REASONING AND REPRESENTATION IN THE ELEMENTARY GRADES: TWO PERSPECTIVES ON ASSESSING THE STANDARDS FOR MATHEMATICAL PRACTICE

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What does it mean to understand mathematics? Thinking about a response to this question leads to additional, related questions such as: What is mathematics? What is understanding? What does it mean to learn mathematics with understanding? The Standards for Mathematical Practice, as articulated in the Common Core Standards for School Mathematics (CCSSM, Common Core State Standards Initiative, 2010), outline more than instructional objectives. In many ways, the Standards for Mathematical Practice describe mathematics and what it means to do mathematics. The purpose of this article is to describe how two perspectives on the Standards for Mathematical Practice can be used to support how teachers assess students. Even though these principles can be used across all grade levels, the examples given in this article focus on assessment in elementary mathematics. Before the discussion of these two perspectives on assessment, I offer a personal account of how I used Standards to inform, affirm, and reveal the purpose and practice of classroom assessment.

Several years into my teaching career, the NCTM *Curriculum and Evaluation Standards* (1989) gave urgency and license to mathematics teachers to engage students in important mathematics that was relevant, authentic, and meaningful. The *Standards* had an influence on how I assessed students as I began to incorporate projects, labs, and problem solving into classroom activities which I also viewed as summative assessments. I also began to dabble with problem contexts in the selection and design of problem solving activities. What I recall vividly at that time was the students' response. Even though the assessments were more complex and challenging, students expressed genuine interest in these opportunities to showcase their skills, reasoning, and understanding of relationships between problem contexts and mathematics.

I found students were willing to commit additional intellectual effort to mathematical problem solving. This was true among students regardless of their differences in identified learning challenges as expressed in their IEPs (i.e., Individualized Education Programs) or their fluency with English. The positive response from students affirmed the somewhat risky choices I made with instruction and assessment, which changed my perspective on what students were capable of accomplishing and what it meant to do mathematics. Even though the accountability landscape was different then compared to today, the influence of a multi-faceted interpretation of assessment on my own classroom practice and conceptions of the purpose of classroom assessment was profound. Without question, the NCTM *Standards* influenced my teaching and what my students learned.

Of the four purposes for classroom assessment articulated in the NCTM *Assessment Standards* (1995) – monitoring student progress, making instructional decisions, evaluating students' achievement, and evaluating programs – the first two relate directly to aspects of teacher planning, interpretation, and decision making that are often characterized in descriptions of formative assessment (Black & Wiliam, 1998; Black et al., 2003). And so, I confess that during this time of experimentation and change in my classroom practice, assessment as I understood it was something that was scored and graded (Wilson, 1994). My assessment practices were represented by my choice and use of summative assessments. And yet, as argued by Norman Webb (1992), "…test results are generally given as a single score or a profile of scores. It is difficult, using only numerical scores, to describe how a student draws relationships between different mathematical concepts… or how a student goes about solving a problem" (p. 663). It wasn't until much later that I was increasingly drawn to understanding methods of monitoring student progress and the role of assessment in informing instruction. These more formative purposes for assessment are the foundation for the following discussion of two perspectives for classroom assessment – in particular, the critical roles of representation and reasoning in assessing students' understanding of mathematics.

Standards for Mathematical Practice

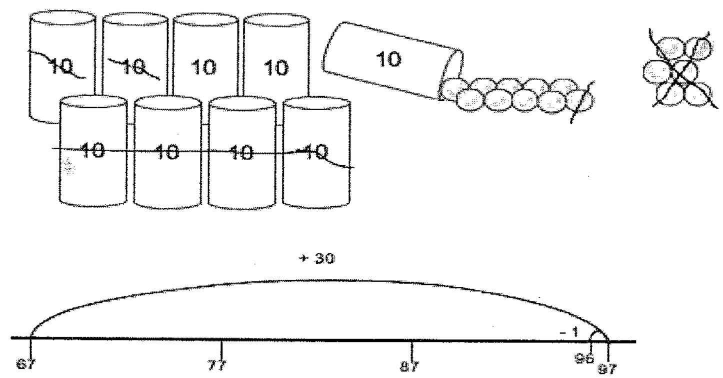
Two perspectives on assessment are discussed here: mathematical reasoning and mathematical representations. They are articulated in various ways in the CCSSM Standards for Mathematical Practice (SMP; CCSSI, 2010). Mathematical reasoning involves thinking about the world and problems in ways that involve numeracy, spatial relationships, patterns, and generalization, and the like. As an example, problem solving often involves mental processes in which mathematically useful features of a problem are identified and related to visual and numerical aspects of the situation. Connections to prior knowledge can lead to further abstraction and idealization of the problem. If needed, additional related strategies can be invoked to pursue a solution. Consider how reasoning is reflected in the following list of SMPs:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Of course, one could say all of these SMPs involve reasoning since each one involves mathematical processes or habits of mind. Constructing viable arguments involves communicating to others, considering your audience, and focusing on the mathematical ideas that need to be shared. The use of appropriate tools, in contrast, is often a more individual undertaking that involves interpreting a problem and recognizing the related strategies that could be used to solve the problem. Indeed, all SMPs involve mathematical reasoning but they have different demands on social and cognitive processes.

From the perspective of mathematical representations particular SMPs become much more prominent. For example, "looking for and making use of structure" can involve visual, numerical and symbolic representations that lead to new representations. Representations of patterns and generalization are also at the heart of ways to "look for and express regularity in repeated reasoning." As shown in Figure 1 below, in order to find 96 minus 67 mathematical representations can involve a diagram for a problem context (modeling), use of a mathematical tool such as an empty number line (appropriate tools), and the proposition of a numerical/visual strategy that relates the number line to a computation strategy (use of structure).

Sam has 67 marbles. He finds some more in the closet.
Now Sam has 96 marbles.
How many marbles did Sam find in the closet?



$$96 - 67 \rightarrow \quad 97 - 67 = 30$$
$$97 - 96 = 1$$

$$\text{so } 96 - 67 = 30 - 1 = 29$$

Figure 1: Representations for subtraction

Knowing the various ways in which students create and use representations is also fundamental to assessment. Representations reveal how students think about a situation mathematically. Representations are abstractions of problem contexts, and they also build upon and connect to other mathematics. Therefore, to assess students' mathematical reasoning it is important to be aware of the various representations students use. Instructionally, representations are also used to support student learning of fundamental concepts in mathematics, and these intersections between instruction and learning are important sites for formative assessment. To interpret student responses to instructional activities and assessment, teachers need to understand how the mathematical content demonstrated in students' representations relate to research-based emergent models of how student learning of mathematics develops over time.

Assessment Pyramid: A Model for Balancing Mathematical Reasoning

To portray the range of reasoning that should be assessed over time in mathematics classes, researchers at the Freudenthal Institute developed the Dutch Assessment Pyramid in 1995 which was later adapted by Shafer and Foster (1997) for use in research studies in the United States (see Figure 2).

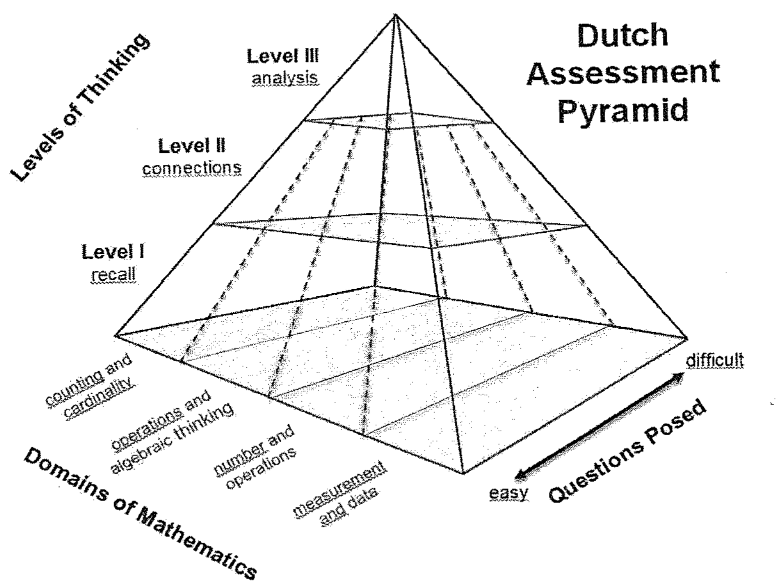


Figure 2: Dutch Assessment Pyramid (adapted from Shafer & Foster, 1997, p. 3).

The relative distribution for the different levels of thinking – recall, connections, and analysis – illustrates that even though recall tasks still form the majority of assessment questions that will be used, they cannot be the only type of task that is used to assess student understanding. Teachers need to move beyond questions designed to assess student recall. They need to include tasks that allow students to show that they can make connections between contexts and mathematics (i.e., modeling, abstract reasoning), communicate their reasoning (i.e., argumentation), and propose general relationships and expressions for patterns. Those familiar with the Programme for International Student Assessment (PISA) may recognize these three "levels of thinking" as the mathematics competencies outlined in the PISA mathematics framework (OECD, 2010).

Recall: The type of thinking represented at this level involves the recall of practiced knowledge and skills. This level deals with knowing facts, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, and operating with statements and expressions containing symbols and formulas in "standard" form. These tasks are quite familiar, as they are the most commonly used tasks on standardized assessments and tend to be the types of tasks that are more easily created for classroom assessment.

Connections: The type of reasoning for this level involves connections within and between the different domains in mathematics. This type of reasoning also involves handling different representations according to situation and purpose. Students need to be able to distinguish and relate a variety of statements. Tasks that elicit this type of reasoning are often placed within a context and engage students in mathematical decisions where they might need to choose from various strategies and mathematical tools in order to solve problems. Therefore, tasks eliciting reasoning at this level are often open to a range of representations and solution strategies.

Analysis: Reasoning at this level is elicited when students are asked to mathematize situations—that is, to recognize and extract the mathematics embedded in the situation and use mathematics to solve the problem. This includes analysis that requires interpretation of a situation and the development of new models and strategies, mathematical conjectures, and generalizations. Problems at this level reveal students' abilities to plan solution strategies and implement them in less familiar problem settings that may contain more elements than those in the connections cluster. Other ways in which reasoning of this type are elicited are through the use of open tasks, tasks that are so open they require students to make assumptions to put boundaries around the task so that it is solvable.

The other two dimensions of the pyramid include the domains of K-5 mathematics as suggested by the CCSSM, and the difficulty of tasks and questions used for assessment. The dimensions of mathematics are meant to convey that all content domains should be included in a comprehensive approach to classroom assessment. Even though numeracy receives significant emphasis in grades K to 5, students should also have opportunities to engage in early algebraic reasoning, measurement, and the interpretation of data.

Lastly, it is worth noting an important distinction between "levels of thinking" and the difficulty of "questions posed." Questions that involve recall (Level I) can be easy or difficult. In fact, some Level I questions can be more difficult than Level II or III questions. Just because questions may require recall only does not mean they are any less difficult than a question that requires the choice of an appropriate strategy. Level II and III reasoning is not necessarily more difficult. It is a different type of reasoning. To describe this in a different way, a question that might involve mathematizing an unfamiliar problem context (Level III) may be easier for students than a multi-digit division problem that requires many steps and, therefore, many opportunities to make computation errors (Level I).

Tasks to Assess Student Reasoning Beyond Recall

To further illustrate the different types of reasoning reflected in an assessment pyramid for elementary mathematics, several tasks are discussed in this section. As you review these examples, and your own assessment tasks, consider the student reasoning that could be elicited for your students (or hypothetical groups of students at different grade levels).

In Figure 3, a partial hundreds chart is given with several blank spaces. Even though a hundreds chart is used to support counting, skip counting, order, and operations in early elementary grades, it can also can be used to assess how students understand place value, order, and how a hundreds chart is organized. The ways in which students complete the chart will be more indicative of their understanding than the numbers they write in the empty boxes: *Do students need to count through (or tap) all of the missing numbers to complete the numbers up to 31? Are students able to work vertically to find 12 and 21?* For a student who is just beginning to use a hundreds chart, the reasoning elicited may best fit Level II (making connections, choosing an appropriate strategy). For students who are quite familiar with the hundreds chart and related extensions through classroom activities, this task would assess recall of procedures.

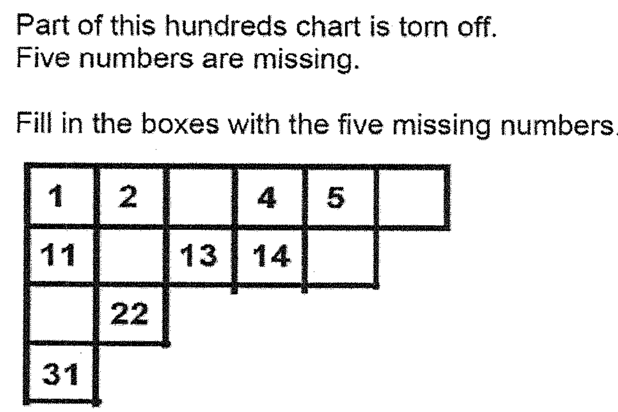


Figure 3. Reasoning about Structure of Number

In Figure 4, part of a rectangular array of tiles in a parking space is hidden by a car. This is a task that could be used with students in Grade 2 or 3 as they transition from direct counting to the use of typical representations, such as arrays, to solve multiplication tasks. Although there are connections to measurement concepts, the primary process being assessed is whether students count, skip count, or multiply to find the total number of tiles in an array. As with the hundreds chart task, particular focus should be given to how students respond to the task in addition to the answer they give to this question.

How many tiles were used to make this parking space?

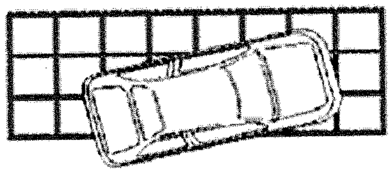


Figure 4. Eliciting New Strategies for Computation

With respect to levels of thinking in the pyramid, a student who is just beginning to explore multiplication may need to count all of the tiles. This reflects Level I reasoning (recall, counting processes). However, a student who notices there are eight groups of three (or visa versa), or chunks the tiles into 15 + 9 (or some related combination) would be demonstrating at least Level II reasoning (making connections between addition, grouping, and multiplication). For similar tasks with larger arrays that are less accessible to counting strategies, some students may develop conjectures about multiplication strategies or even invent other strategies involving combinations of skip counting and grouping (Level III, generalization).

The prompt and the series of related questions in Figure 5 assess students' relational reasoning with multiplication facts. The three multiplication problems that are given are organized in a way to promote inductive reasoning with a sequence of problems in which the answers are already provided. The three questions that follow can all be answered using information from the set of questions and answers.

In Figure 5, students who choose to subtract 13 from 338 to find the answer to (a) and add 13 to 390 to find the answer to part (c) demonstrates Level II reasoning, since they are strategically relating the given information to the problems that follow.

On a chalkboard is the following list of math problems:

13 X 26 = 338

14 X 26 = 364

15 X 26 = 390

What are the answers to the following problems?

a) 16 X 26 = __________

b) 2600 X 14 = __________

c) 12 X 26 = __________

Figure 5: Assessing Relational Reasoning

What may be surprising to some is that if students use the standard algorithm for multiplication to find the answers, they are simply demonstrating recall of procedures (Level I). Once again, the students' strategies are more revealing of the level of thinking elicited than whether or not the answers are correct. While correct answers are certainly important, if mathematical reasoning is valued and included as part of classroom assessment practices, asking students how they find those answers is critical for determining the reasoning elicited. This leads to further consideration of how instructional practices influence how students choose to answer these questions. If the standard algorithm is promoted at the expense of other appropriate strategies, students will be less likely to take risks and will be less likely to apply the patterns they see in written or verbally administered assessments.

To conclude this section, Figure 6 showcases a task that involves two multiplication problems with the same answer. In this problem the answer is not given, although for upper elementary students (i.e., Grade 4 or 5) finding 8 x 40 should not be too difficult. In this task students are asked to find two more multiplication problems that have the same answer. Certainly, some students might take advantage of the "give away" problem and write 1 x 320. This would then lead to a related second problem, 2 x 160. However, even though this seems to be an easy solution path, students who choose this approach may recognize that 2 x 160 looks similar to 16 x 20, leading them to contemplate why those two problems must be equal (Level II). This approach could also lead to related conjectures that could be explored further with other multiplication sequences (Level III).

> Here are two multiplication cards that have the same answer.
>
> 16 x 20 | 8 X 40
>
> Find two other multiplication problems that have the same answer as these two cards.

Figure 6: Extending Relational Reasoning

Another expected response, perhaps more common, would be for students to notice that as one factor is halved, the other factor is doubled. This insight could be further extended to identify other products such as 4 x 80 and 2 x 160 (or in the other direction, 32 x 10 and 64 x 5). Assuming students do not regularly practice these types of problems, these methods demonstrate reasoning that is at least Level II (making connections) or even Level III (generalization, new strategies).

Mathematical Representations and the Iceberg Model

To interpret, promote, and assess student reasoning, it is important to be able to recognize the role different representations play. Learning mathematics in the elementary grades often involves students making sense of representations that range from concrete supports for counting, to intermediary models that support computation, to standard algorithms that convey structure in more abbreviated forms. Instructional materials designed for primary grades mathematics often include a multitude of representations and strategies that can be used to convey concepts

and skills. However, the instructional value of representations can vary, from modeling a specific problem to modeling a host of situations involving similar processes. For instance, a visual or diagram for a problem can focus students on the specific mathematical goals for the problem, and can serve as a scaffold for language and/or mathematical demands. In this case the representation is useful, but only for a specific task. More broad reaching representations include ten frames, number grids, beaded number lines, rekenreks, empty number lines, and so on.

Progressive formalization, an instructional design approach that draws from decades of developmental research using principles of Realistic Mathematics Education (RME; van den Heuvel-Panhuizen, 2001), suggests that learners should access mathematical concepts initially by relating their informal reasoning about problem contexts to more structured, pre-formal models, strategies, and representations for solving problems (e.g. array models for multiplying fractions, ratio tables for proportions, percent bars for solving percent problems, combination charts for solving systems of equations, etc.).

The Iceberg Model is a visual metaphor that illustrates enacted features of progressive formalization through informal, pre-formal, and formal representations (see Figure 7). The iceberg consists of the "tip of the iceberg" and a much larger area below the surface, designated the "floating capacity." The top of the iceberg represents the formal procedure or symbolic representation of interest. However, before this formal level is reached, students should have an opportunity to engage in informal reasoning and use pre-formal representations.

Informal reasoning: Students' experiences with real or imaginary contexts are often the basis for informal mathematical reasoning. The iceberg model in Figure 7 illustrates some contexts that could inspire students' informal reasoning about two-digit multiplication. A mathematical context would be the approach of multiplication as repeated addition. Even though this pre-cursor to multiplication does not represent multiplicative reasoning (Clark & Kamii, 1996), it is a common approach for students to use before they transition to grouping strategies. Students' encounters with computing with money, dice, or other objects with associated value are opportunities for students to consider groupings and ways to coordinate, skip-counting, number facts, and multiplication. For example, a stack of 18 quarters might lead a child to think about grouping coins by fours (dollars) or tens ($2.50), and the relationship between known groupings and multiplication. Informal reasoning tends to be bound to specific contexts, but can lead to other ways to structure mathematics problems.

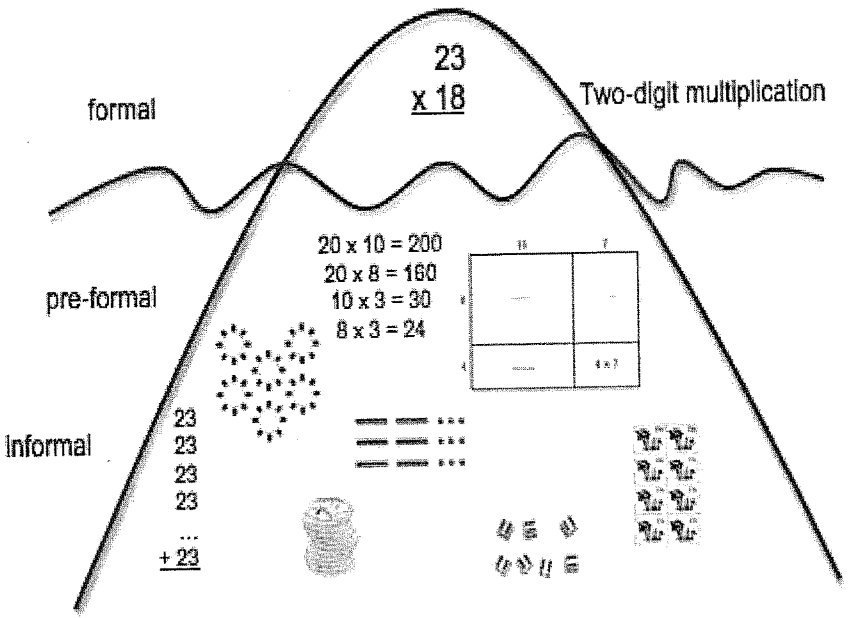


Figure 7: Iceberg Model for Two-Digit Multiplication

Pre-formal representations and strategies: Pre-formal representations build on students' informal representations, or reasoning, and offer greater mathematical structure that can be used across many problem contexts. Some examples included in the iceberg for multiplication include: making consistently-sized groups to support multiplication (e.g., seating arrangements), using base ten blocks and various related models, using an area model to illustrate the structure of two-digit numbers and related products, and the related use of partial products, as another example of how the distributive property is applied with multi-digit multiplication. While the seating arrangement may be more

accurately placed between informal and pre-formal sections of the iceberg model, the other three models are more meaningful approaches to support student understanding of the standard algorithm for two-digit multiplication. As noted by Webb, Boswinkel, and Dekker (2008),

> Most pre-formal representations are rarely developed by students on their own to solve a problem. Instead, students are guided by teachers or instructional materials to use pre-formal representations and strategies that can be applied across many situations and contexts. Pre-formal representations offer greater opportunities to empower students' sense-making, but they often have limitations in the scope of problems that can be solved using the chosen representation (p. 112).

Informal and pre-formal representations play a fundamental role in the development of student understanding of formal mathematics. When the "floating capacity" of the iceberg is bypassed to expedite instruction of formal mathematical goals, students are left with limited opportunities to make sense of why these formal representations work. While other curricular demands and limitations of time may restrict the representations that one might introduce to students, even the consideration of trade-offs between including or excluding specific representations in mathematics instruction will support more informed instructional decisions and formative assessment.

The Iceberg Model and Formative Assessment

Through careful attention to students' prior knowledge, expected informal strategies, and known pre-formal models, teachers can select and adapt assessment tasks that are more likely to elicit representations that are accessible to students. Progressive formalization serves as a principle for organizing instruction and assessment so that student responses can be interpreted with respect to how student learning for a given mathematical domain is developed over time. Subsequent prompts, scaffolds, representations, and questions are informed by how student responses relate to other relevant strategies and representations, and proposed instructional sequences.

When appropriate, teachers can adapt instruction to build upon students' less formal representations by either drawing upon strategies by other students in the classroom that are progressively more formal, or introducing the students to new pre-formal strategies and models. Because pre-formal models are usually more accessible to students who struggle with formal algorithms, they are encouraged to refer back to the less formal representations when needed to deepen their understanding of a procedure or concept.

Teacher educators and researchers who have investigated formative assessment have characterized this practice as a response to these three questions:

- Where are they going?
- Where are they now?
- How will I help them get there?

The first question relates to goal setting and curriculum, and includes content and process goals established by the state, district, or school. The second question relates to knowing "where students are" at the moment which requires some combination of observing and listening to student responses and interpreting those responses – i.e., assessment. The most challenging question to answer is the third. How will you help students move from their current understandings of mathematics to be able to demonstrate established mathematical goals? One of the reasons for thinking about assessment from the perspective of mathematical representations is that it provides potential instructional pathways that are based on student responses and hypothetical learning trajectories (Simon & Tzur, 2004; Clements & Sarama, 2004).

Intersections Between the Pyramid and Iceberg Models

Teachers who have used these two models to inform the design and use of assessment have noticed a relationship between the upper levels of the pyramid and the floating capacity of representations below the water line in the iceberg. To select, adapt, and design assessment tasks that are open to student choice of solution strategy, one has to be mindful of the various solution strategies that are possible. To emphasize further, *the choice of assessment tasks is informed by knowledge of the expected student responses that could be elicited.* Knowledge of representations for a given mathematics topic leads to better selection and design of

assessment tasks that assess reasoning beyond recall. On the other hand, the recall of skills and procedures reflected in the base of the assessment pyramid are, more often, the facts, algorithms, and procedures that are the formal mathematical goals at the tip of the iceberg.

A comprehensive approach to mathematical reasoning in classroom assessment, therefore, needs to take into account the representational pathways that lead to student understanding of the formal mathematical goals. Of course, the formal goals are not the only instructional objectives that are worth assessing. The manner in which students choose and use representations and models demonstrates a deeper understanding of mathematics than might be portrayed in responses that reflect only recall of skills and procedures.

Closing Thoughts

Reasoning and representation are complementary perspectives that inform the assessment of the CCSSM Standards for Mathematical Practice. Students first encounter mathematics as they make sense of their world. Later, in school, they are introduced to problems that require use of more structured representations and strategies to advance their knowledge and understanding of mathematics. Classroom assessment practices that are consistent with how students learn mathematics present certain challenges, but support mathematical reasoning that is more generative, productive, and enduring.

One challenge to incorporating assessment practices that utilize these dual perspectives is the need to develop one's knowledge of the mathematical terrain. What are the models and strategies that represent the "floating capacity" for various mathematics topics? While there are many instructional resources and professional development materials that outline useful models and strategies, being able to articulate the pros and cons for each of these representations takes years of classroom experience.

Prior experiences with teachers in assessment-related professional development activities using the assessment pyramid (e.g., Webb et al., 2004) and the iceberg model (Webb, Boswinkel & Dekker, 2008), have shown that developing an iceberg of representations and discussing various options for good assessment tasks is best accomplished with the collective insight and support of colleagues. As we observed first hand over a decade ago, teachers who collaborate to adapt and design questions that assess more than recall promote ongoing professional discussion and deliberation of the essential concepts and mathematical connections that are at the intersection of teaching and learning. This type of collaboration should continue as teachers share student work and negotiate how students' responses illustrate tentative claims about student understanding of mathematics as well as features of task design that support or constrain student expression.

As you review the assessment tasks you use with your students and analyze the extent to which they "fill the pyramid," you will identify opportunities to include questions that elicit reasoning beyond recall. As you begin to shift to greater use of Level II and III tasks, other forms of student reasoning will be revealed and should lead to instruction that is more consistent with student understandings. When there appears to be a gap between where students are now and where you want them to go, consider how different problem contexts and pre-formal representations might, respectively, provide more accessible anchors and illustrative connections that have lead to an understanding of formal mathematics that makes sense. While there is much to learn, there is much that can be learned when we use assessment to open up windows into students' mathematical reasoning.

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