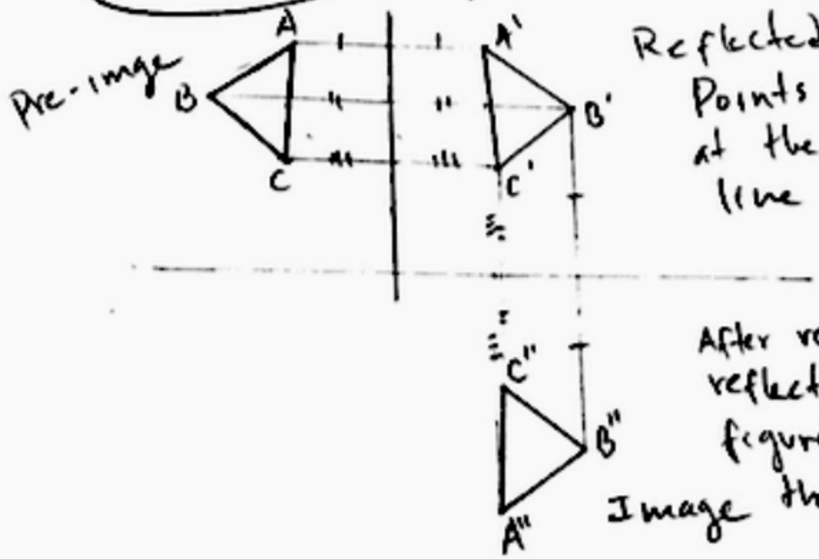
???

Isometric rotations are performed by "spining" the figure with respect to a slope in order to create a new image. The distance from one point in the figure to the slope is the same in both the image and the pre image. We use degrees to refer to a rotation

Translations are performed by reflecting the image over a line or slope. The movement distance is once again the same from the image to the slope or line of translation. This, however, does not apply to a double translation over parallel lines.

?

Why? Isometric rotations can be made by reflection an object across a vertic line and then across a horizontal line, or viceversa.

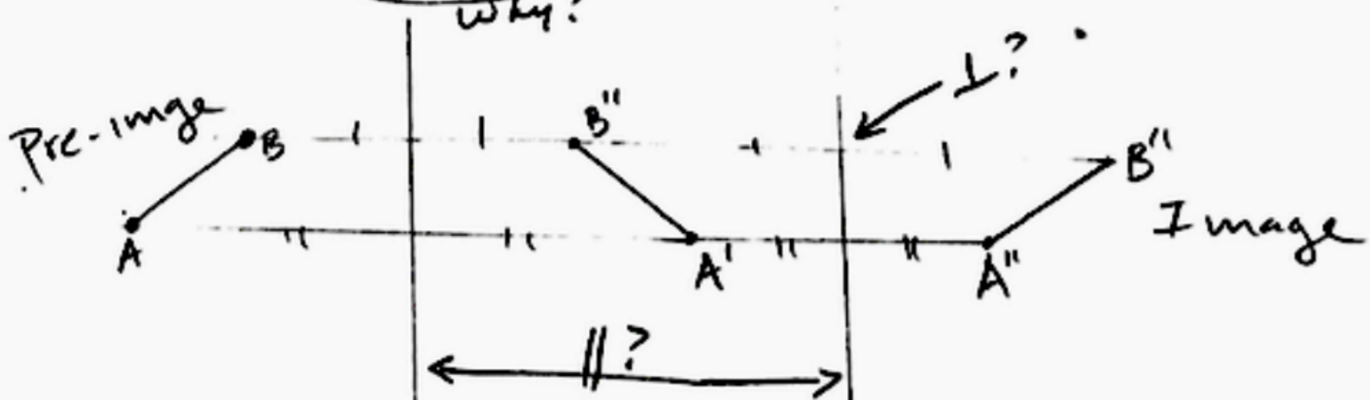


Reflected across vertical line

Points A, B, C are exactly at the same distance from reflection line as points A', B', C'

After reflecting it across the horizontal reflection line, we get a rotated figure that is congruent to the pre-image.

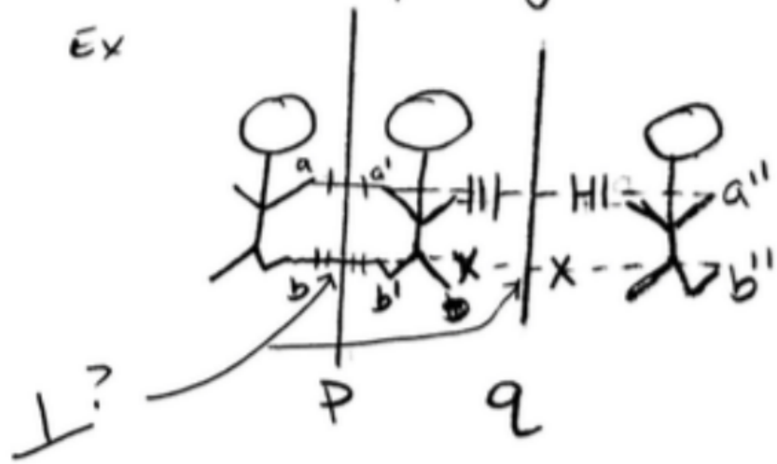
Isometric Translations occur when an object is reflected across two vertical reflection lines. Why?



Pre-Image is translated to Image to $\overline{A''B''}$ after double reflection across vertical lines

When you reflect an image over ~~the x~~ (the same) an axis or (point), twice you get the translation as the third image.

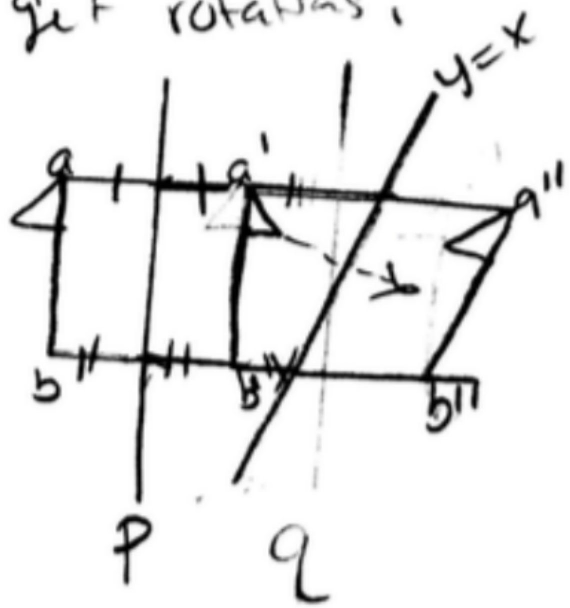
Ex



The third image shows that the first image ~~transformed~~ moved over the distance from a to a''.
You can see that a + a' are equal dist from ~~line~~ line p and a' and a'' are equ distance from q.
This is an example with just twice reflections more can be done.

Is p // q ?

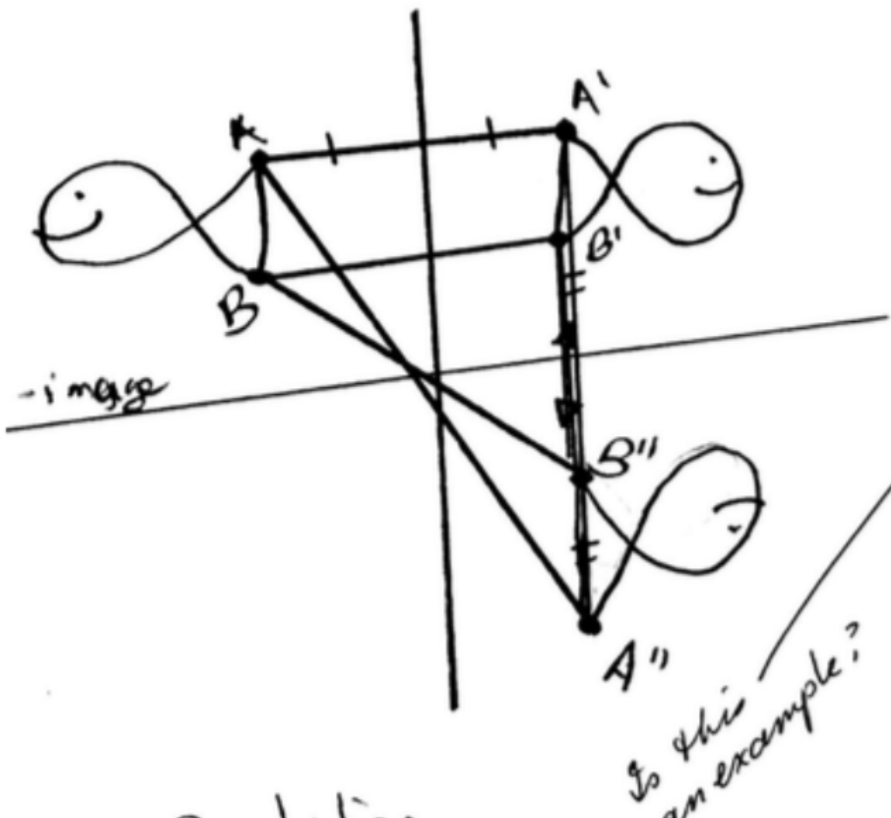
When you reflect the image across two differt lines. (non-//?) (intersecting??)
get rotations.



Here the picture is moving ~~but~~ and facing the same as in the translation but its also Rotated on a line at y=x.
You can also rotate around a pt such as pt b.

Translations

Translation can be performed by reflec[ting] twice ← across ∥ lines?
The distance from A to the reflection line is congruent to the distance from A' to the reflecti[on] line, the distance from A' to the second reflection line is congruent to the distance from A'' to the second reflection line. Total distance A → ...

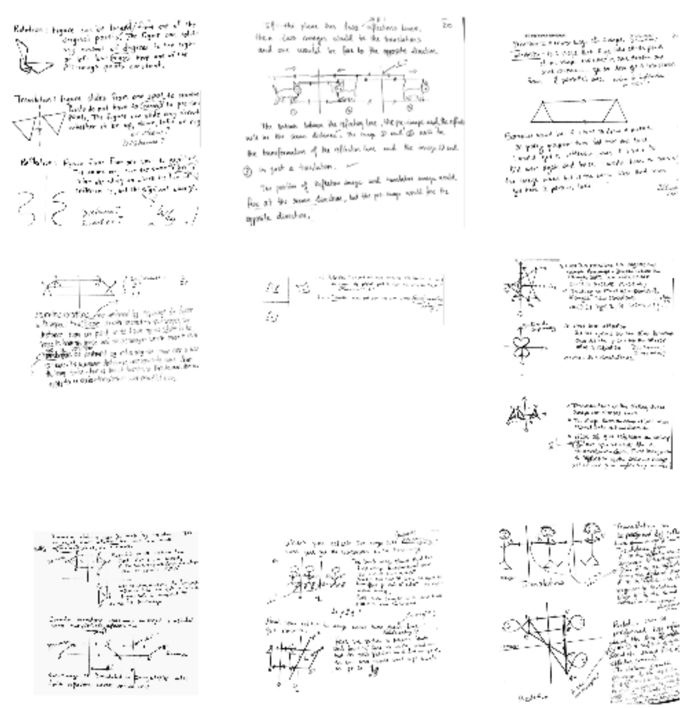


Rotation

Rotation can be performed by reflec[ting] about the line of reflec[tion] (y-axis) & reflecting agai[n] about the second line of reflection (x-axis).
The distance from the pre image to the reflection line (y-axis) is the same ... the distance from the reflecte[d] image. The distance from the final image to the reflection line ... the distance from second ... reflection line (x-axis)

Student Work

Describe how isometric rotations and translations can be performed using the properties of isometric reflection.
Be sure to describe these transformations in terms of distance and direction that the image "moves" in relation to the position of the pre-image.
Please provide pictures in your descriptions.



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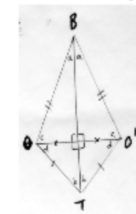

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Circle A ce
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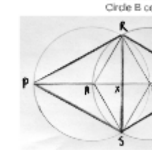

Undergraduate Preservice Teachers

Jacqueline Sack
Associate Professor

sackj@uhd.edu

Judith Quander
Assistant Professor

quanderr@uhd.edu

University of Houston DOWNTOWN