Emergent modeling: From chains of signification to cascades of artifacts

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Boulder CO

Agenda

Emergent modeling…

…From chains of signification…

- History and use in RME

- Some problems

…To cascades of artifacts

- Resolution of problems

- A new way to think about emergent modeling

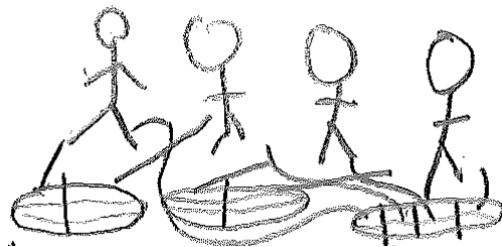**Q:** How can formal mathematics become "experientially real" for students?

A: Formal mathematics should *emerge* from experientially real activity via *modeling*

Experientially real activity:
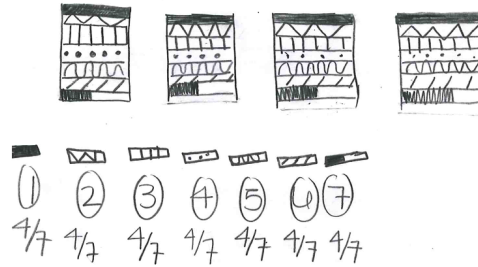Sharing

Sharing three sub
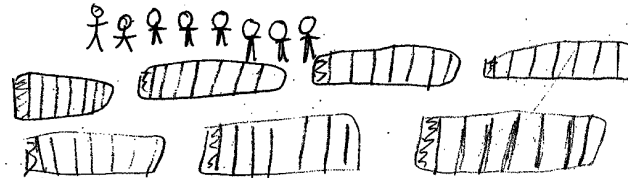sandwiches among four
people

Model *of*
activity

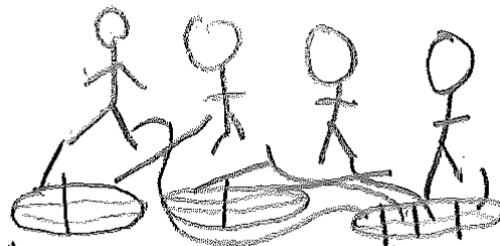Experientially real activity:
Sharing

Sharing four pounds of chicken food among seven chickens

Sharing seven sub sandwiches among eight people

Sharing three sub sandwiches among four people
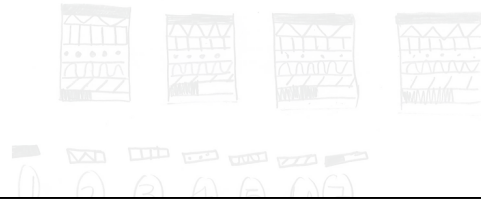
Experientially real activity: Sharing

Model *for* activity

Model *of* activity
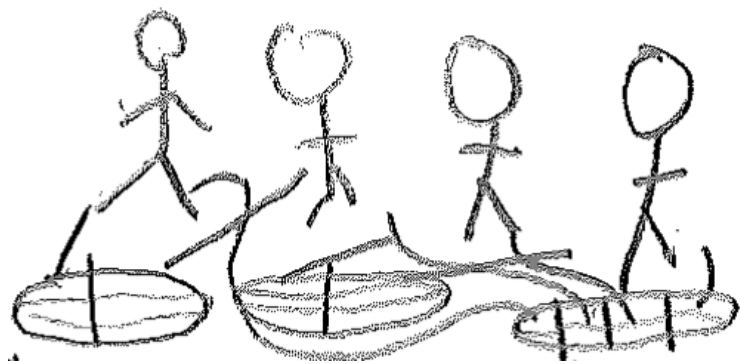
In practice, the model in the emergent modeling heuristic is actually shaped as a series of consecutive symbolizations or tools that can be described as […] a *chain of signification*

(Gravemeijer, 2004, p.117, emphasis added)

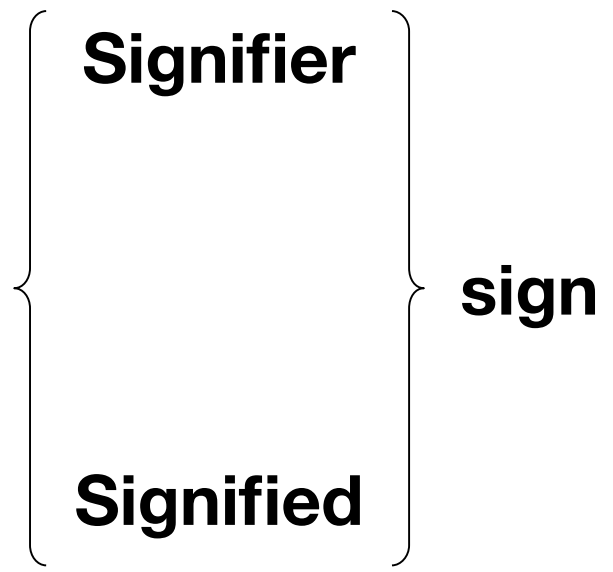The picture **Signifier**

signifies

the activity **Signified**
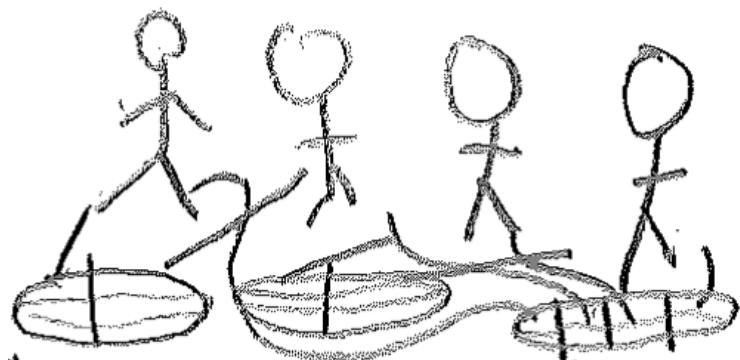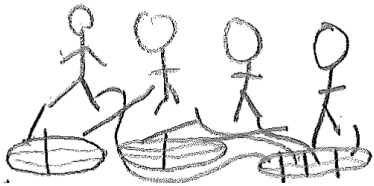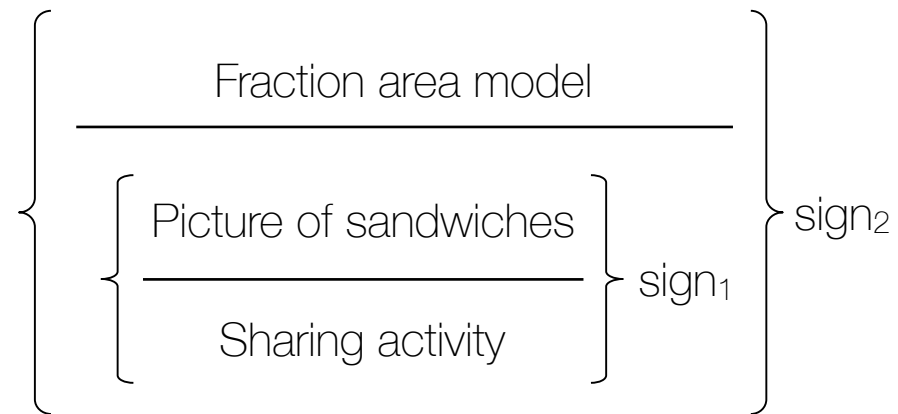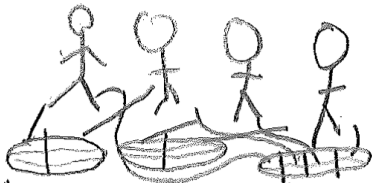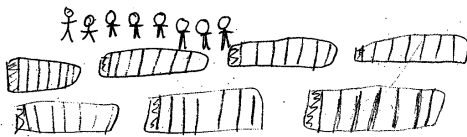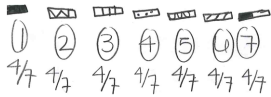
Signifier

sign

Signified

$$\left\{ \frac{\text{Picture of sandwiches}}{\text{Sharing activity}} \right\} \text{sign}_1$$

Fraction area model

Picture of sandwiches

Sharing activity

$sign_1$

$sign_2$

Fraction-as-quotient

Fraction area model

Picture of sandwiches

Sharing activity

$sign_1$

$sign_2$

$sign_3$
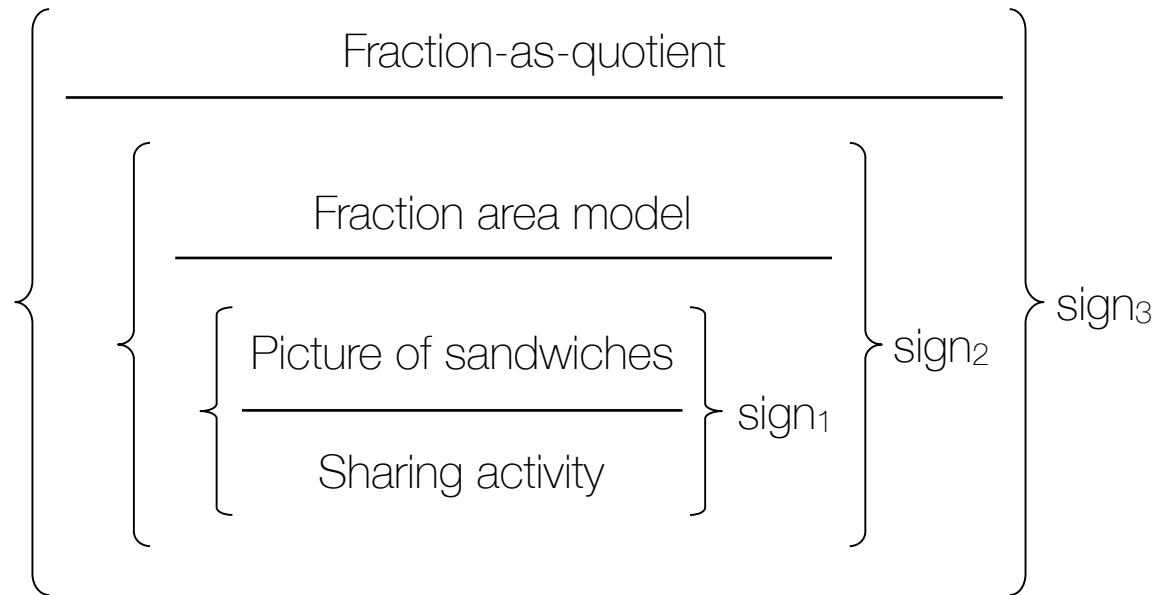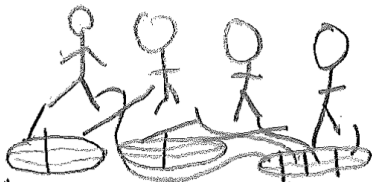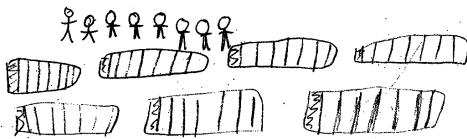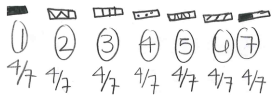
Chains of signification

Saussure (late 19th)

a dyadic sign

$$\left\{ \frac{\text{Signified}}{\text{Signifier}} \right\} \text{sign}$$

Chains of signification

Saussure (late 19th)

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Lacan (mid 20th)

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Chains of signification

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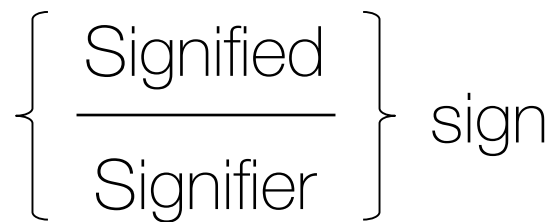
$$\left\{ \frac{\text{Signifier}_2}{\left\{ \dfrac{\text{Signifier}}{\text{Signified}} \right\} \text{sign}} \right\} \text{sign}_2$$
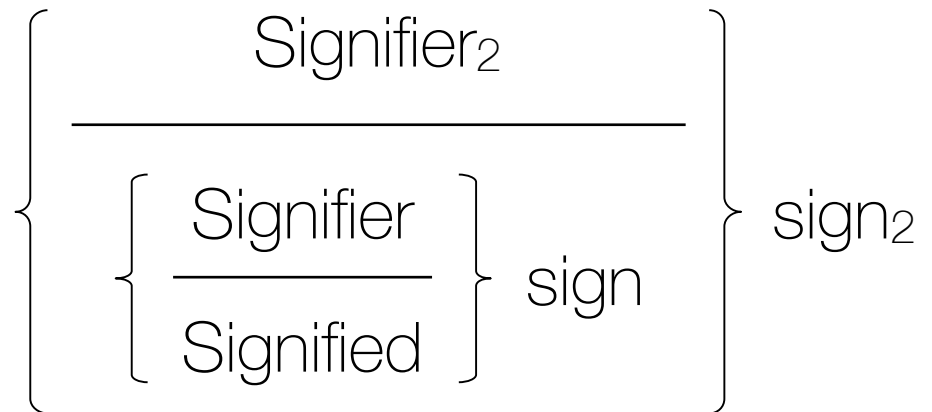
Chains of signification

Lacan (mid 20th)
a chain of signifiers
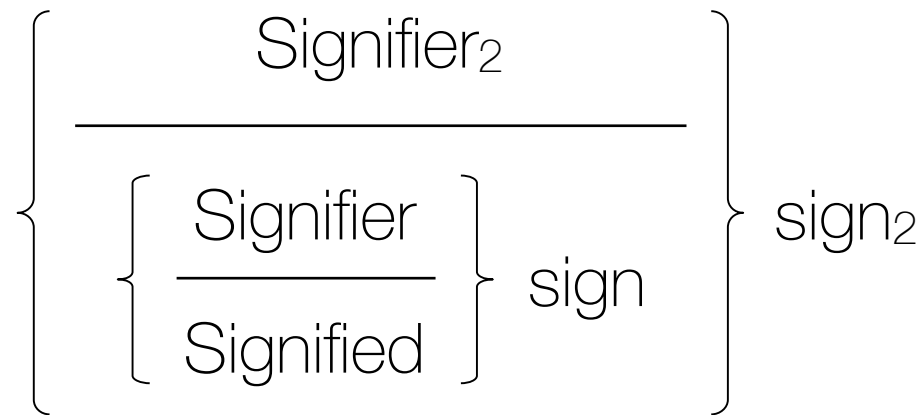
$$\left\{ \frac{\text{Signifier}_2}{\left\{ \frac{\text{Signifier}}{\text{Signified}} \right\} \text{sign}} \right\} \text{sign}_2$$

Walkerdine (1988)

- "introduced" chain of signification to mathematics education

- Signs <—> discourse

- How *children* get produced as signifiers in a chain of signification

Chains of signification

Walkerdine (1988)

- "introduced" chain of signification to mathematics education

- Signs <—> discourse

- How produced as signifiers in a chain of signification

RME (1997)

- Emergent modeling constitutes a chain of signification

- Chain of signification is reflexively related to mathematical reality

Chains of signification

RME (1997)

- Emergent modeling constitutes a chain of signification

- Chain of signification is reflexively related to mathematical reality

Early number (e.g., Cobb et al., 1997)

Statistics (e.g., Cobb, 2002)

Calculus (e.g., Doorman and Gravemeijer, 2009)

Differential equations (Keene et al, 2012)

Some problems

Process

Product

Some problems

Process

new productions emerge as

One-to-one mapping

Product

Some problems

Process

new productions emerge as ~~One to one mapping~~ Coordinated assemblies

Product

Some problems

Process

new productions
emerge as

~~One to one
mapping~~

Coordinated
assemblies

Product

what emerges
from activity?

One model
Consecutive symbolizations

Some problems

Process

new productions
emerge as

~~One to one mapping~~

Coordinated assemblies

Product

what emerges
from activity?

~~One model~~
~~Consecutive symbolizations~~

Multiple models, tools, & strategies

Some problems

Process

new productions
emerge as

~~One to one
mapping~~

Coordinated
assemblies

Product

what emerges
from activity?

~~One model~~
~~Consecutive symbolizations~~

Multiple models,
tools, & strategies

the mathematical
world is

Hierarchical
and siloed

Some problems

Process

new productions emerge as	~~One to one mapping~~	Coordinated assemblies

Product

what emerges from activity?	~~One model~~ ~~Consecutive symbolizations~~	Multiple models, tools, & strategies
the mathematical world is	~~Hierarchical and siloed~~	Relational and web-like

$\div 12 \overline{)\,3\,}$

$\dfrac{3}{12}$

Fraction-as-quotient

Fraction area model

Picture of sandwiches

Sharing activity

$sign_1$

$sign_2$

$sign_3$

$$\div 12 \overline{)}^{3}$$

$$\frac{3}{12}$$

① ② ③ ④ ⑤ ⑥ ⑦
4/7 4/7 4/7 4/7 4/7 4/7 4/7

$\div 12 \overline{)}\,3$

$\dfrac{3}{12}$

Coordinating: Single-item partitioning

Coordinating: partitioning by progressive parts using benchmark fractions

Pre-coordinating strategies

3 pizzas cost 12 dollars

$\div 12 \Big($ 3 pizzas cost 12 dollars $\Big) \div 12$

$\frac{3}{12}$ pizzas cost 1 dollar

Coordinating: Single-item partitioning

Coordinating: partitioning by progressive parts using benchmark fractions

Pre-coordinating strategies

$\div 12 \overline{)} \quad 3$

$\dfrac{3}{12}$

Coordinating: Single-item
partitioning

$\div 12 \overline{)}$ 3 pizzas cost 12 dollars $\Big) \div 12$

$\dfrac{3}{12}$ pizzas cost 1 dollar

Coordinating: partitioning by
progressive parts using
benchmark fractions

There are 6 bottles of Coca-Cola in 1 case

There are 12 bottles of Coca-Cola in 2 cases

There are 24 bottles of Coca-Cola in 4 cases

There are 48 bottles of Coca-Cola in 8 cases

Pre-coordinating strategies

$$\div 12 \overline{)3}$$

$$\frac{3}{12}$$

Coordinating: Single-item partitioning

"Find one" strategy

$\div 12 \overline{)\text{3 pizzas cost 12 dollars}}$ $\div 12$

$\frac{3}{12}$ pizzas cost 1 dollar

Multiplicative: any factor

Coordinating: partitioning by progressive parts using benchmark fractions

There are 6 bottles of Coca-Cola in 1 case

There are 12 bottles of Coca-Cola in 2 cases

There are 24 bottles of Coca-Cola in 4 cases

There are 48 bottles of Coca-Cola in 8 cases

Multiplicative: doubling and halving

Additive strategies

Pre-coordinating strategies

Area model

Partitioning strategies

Ratio table

Proportional reasoning strategies

Coordinating: Single-item partitioning

Coordinating: partitioning by progressive parts using benchmark fractions

Pre-coordinating strategies
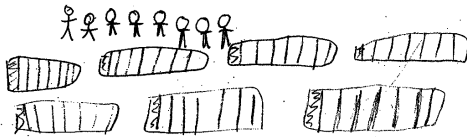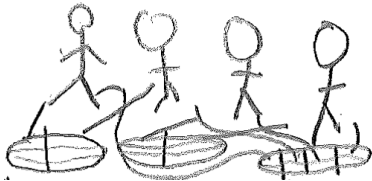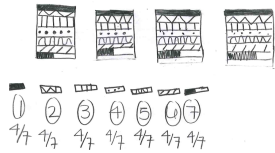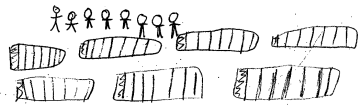
$\div 12 \begin{pmatrix} 3 \\ \\ \dfrac{3}{12} \end{pmatrix}$

$\div 12 \begin{pmatrix} 3 \text{ pizzas cost 12 dollars} \\ \div 12 \\ \dfrac{3}{12} \text{ pizza cost 1 dollar} \end{pmatrix}$

There are ___6___ bottles of Coca-Cola in ___1___ case

There are ___12___ bottles of Coca-Cola in ___2___ cases

There are ___24___ bottles of Coca-Cola in ___4___ cases

There are ___48___ bottles of Coca-Cola in ___8___ cases

"Find one" strategy

Multiplicative: any factor

Multiplicative: doubling and halving

Additive strategies

Area model ↔ Partitioning strategies — Ratio table ↔ Proportional reasoning strategies

$\div 12 \overline{\smash{\big)}\,} \quad 3$

$\dfrac{3}{12}$
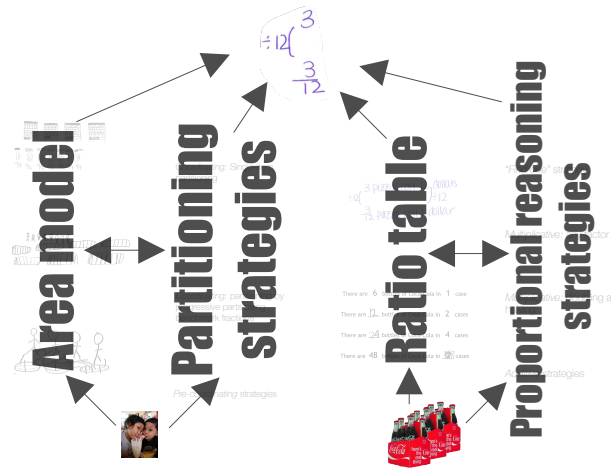
4 pizzas can feed 7 people

_.57_ pizzas can feed 1 person

$) \div 7$

Show your work or explain your reasoning:

1 Person

$= \dfrac{4}{7}$ of a

$\dfrac{4}{7} = .57$ of a Pizza

of Pizza
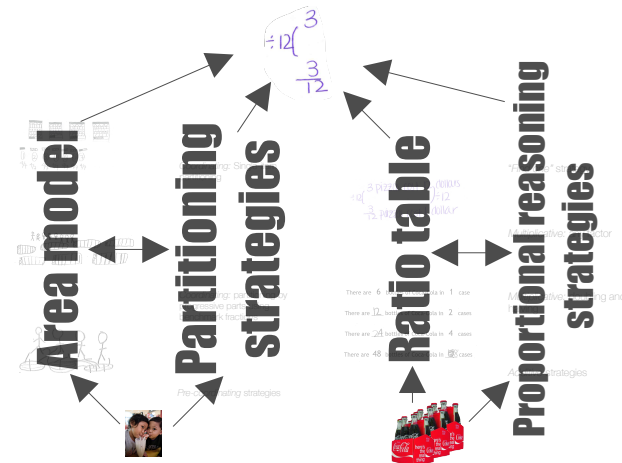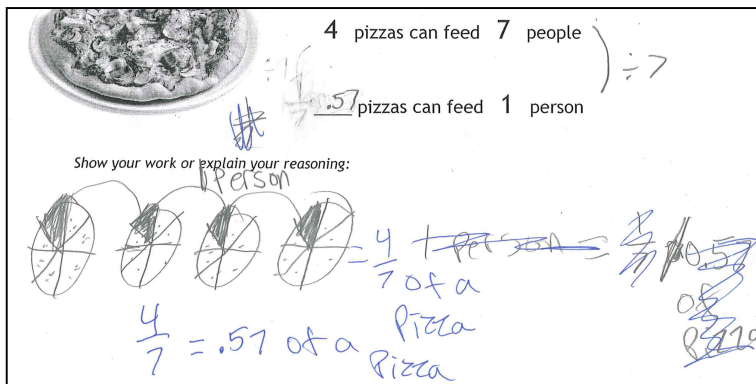
Process
new productions emerge as ~~One-to-one mapping~~ Coordinated assemblies

Product

what emerges from activity? ~~One model~~ ~~Consecutive symbolizations~~ Multiple models, tools, & strategies

the mathematical world is ~~Hierarchical and siloed~~ Relational and web-like

Towards a new description

In practice, the model in the emergent modeling heuristic is actually shaped as a series of consecutive symbolizations or tools that can be described as a *cascade of inscriptions* or a *chain of signification*

(Gravemeijer, 2004, p.117, emphasis added)

Towards a new description

In practice, the model in the emergent modeling heuristic is actually shaped as a series of consecutive

cascade of inscriptions

or a

(Gravemeijer, 2004, p.117, emphasis added)

cascade of inscriptions

I was struck, in a study of a biology laboratory, by the way in which many aspects of laboratory practice could be ordered by looking not at the scientists' brains (I was forbidden access!), at the cognitive structures (nothing special), nor at the paradigms (the same for thirty years), but at the transformation of rats and chemicals into paper.

(Latour, 1986, p.3)

cascade of inscriptions

- The work of science is to produce inscriptions.

(Latour, 1986, p.3)

cascade of inscriptions

- The work of science is to produce inscriptions.

- Inscriptions come from the world

(Latour, 1986)

cascade of inscriptions

- The work of science is to produce inscriptions.

- Inscriptions come from the world

- Material: Immutable, mobile, combinable

(Latour, 1986)

cascade of inscriptions

Domains which are far apart become literally inches apart … [files] can be arrayed in cascade: files of files can be generated and this process can be continued until a few men consider millions as if they were in the palms of their hands. … It is hard to overestimate the power that is gained by concentrating files written in a homogeneous and combinable form

(Latour, 1986, p. 28)

cascade of inscriptions

- The work of science is to produce inscriptions.

- Inscriptions come from the world

- Material: Immutable, mobile, combinable

- New inscriptions are produced as coordinated assemblies of old inscriptions in a cascade

(Latour, 1986, p. 28

cascade of inscriptions

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(Latour, 1986, p. 28

Artifact

Accomplishments of past human activities that serve as resources for future activities. (Cole, 2006, 2010)

Artifact

Accomplishments of past human activities that serve as resources for future activities. (Cole, 2006, 2010)

Ancient?

Material?

Artifact

Accomplishments of past human activities that serve as resources for future activities. (Cole, 2006, 2010)

~~Ancient?~~ Emerge and change on many timescales— including the microgenetic time of a classroom (Schwarz & Hershkowitz, 2001)

~~Material?~~ Simultaneously material and conceptual (Cole & Levitin, 2000; Ilyenkov, 1977, 1979)

Artifact

In being created as an embodiment of purpose and incorporated into life activity in a certain way – being manufactured for a *reason* and put into *use* – the object acquires a significance. This significance is the 'ideal [conceptual] form' of the object, a form that includes not a single atom of the tangible physical substance that possesses it.

(Bakhurst, 1991, p. 182)

Artifact

Accomplishments of past human activities that serve as resources for future activities. (Cole, 2006, 2010)

Emerge and change on many timescales—including the microgenetic time of a classroom (Schwarz & Hershkowitz, 2001)

Simultaneously material and conceptual
(Cole & Levitin, 2000; Ilyenkov, 1977, 1979)
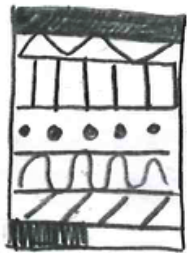
Coordinating: Single-item partitioning

$$\div 12 \left(\begin{array}{l} 3 \text{ pizzas cost } 12 \text{ dollars} \\ \frac{3}{12} \text{ pizzas cost } 1 \text{ dollar} \end{array} \right) \div 12$$

"Find one" strategy

$$\frac{3}{12}$$

Emergent modeling as a **cascade of artifacts**

coordinate & assemble

Mathematical activity

Social interaction
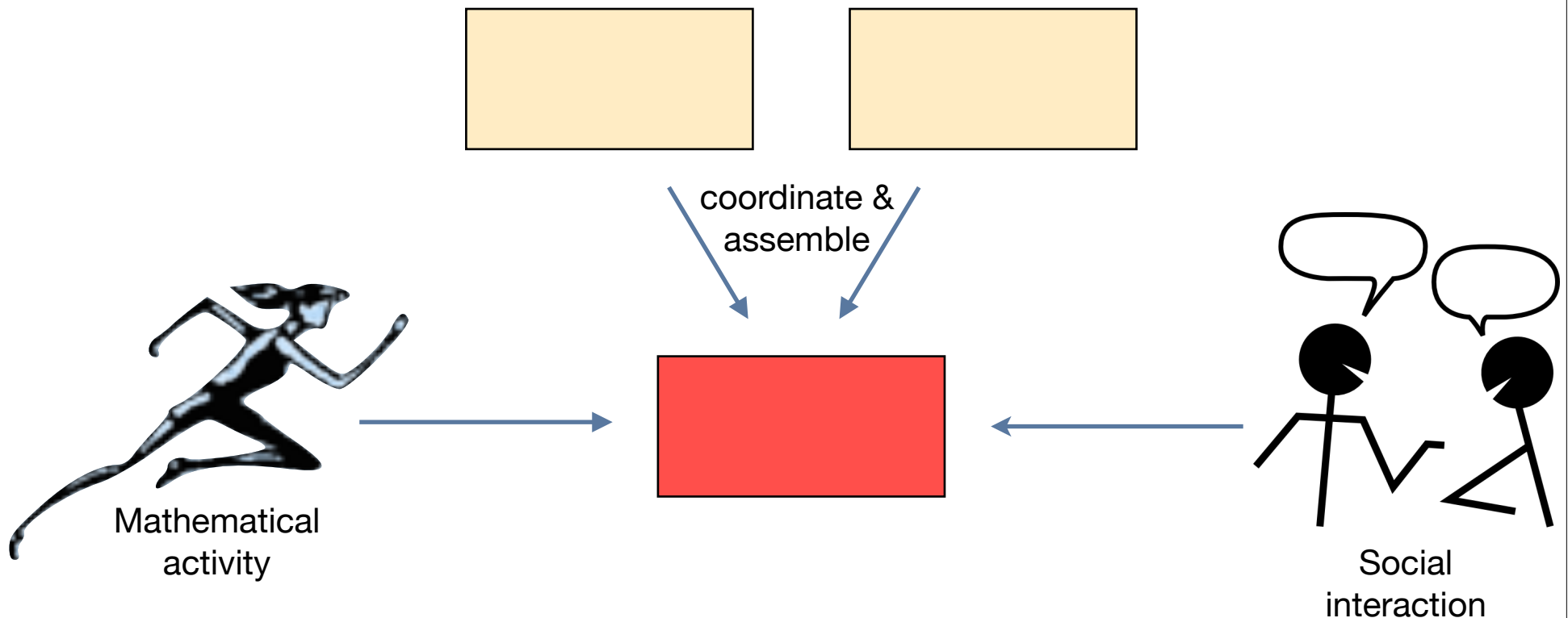
Example:
A cascade of artifacts for slope

slope

meaningful?

procedural

physical

Stump, 2001
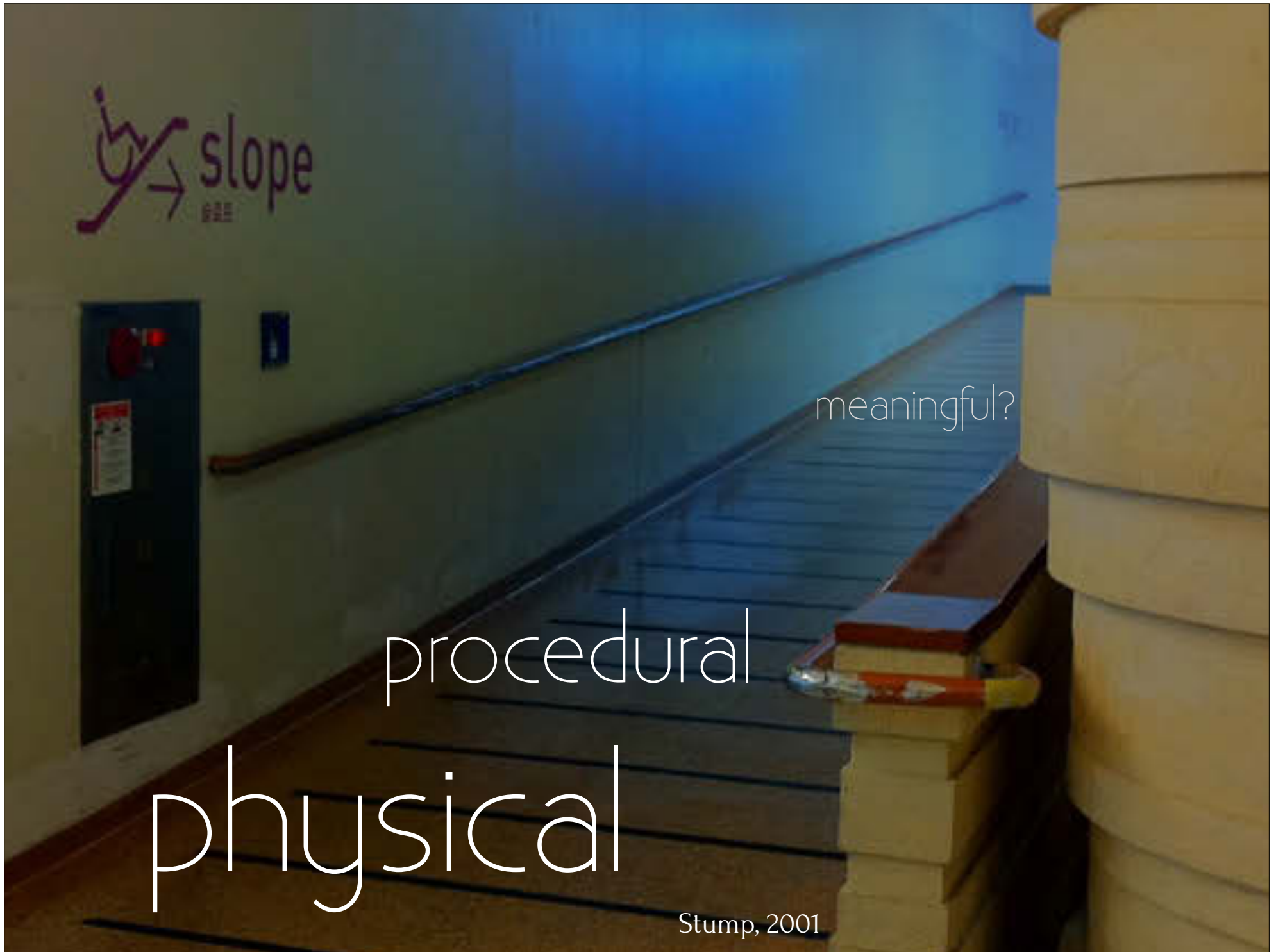
How do students make slope meaningful?

slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property |

Stump, 1999

slope

Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property

$$y = ax + b$$

Stump, 1999

slope

Parametric coefficient	**Algebraic ratio**	Rate of change	Geometric ratio	Physical property

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Stump, 1999

slope
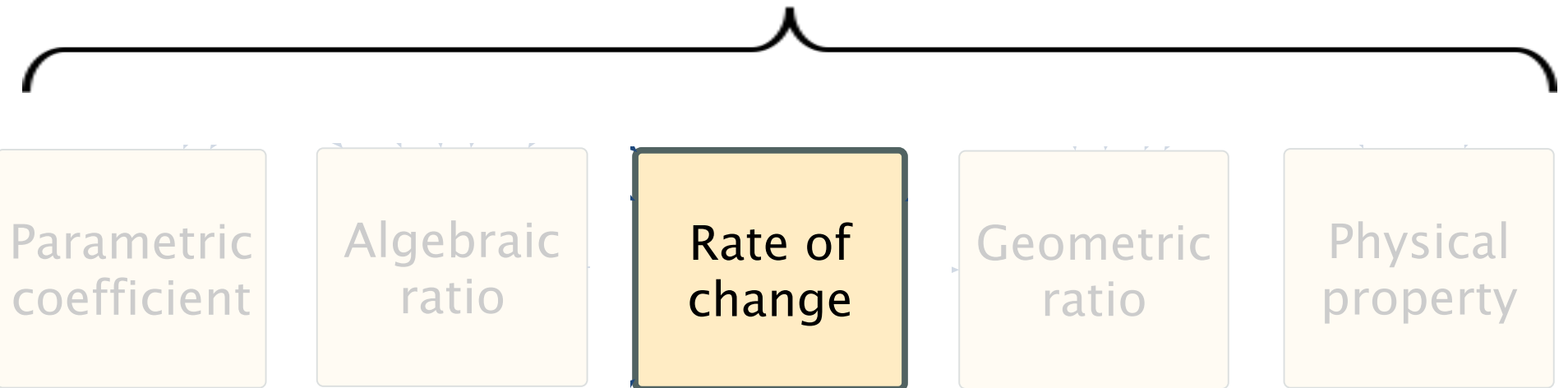
Parametric coefficient | Algebraic ratio | **Rate of change** | Geometric ratio | Physical property

Stump, 1999

slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | Physical property |

$$\frac{\text{rise}}{\text{run}}$$

Stump, 1999

slope

Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | **Physical property**
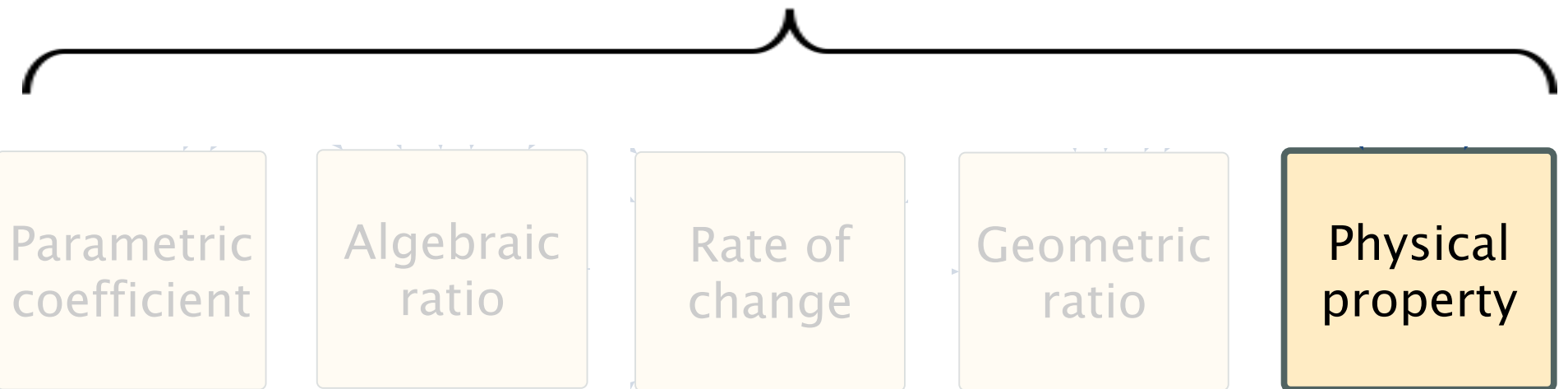
"steepness"

Stump, 1999

slope

```
┌──────────────┐ ┌──────────────┐ ┌──────────────┐ ┌──────────────┐ ┌──────────────┐
│  Parametric  │ │  Algebraic   │ │   Rate of    │ │  Geometric   │ │   Physical   │
│ coefficient  │ │    ratio     │ │   change     │ │    ratio     │ │   property   │
└──────────────┘ └──────────────┘ └──────────────┘ └──────────────┘ └──────────────┘
```

Stump, 1999

slope

| Parametric coefficient | Algebraic ratio | Rate of change | Geometric ratio | **Physical property** |

Why not steepness?
- Begging to be organized?
- Can be organized by all five sub-constructs?
  - y=ax+b

slope

| Parametric coefficient | Algebraic ratio | **Rate of change** | Geometric ratio | Physical property |
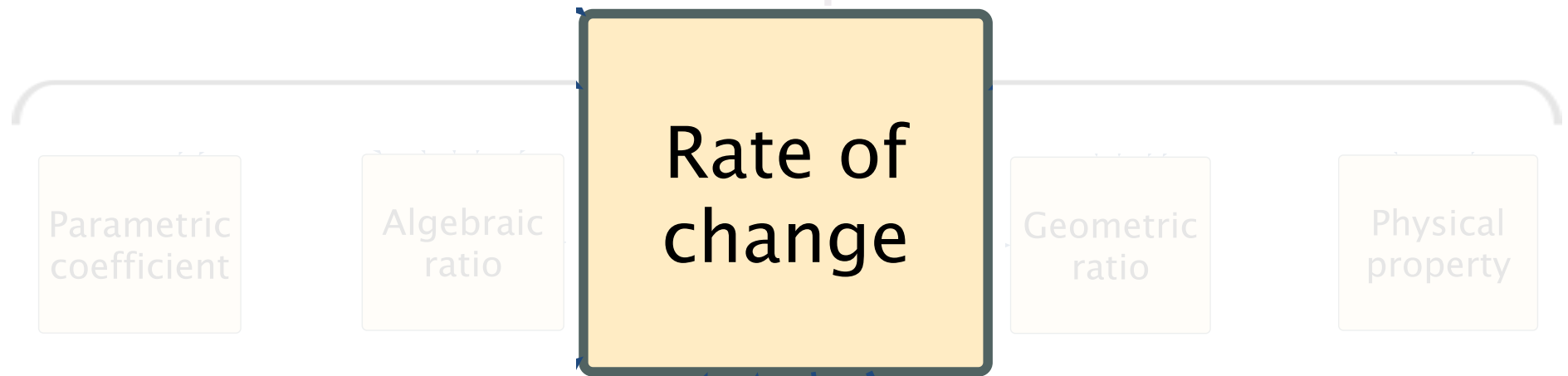
Why rate of change?
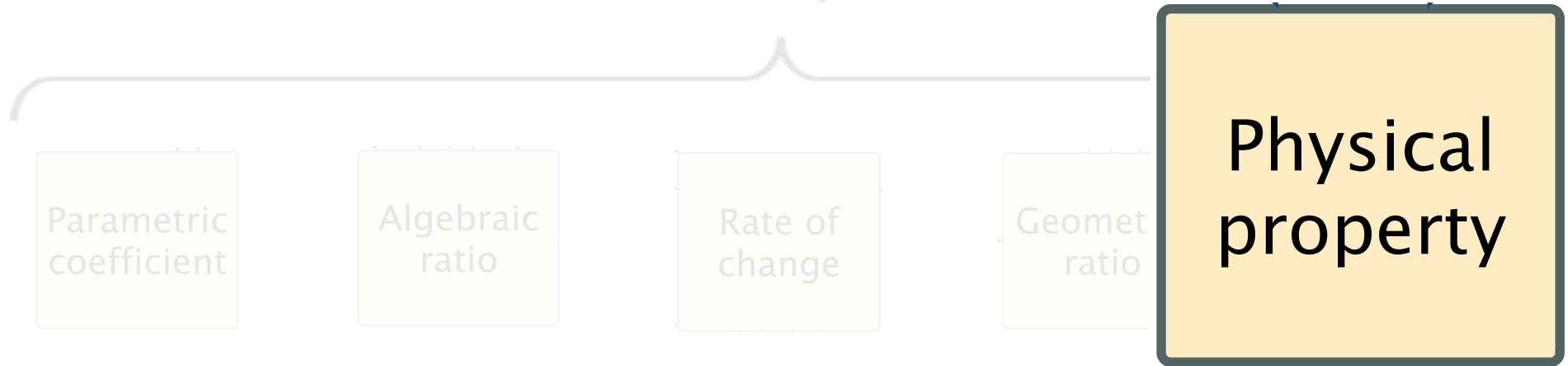
- Begging to be organized :: predicting the future
- Robust :: one of five NCTM "key concepts"
  - Starts with proportional reasoning
  - All sub-constructs of slope can be built from there

instructional
starting point

Ratio table	"Find one" strategy	Intensive units	Fraction as quotient

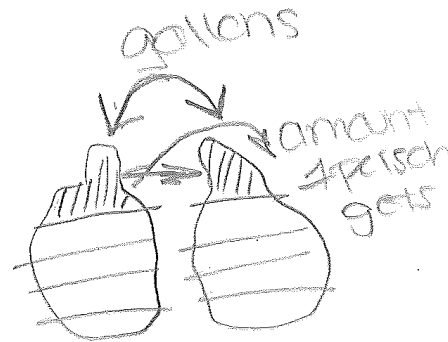1. After a race, five people shared two gallons of water equally. How much water did each person receive?

Show your work or explain your reasoning:

State your final answer using units: __2/5__ __gallons__ *per* __person__.

cascade of artifacts

cascade of artifacts

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

Unit rate strategy

Parametric coefficient

Algebraic ratio

Geometric ratio

Physical property

cascade of artifacts

Ratio table

"Find one" strategy

Intensive units

Fraction as quotient

Number line

Algebraic equations

Function table

Graphs in coordinate plane

Rate of change

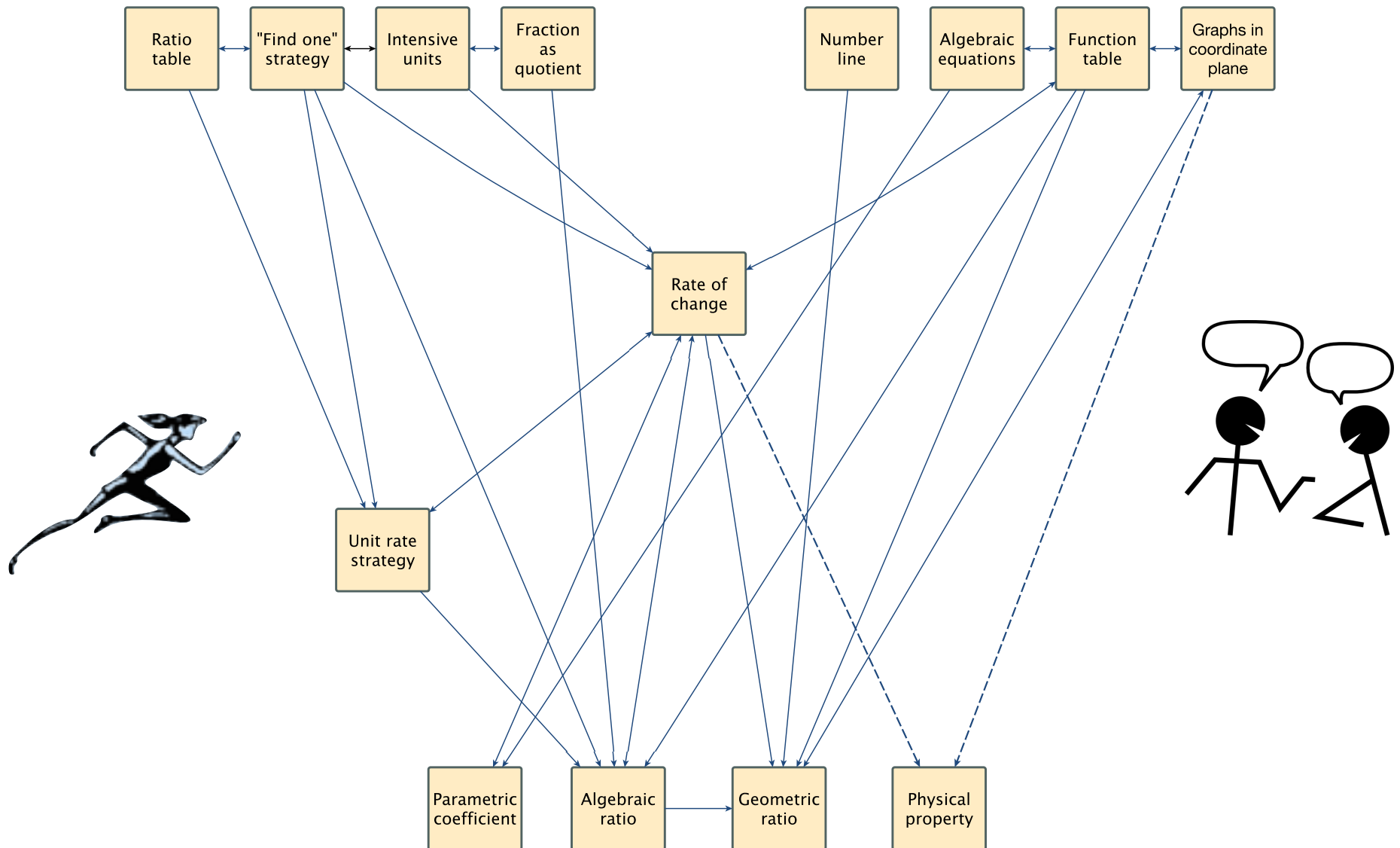Unit rate strategy

Algebraic ratio

Parame coeffici

Physical property
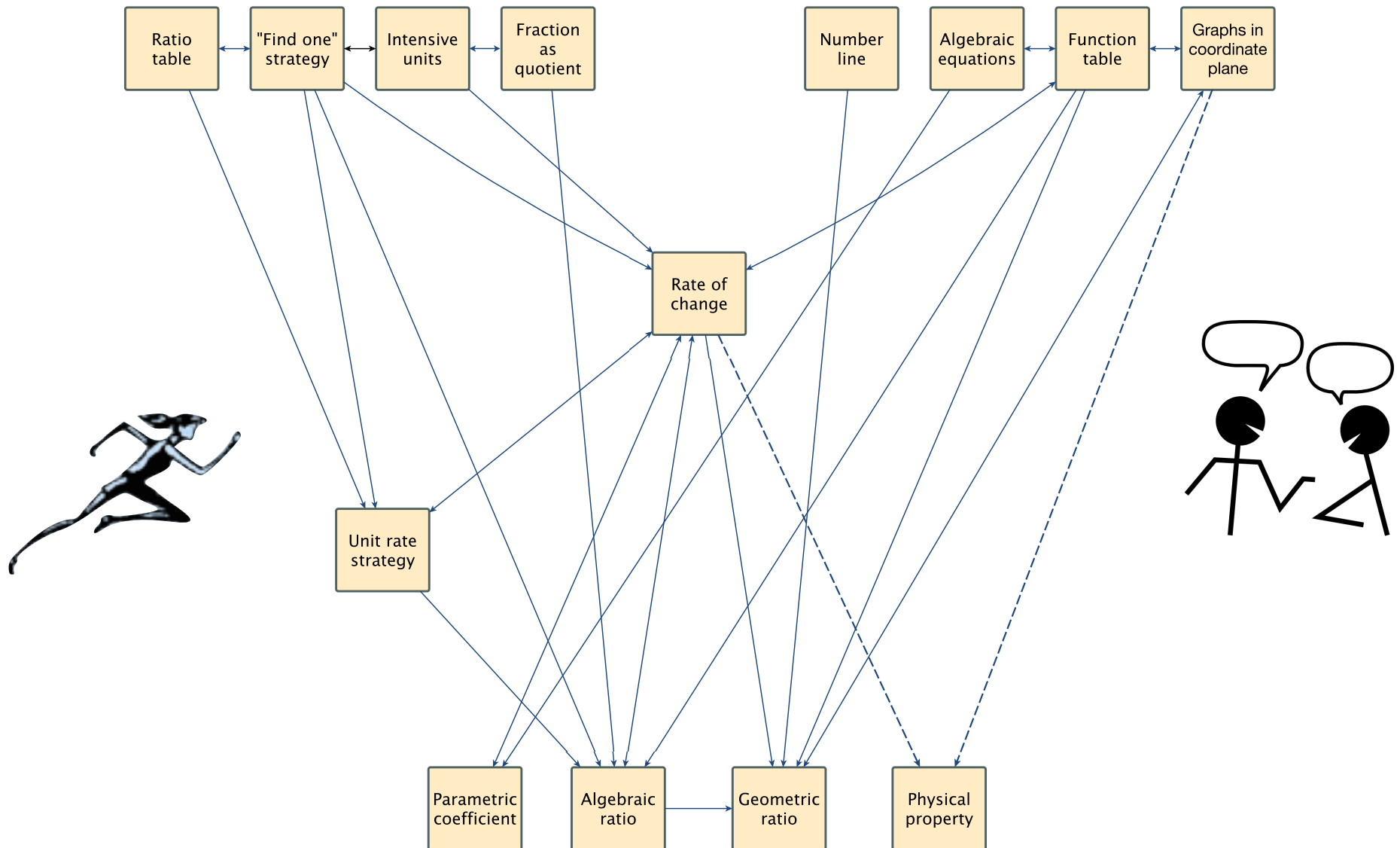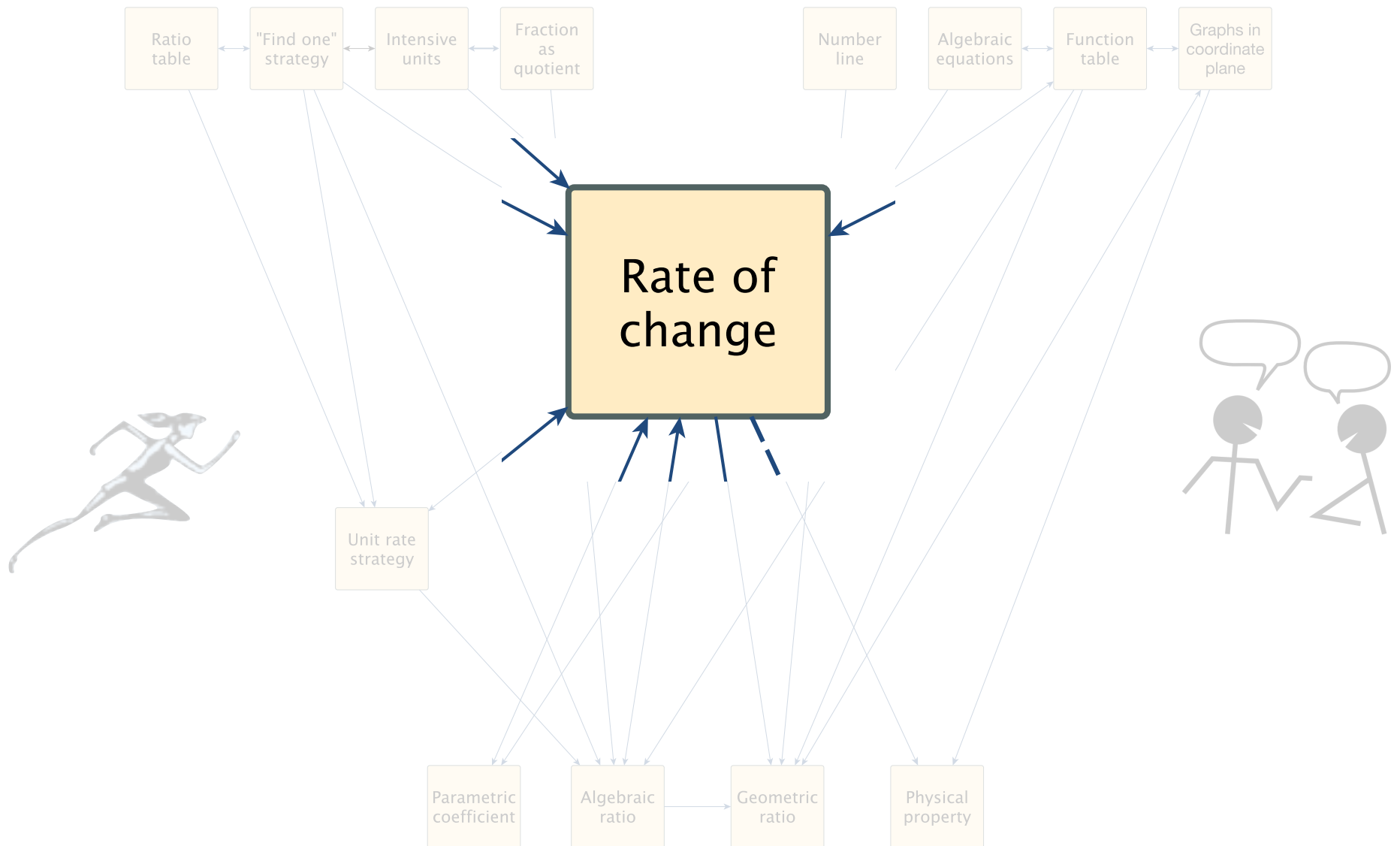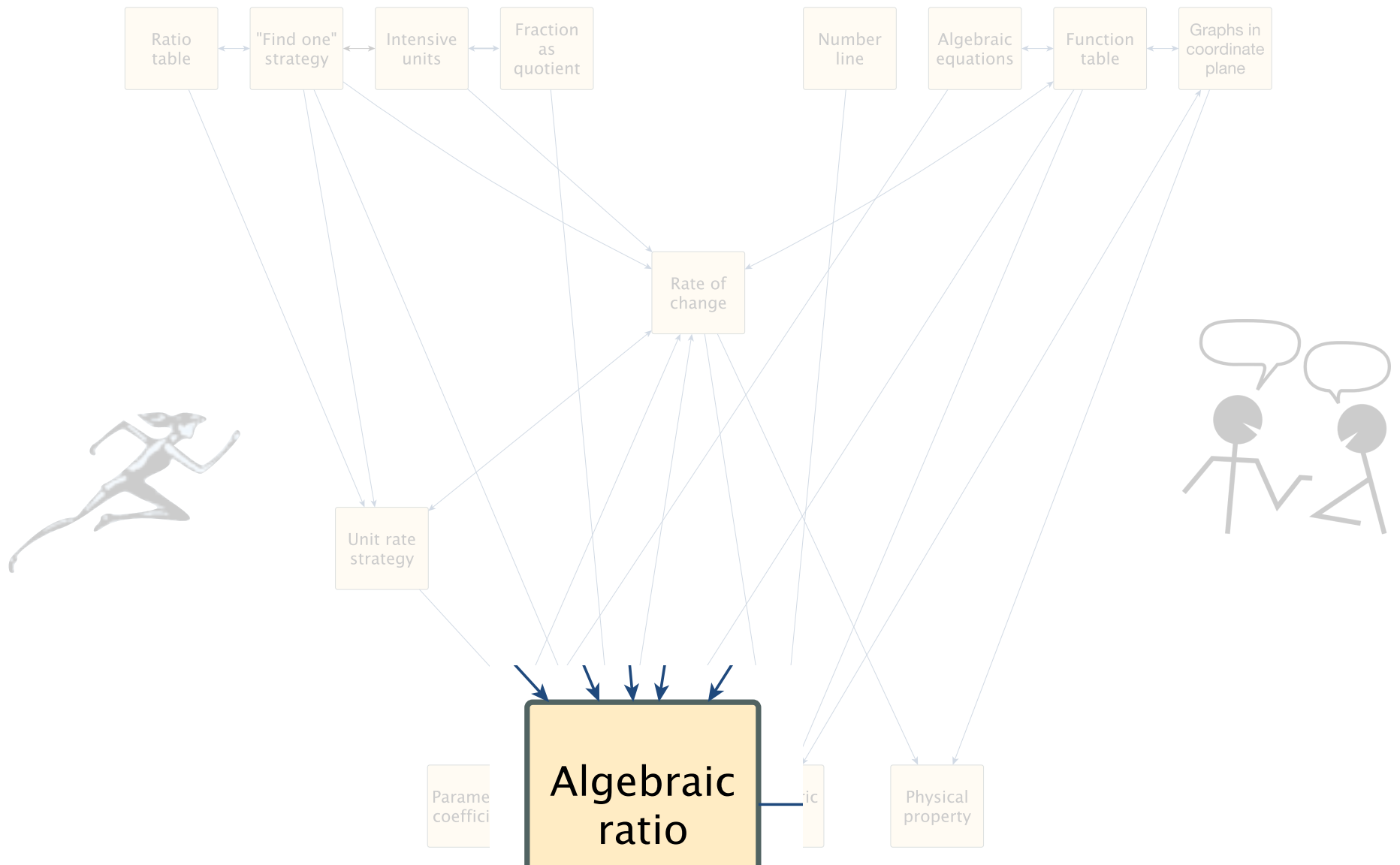
cascade of artifacts
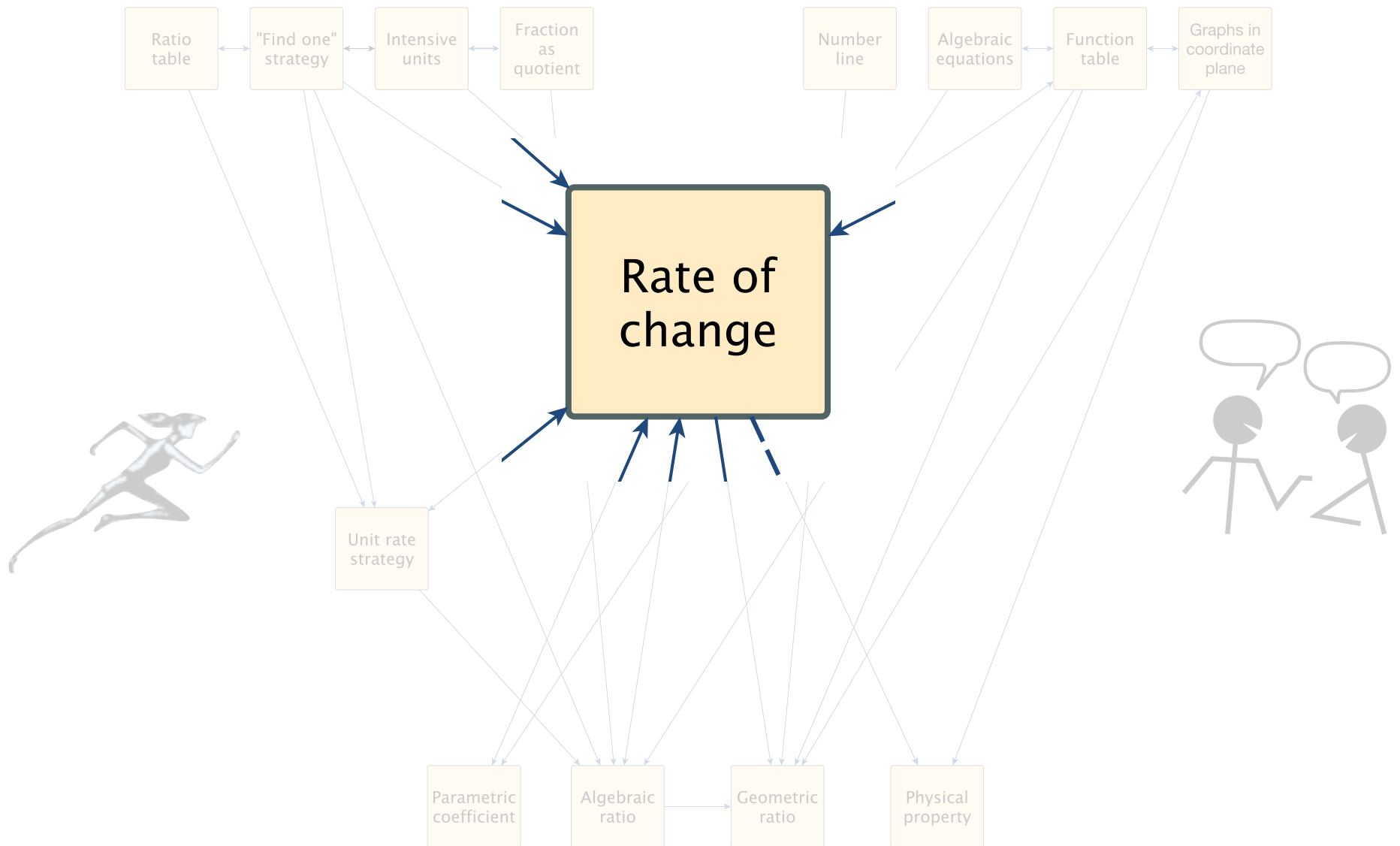
cascade of artifacts

Rate as an emergent artifact

Monday, August 04, 2008, 07:00 am PT (10:00 am ET)

Apple already building iPhones at rate of 40 million a year?

By Slash Lane

Apple is reportedly testing the limits of its overseas manufacturing facilities in order to keep up with demand for the new iPhone 3G, with production already cranked nearly sevenfold compared to the first-generation model.

Foxconn, the company's Taiwanese handset and iPod manufacturer, has recently ramped production of the new iPhone to 800,000 units per week, says *TechCrunch*, citing a person "close to Apple with direct knowledge of the numbers."

The build rate is said to be "above current full capacity" for the Foxconn facilities alloted to Apple's handset business, which has led to concerns that quality control may suffer. At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

Rate as an emergent artifact

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That paces well ahead of analysts' estimates (1, 2, 3) and early reports that suggested Apple's initial iPhone 3G orders spanned only 25 million units through the expected lifespan of the product.

TechCrunch believes Apple's initial order was actually 40 million units over the course of the first twelve months, but is now hearing that "those numbers are being revised upwards sharply."

Apple said it sold 1 million iPhones in the first 72 hours the new iPhone 3G was put on sale, but has not provided an updated sales tally since. The iPhone is currently on sale in 23 countries, with 20 more expected to be added on August 22nd, and another 30 by the end of the calendar year.

... 800,000 units per week ...

... At the current rate, Apple stands to produce more than 40 million iPhone 3Gs over the course of twelve months ...

stands to produce more than 40 million iPhone 3Gs over the course of twelve months.

That paces wo... ...s initial
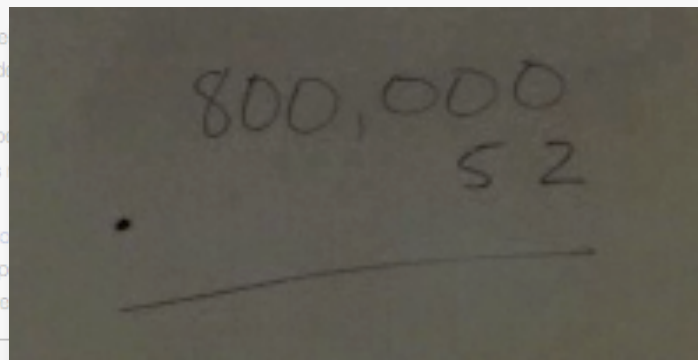iPhone 3G ord...

TechCrunch b... ...rst twelve
months, but is...

Apple said it so... ...t has not
provided an up... ...more
expected to be...

800,000
52

Rate as an emergent artifact

FAP: Randy why is that [multiplication] going to get us a prediction for the number of iPhones in a year? How does weeks turn into iPhones?
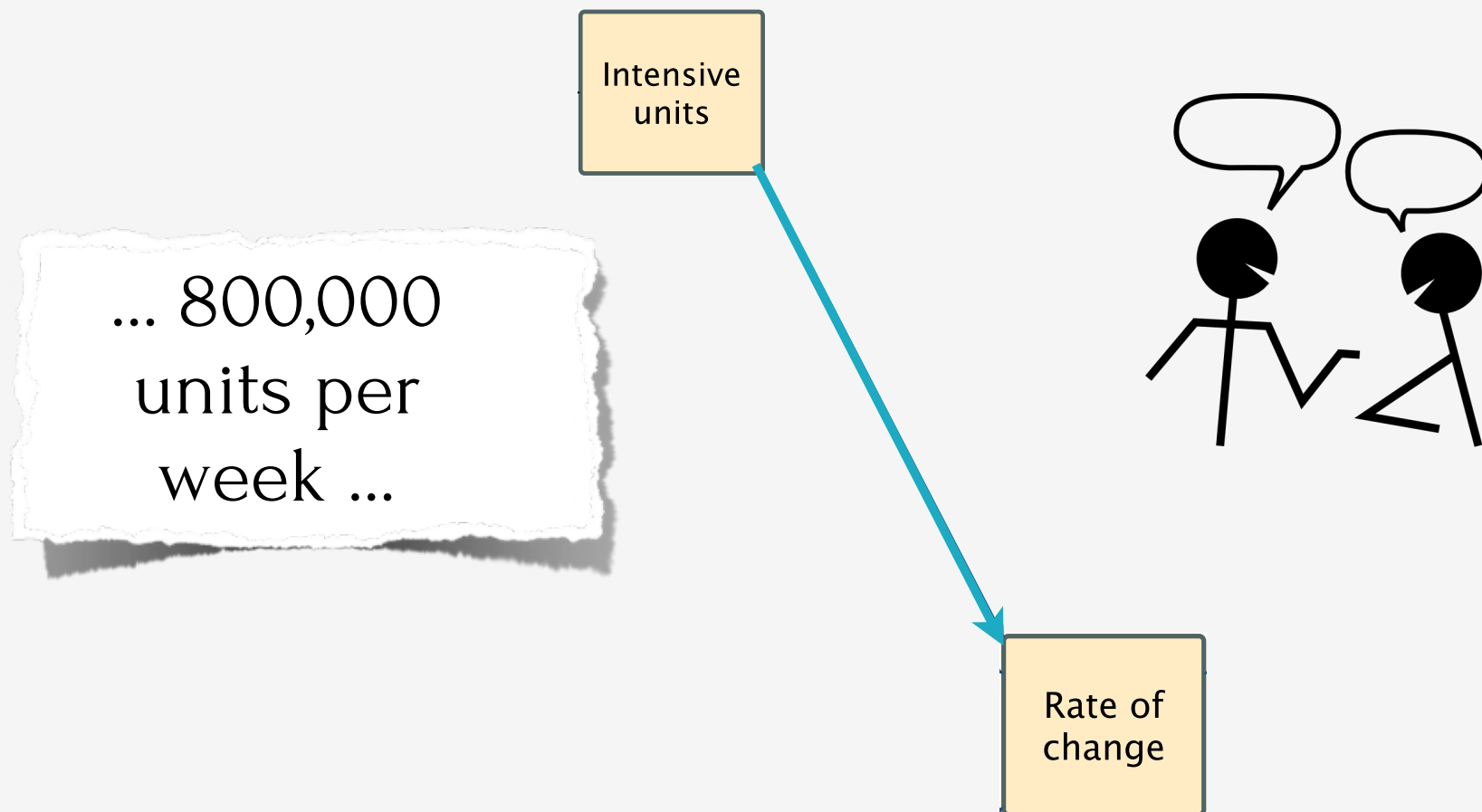
Randy: Because for every week you have, you produce a certain amount of iPhones, so if you multiply it by a certain amount of weeks, the amount of iPhones will go up. [The reason-

FAP: [For every-

Randy: -that might be important is for (investors to know)

Rate as an emergent artifact

Rate as coordination of many-per-one

FAP: Randy why [on] going to get us a predic[t] f iPhones in a year? How does w iPhones?

Randy: Because **for every week you have, you produce a certain amount of iPhones,** so **if you multiply it by a certain amount of weeks, the amount of iPhones will go up**. [The reason-

FAP: [For every–

Randy: –that m r (investors to know)

Rate as number that can be iterated and accumulated

Rate as an emergent artifact

... 800,000 units per week ...

Intensive units

Rate of change

Up and down in the cascade
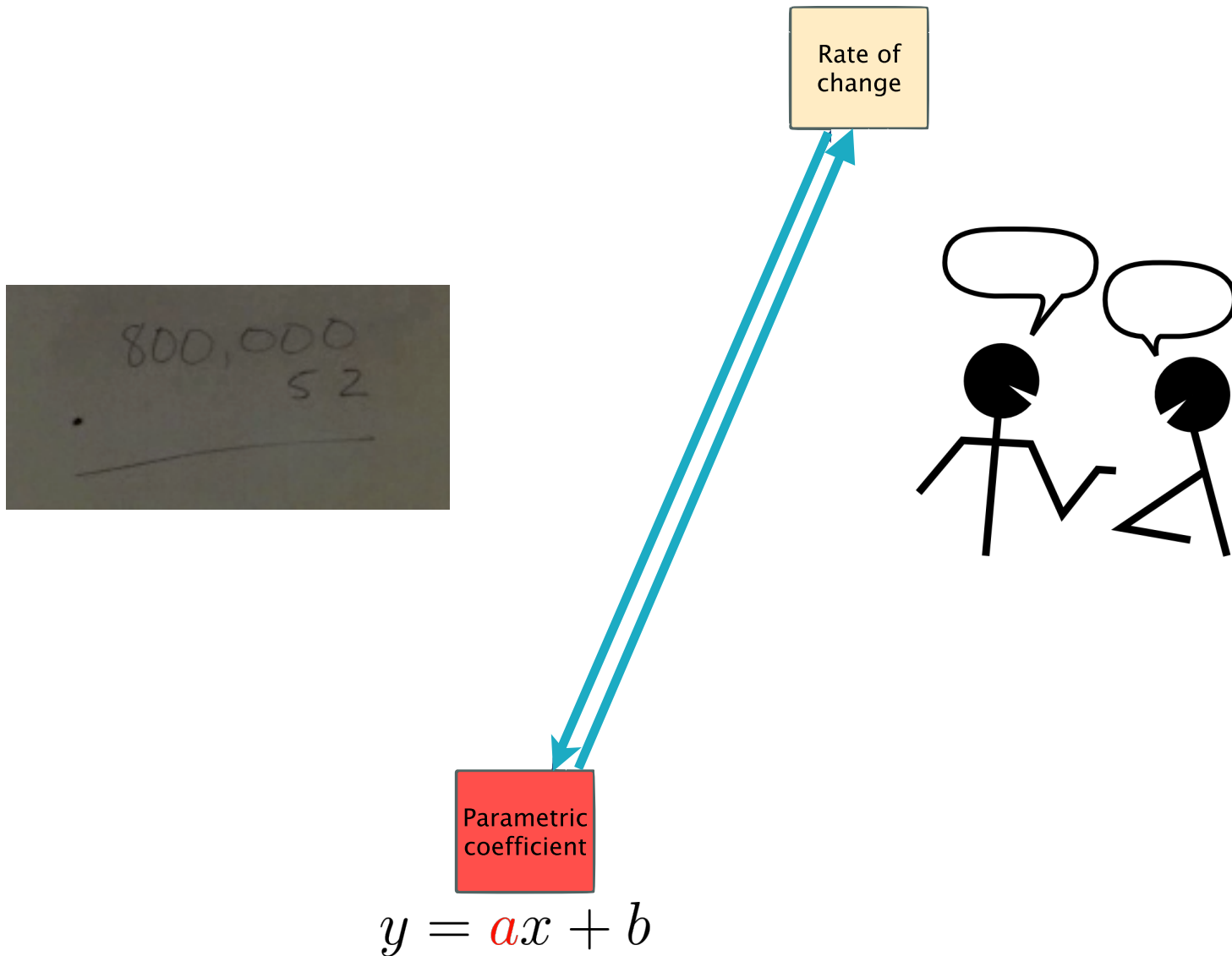
Rate of
change
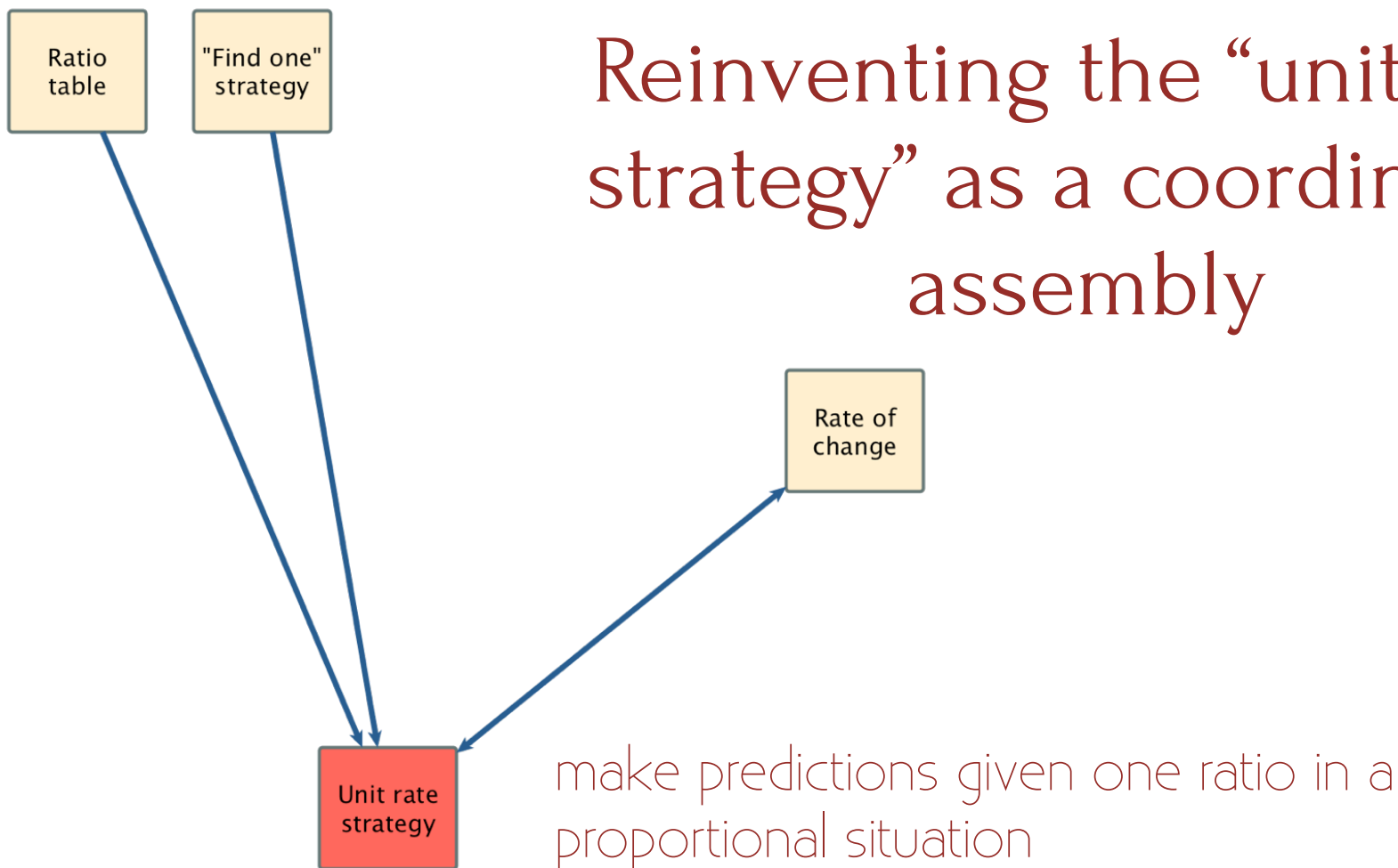
Parametric
coefficient

800,000
S 2

$$y = ax + b$$

Reinventing the "unit rate strategy" as a coordinated assembly

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

make predictions given one ratio in a proportional situation

Reinventing the "unit rate strategy" as a coordinated assembly

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

$6 \div ($ 6 mile $>$ 1 mile

54 min $) \div 6$ 9 min
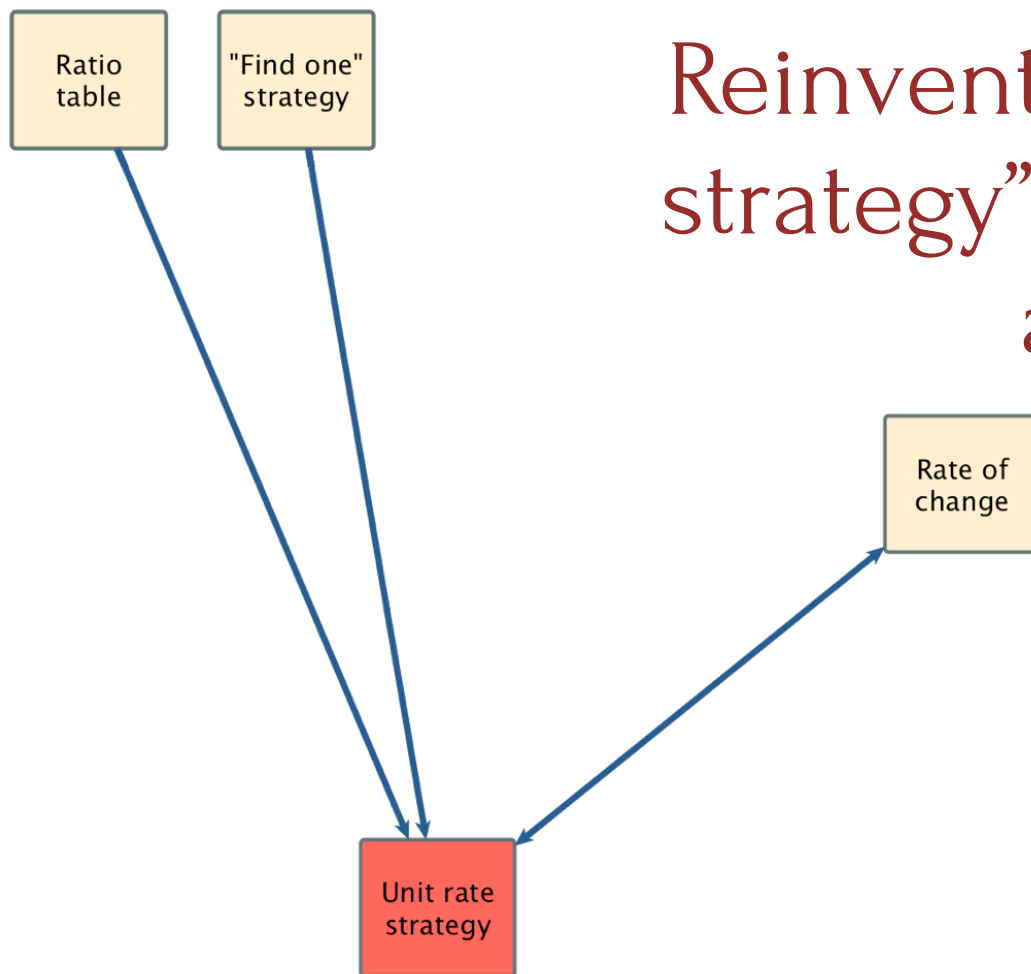
$9 \times 11 = 99$.

Takes 99 minutes

Ratio table

"Find one" strategy

Unit rate strategy

2. In **7** minutes, a hot-air balloon rose **12** meters

In **1** minute, the hot-air balloon rose ___ meters

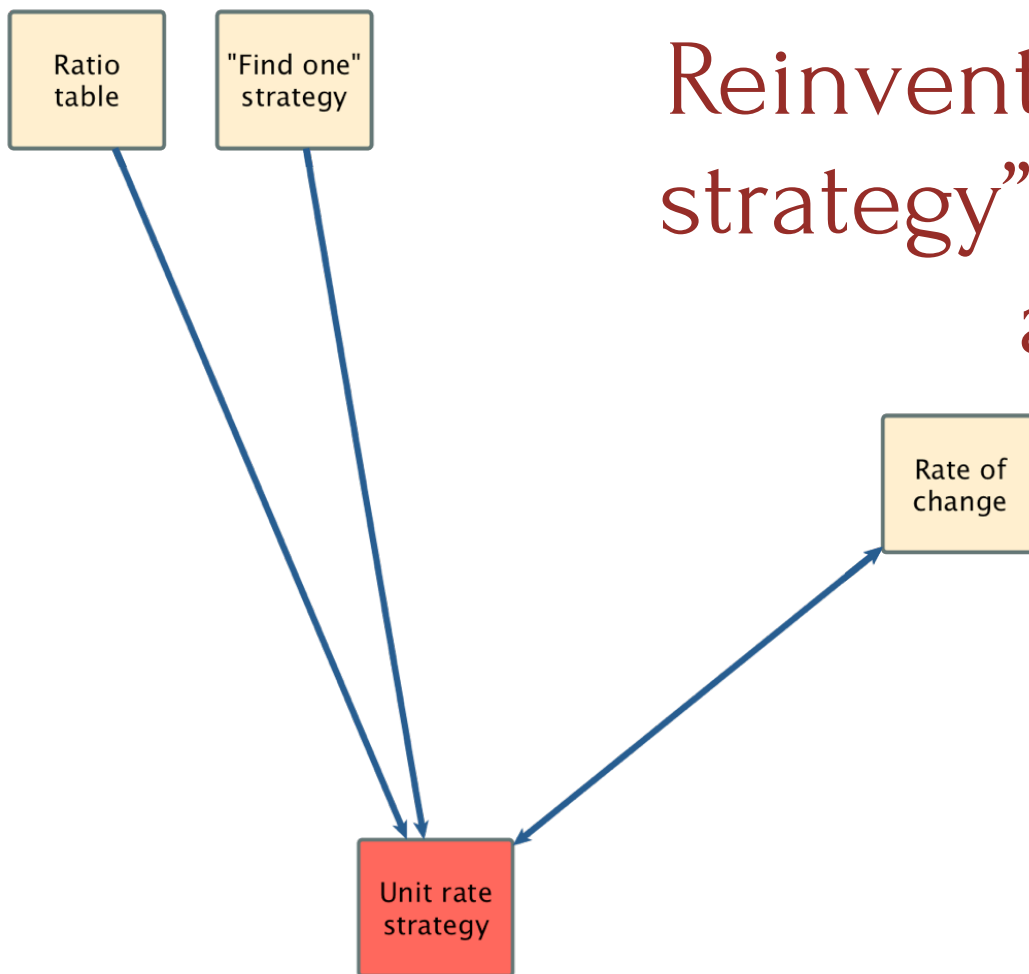*Show your work or explain your reasoning:*

Ms. Magro runs 6 miles every day. On average, she can run six miles in 54 minutes. At this rate, how long would it take Ms. Magro to run an 11-mile race?

6 miles > 54 min
6 ÷ (1 mile   ) ÷ 6
         9 min

9 × 11 = 99.

Takes 99 minutes

Ratio table

"Find one" strategy

Because **for every week you have, you produce a certain amount of iPhones,** so if you multiply it by a certain amount of weeks, the amount of iPhones will go up.
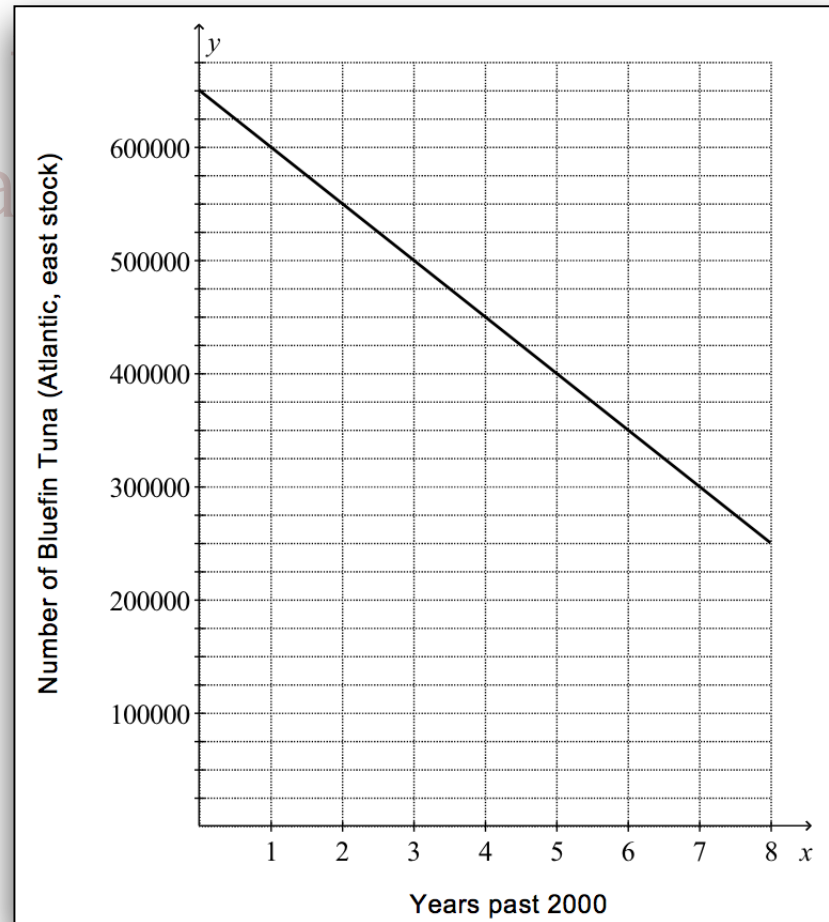
Rate of change

800,000
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Unit rate strategy

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Reinventing the "unit rate strategy" as a coordinated assembly

Ratio table

"Find one" strategy

Rate of change

Unit rate strategy

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1 mile 9 min $) \div 6$

$9 \times 11 = 99.$

Takes 99 minutes

Reinventing the geometric ratio: adding artifacts to the assembly

Graphs in coordinate plane

Rate of change

Algebraic ratio

Geometric ratio

make predictions given a graph

Reinventing
adding artifa

Graphs in coordinate plane

x	y
2	550,000
4	450,000

450,000
− 550,000
− 100,000
÷ 2
− 50,000

We picked 2 points of the graph, subtracted output₁ from output₂ and then ÷ that by 2 which was Input₂ − Input₁.
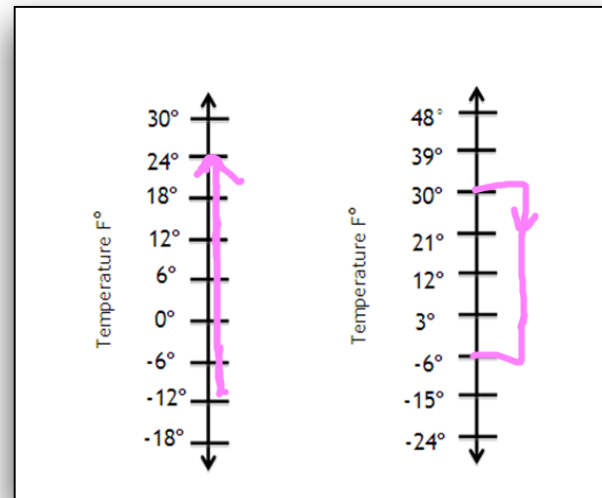
Reinventing the geometric ratio: adding artifacts to the assembly
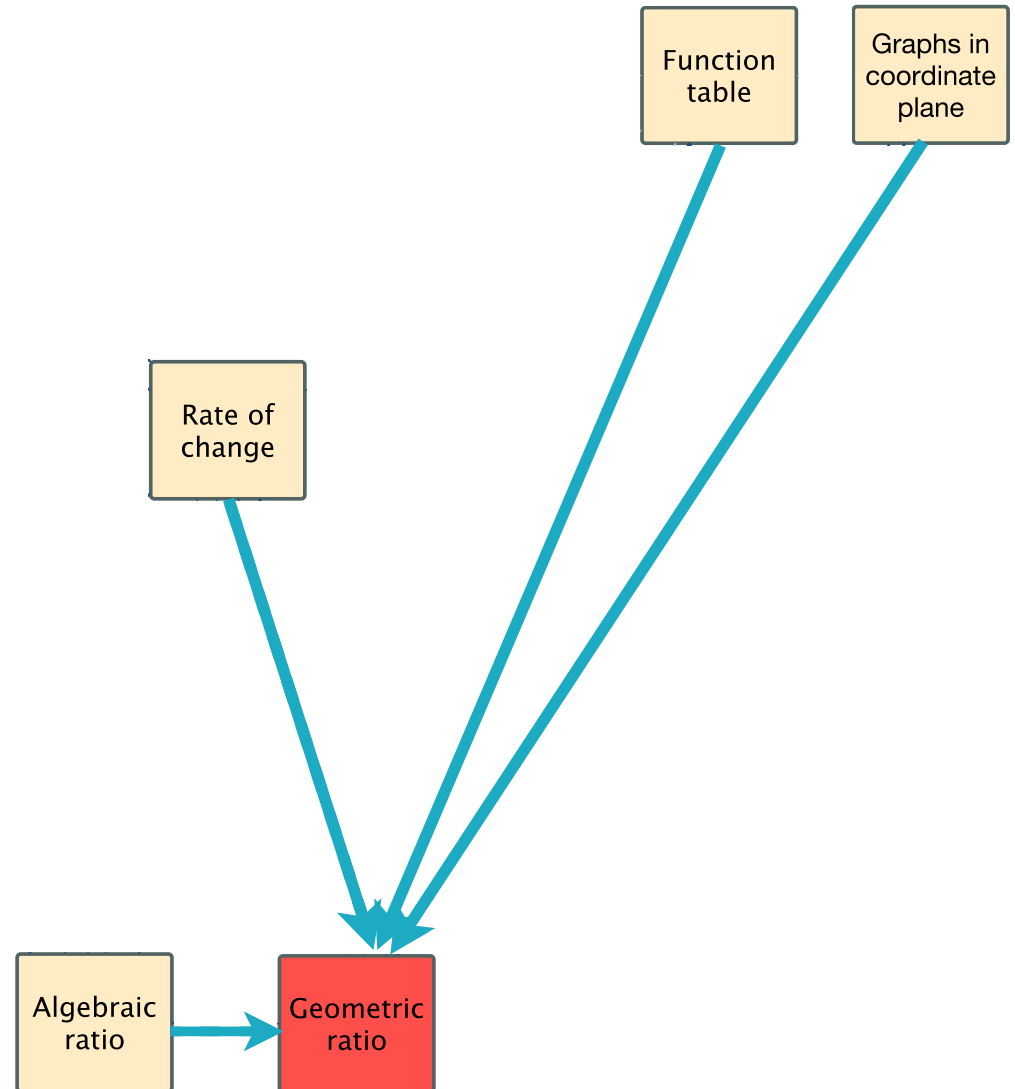
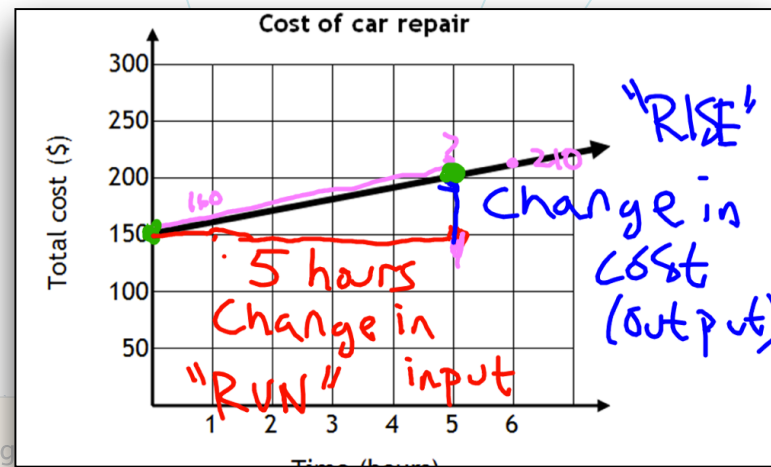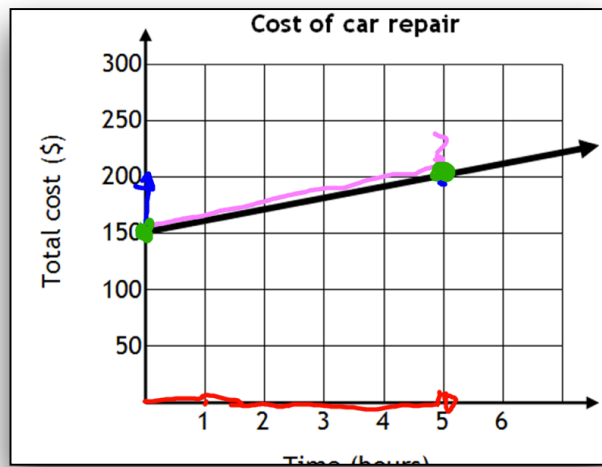# Reinventing the geometric ratio: adding artifacts to the assembly
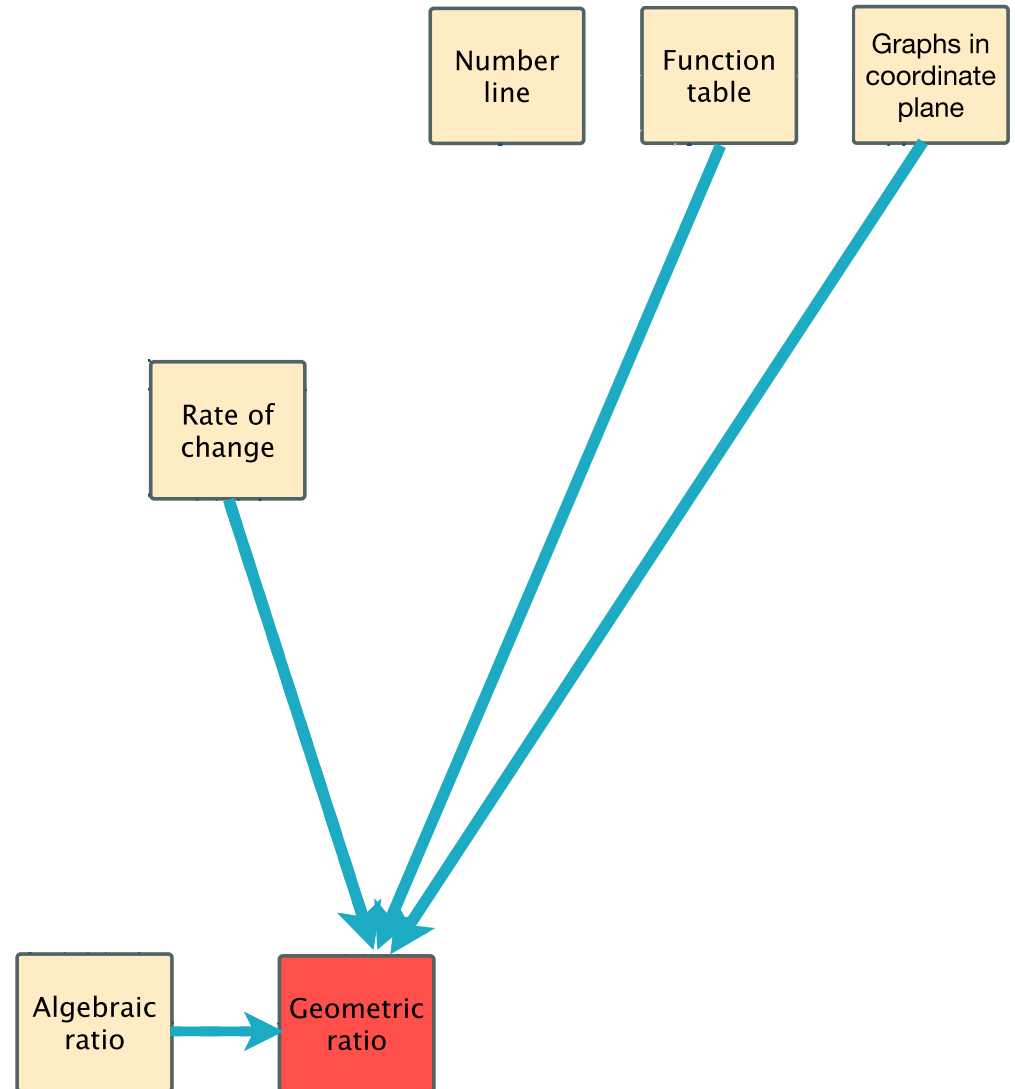
Reinventing the
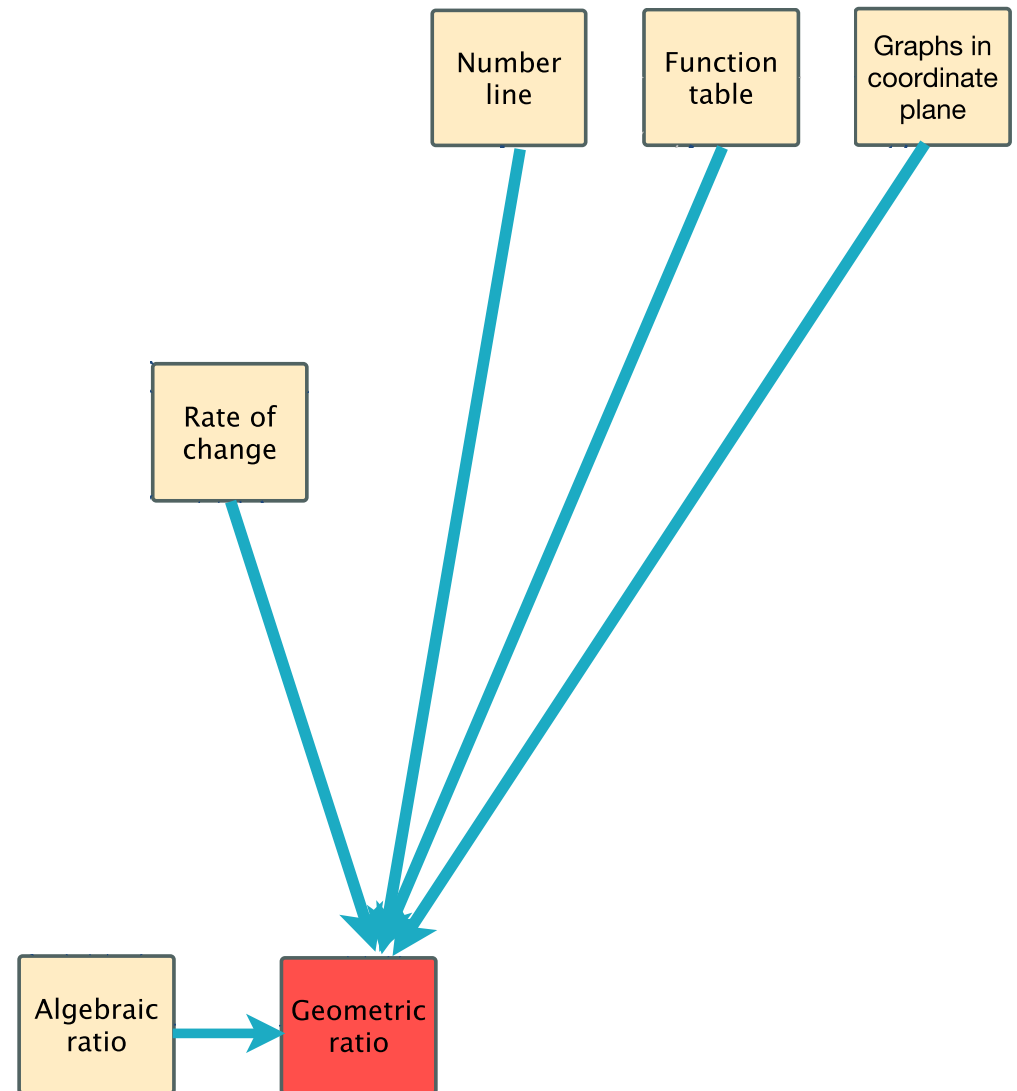# adding artifacts to the assembly

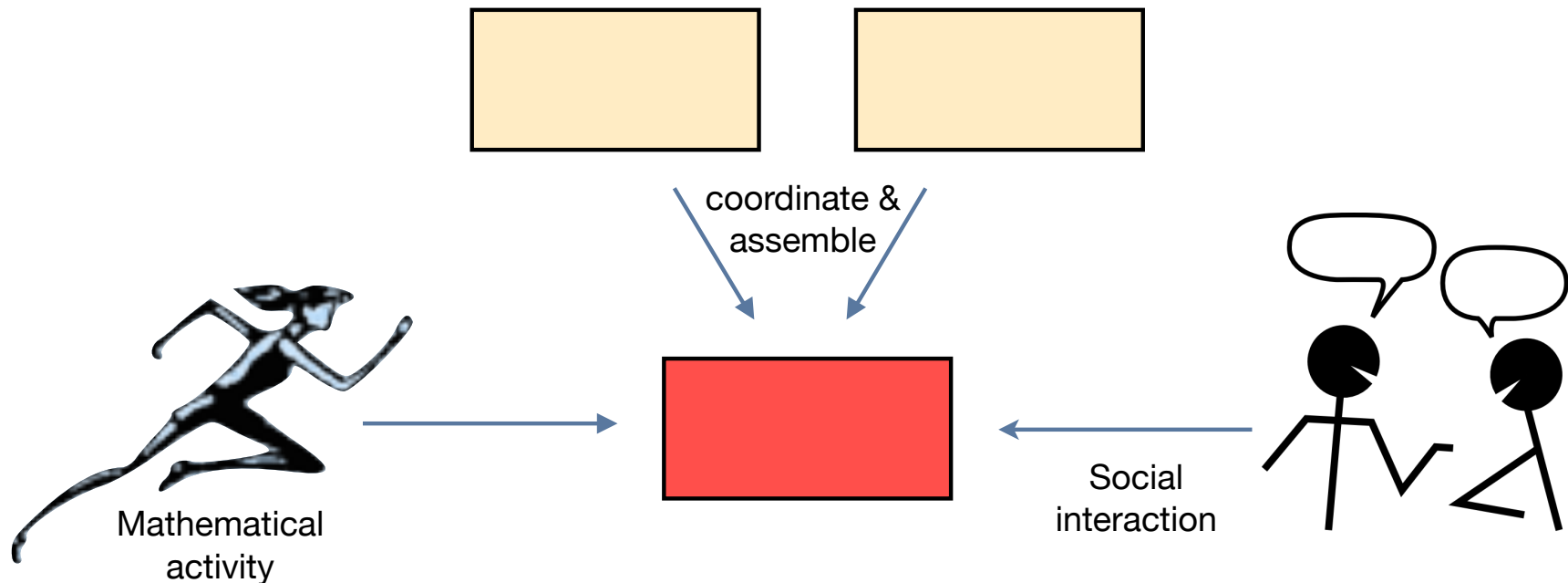

Number line

Function table

Graphs in coordinate plane

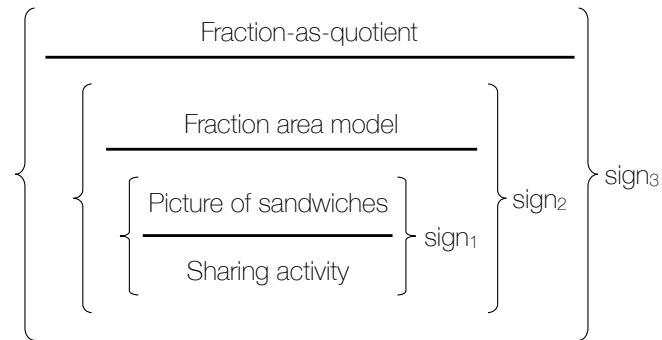# Reinventing the geometric ratio: adding artifacts to the assembly

Emergent modeling

New artifacts emerge
from activity and social interaction
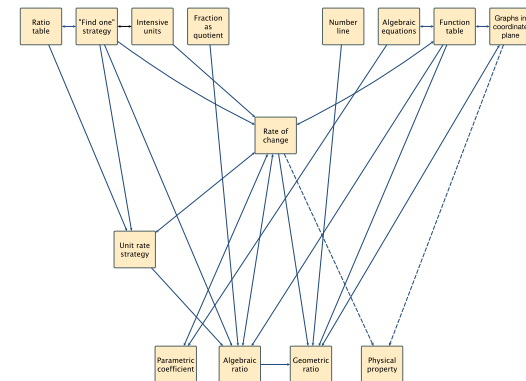as *coordinated assemblies*
of existing artifacts

coordinate &
assemble

Mathematical
activity

Social
interaction

Emergent modeling

New artifacts emerge
from activity and social interaction
as *coordinated assemblies*
of existing artifacts

From

Fraction-as-quotient

Fraction area model

Picture of sandwiches

Sharing activity

$sign_1$

$sign_2$

$sign_3$

To

Emergent modeling

New artifacts emerge
from activity and social interaction
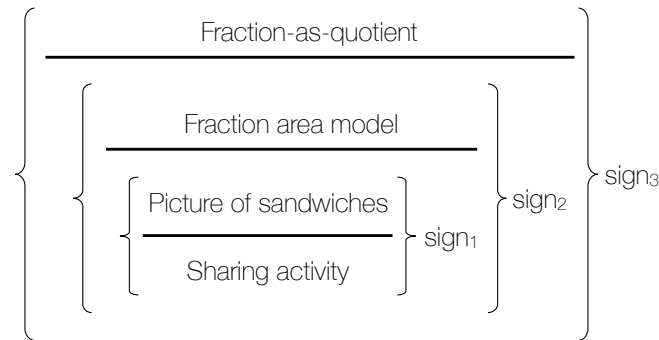as *coordinated assemblies*
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Ratio table · "Find one" strategy · Intensive units · Fraction as quotient · Number line · Algebraic equations · Function table · Graphs in coordinate plane · Rate of change · Unit rate strategy · Parametric coefficient · Algebraic ratio · Geometric ratio · Physical property
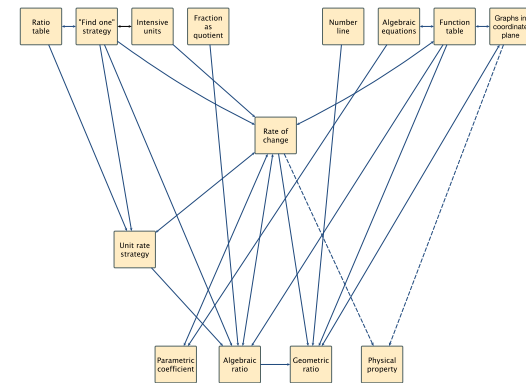
~~One model~~

~~Consecutive symbolizations~~

~~One-to-one mapping~~

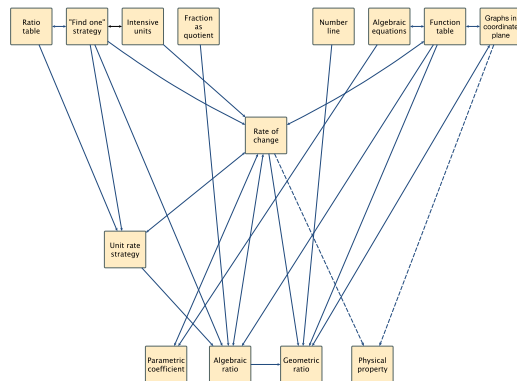~~Hierarchical and siloed~~

Multiple models, tools, & strategies

Coordinated assemblies

Relational and web-like

Emergent modeling

New artifacts emerge
from activity and social interaction
as *coordinated assemblies*
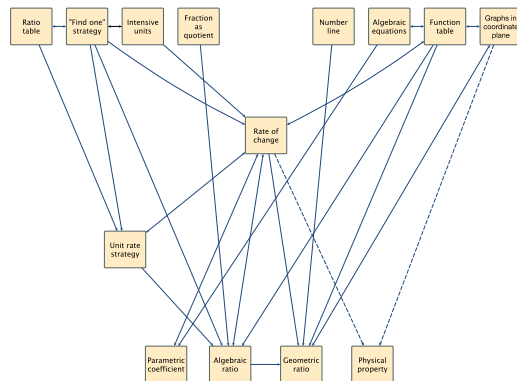of existing artifacts

Cascade of artifacts

- Powerful and flexible way to describes the process and product of emergent modeling

- Shows how the mathematical world is relational, not hierarchal

Emergent modeling

New artifacts emerge
from activity and social interaction
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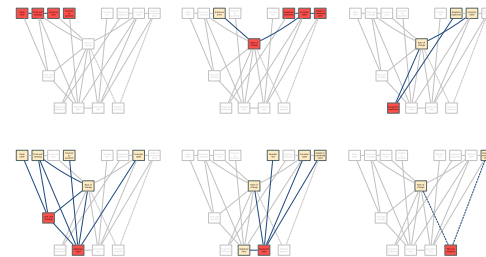of existing artifacts

Cascade of artifacts

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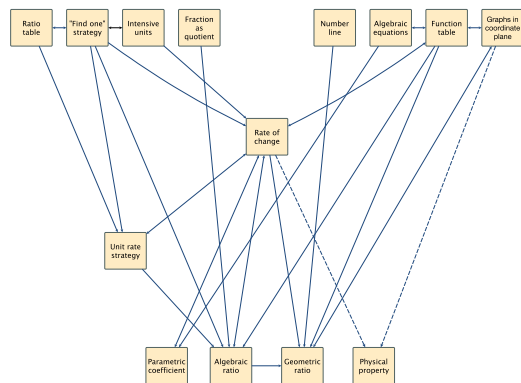- Shows how the mathematical world is relational, not hierarchal

Local instructional theory

- Progression of learning

- Kinds of activities to bring about learning

- Rationale

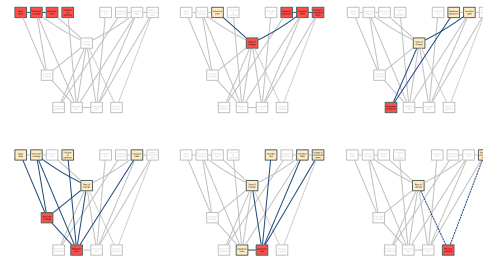- Describes how the cascade is built over time through activity and social interaction

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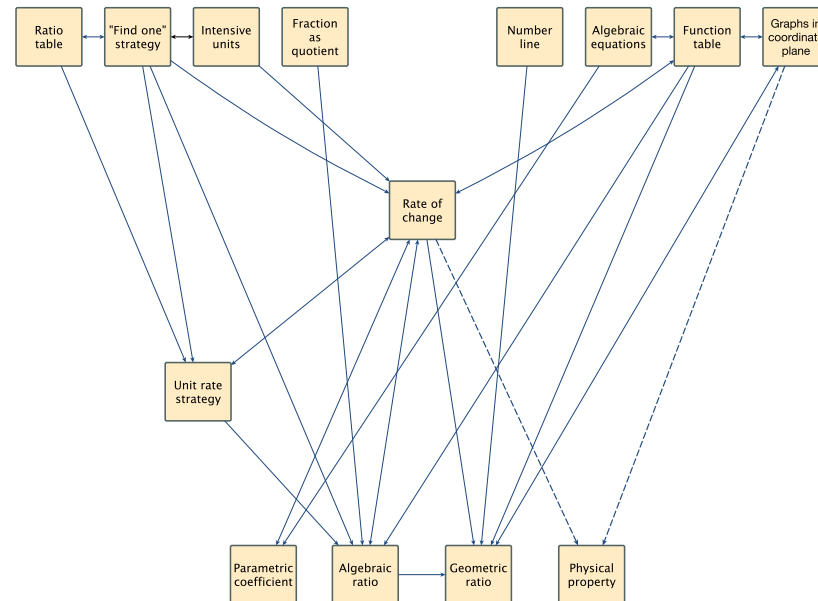
- Progression of learning

- Kinds of activities to bring about learning
- Rationale

- Describes how the cascade is built over time through activity and social interaction

means to account for a mathematical world that that is relational even as learning activities take place in linear time

Emergent modeling: New artifacts emerge from activity and social interaction as *coordinated assemblies* of existing artifacts

Cascade of artifacts