Promoting mathematics and language learning in interaction

This chapter offers a first introduction in the issues at play in the two mathematics classrooms on which this volume reports. Pupils in these classes are often non-native speakers of Dutch, and the question is therefore whether and how this plays a part in the way the teachers teach maths. This chapter first shows that normative theories on Realistic Mathematics Education (RME) and on teaching a second language largely confer on the idea that active student participation is a prerequisite for learning. Next, it shows how the teachers in the two classrooms deal with this requirement in a very different way. The chapter discusses examples of good practice and illustrates the pedagogy of active student participation in maths learning.

1 *Introduction*

The decision to focus the research on the content area of mathematics was not a coincidence. Mathematics is a core subject in the secondary school curriculum and surveys have shown that minority pupils have less success than their native-Dutch peers.[1] Mathematics is often considered to be a school subject which is 'language independent' and in which no problems are expected for minority pupils. Since more than a decade, this is no longer true for mathematics teaching in Dutch schools: with the implementation of the modern teaching approach in this content area, called 'Realistic Mathematics Education' (RME) the linguistic demands put on learners seem to have increased considerably. Several empirical studies suggest a relation between poor results and the didactic approach chosen by content teachers in multiethnic classrooms, such as Hajer[2] in five different school subjects, Hajer et al.[3] and Van den Boer[4] in mathematics. They both looked at the ways in which teachers convey their content matter to pupils with heterogeneous linguistic and ethnic backgrounds.

Van den Boer shows that pupils' language problems go deeper than simply not knowing certain mathematical concepts: teachers and pupils are not aware of this barrier.[5] They all use strategies to compensate for language difficulties and presume that they understand each other. Teachers expected learners to indicate troubles, whereas the learners did not want to disturb lessons and remained quiet in the classrooms. Teachers' didactic approaches seemed to discourage pupils from being active learners. Hajer studied teachers in mathematics and four other content subjects, teaching the same group of pupils and found remarkable differences in their way of dealing with language and learning.[6] The interaction patterns in the classroom were related to the depth of the content matter discussed. She describes a watershed between teachers who avoid language problems in their content lessons by lowering linguistic and cognitive demands and those who keep high expectations and look for ways to overcome linguistic barriers. The different approaches led to observable differences in interaction patterns in these teacher's classrooms. Both Van den Boer and Hajer argue that lessons should be carefully designed to promote classroom interaction if language is to be used as an essential tool for high quality learning, and conclude that multidisciplinary studies are required in this field.

Against this background, the aim of the study presented in this chapter is to deepen our understanding of the role of language in learning mathematics in the classroom in connection to the active role of teachers and their didactical choices. What happens in maths classes and what observable efforts do teachers make to stimulate the interactive process of meaning construction, while using a language that is not the mother tongue of many pupils?

As learning processes of individual learners are difficult to observe in classroom interaction we will focus on the teachers and their observable efforts to stimulate the interactive process of meaning construction.

The next section 2 provides the theoretical background and in section 3 we will present our research questions and method. In 4 the results will be presented: findings on meaning construction as part of larger classroom discourse structures, followed by an in-depth analysis of a mathematics lesson. After that we give in 5 our reflections on the opportunities for (second-) language learning as observed in the classroom data. On the basis of interviews with the teachers we will then in 6 make an attempt to account for differences between the two teachers at The Sun and The Rainbow. Section 7 contains the conclusion and discussion of our findings.

2 *Theoretical background*

As we discussed in the introduction to this volume, theories on learning in interaction form the basis of our research. The key idea of interaction between children and adults as a major source for learning has influenced research in many disciplines within the educational field. This resulted in forceful theories on the characteristics of mathematics teaching as well as (second) language teaching, two disciplines that meet in the multilingual mathematics classroom in our project. The theoretical viewpoints in both disciplines underline the importance of active participation of pupils in classroom interaction. Few studies, though, integrate these viewpoints in empirical classroom research. In this section we discuss both theories in search for a useable definition and operationalisation in observable aspects of 'meaning construction'.

Realistic Mathematics Education

In Realistic Mathematics Education (RME), mathematics is not considered to be a product that can be passed on by a teacher to pupils in one-way instruction, but something that pupils themselves have to build up through a process of guided reinvention. Starting from the assumption that mathematics has its roots in real life, children should be guided to reinvent the mathematics that was developed in the past.[7] In carefully constructed tasks they learn to transform a meaningful context problem into a representation that one can manipulate mathematically. These tasks that challenge children to look for solutions give meaning to the learning process. Informal knowledge and procedures of children form the starting point for a gradual development of more general mathematical concepts through a process of reinvention and reconstruction.[8]

The interactive teaching process is one of the main characteristics of RME. Ideally, the interaction is organised around three activities: problematisation, construction and reflection. The teacher is the activator in the process of problematisation and tutor in the process of knowledge construction, taking pupils informal strategies as a starting point for the interactional development of mathematical concepts and insights. It is desirable that teachers give pupils the opportunity to verbalise and justify their solutions and stimulate pupils to listen to each other's solutions, to compare and criticise these and to ask for clarifications; they should be 'pushing discourse'. In this negotiation of (mathematical) meaning, pupils construct common knowledge.[9] Classroom interaction thus forms a critical resource for learning.

Two different forms of interaction can be distinguished. Following Mercer vertical interaction takes place when a teacher leads a discussion with a group of children or talks with an individual child.[10] When children talk among themselves or react upon each other we call it horizontal interaction.

This view on RME implies a change of 'social norms', in relation to traditional maths.[11] Traditionally, the implicit rules for interaction are roughly that the teacher asks (closed) questions, that one pupil answers and the teacher gives feedback. RME requires a classroom climate where pupils have confidence to say what they think and have the patience to listen to their classmates and react to what they say. The idea is that pupils understand that they can learn from each other. The teacher stimulates and guides the discussions as a means for the interactive learning process.[12] The new social norms imply that a teacher stimulates pupils to bring in (alternative) solutions, encourage pupils to listen to each other, that the teacher does not accept an answer without an explanation and gives no direct feedback. In short: pupils must no longer just try to understand what the teacher means, but the teacher must try to understand what the pupils mean.[13]

In addition to social norms concerning the rules for interaction, teachers need norms to determine to what extend strategies and solutions contribute to mathematics learning. These socio-mathematical norms relate to the quality of mathematical solutions: what counts as an acceptable or a different mathematical solution, an efficient solution, or a sophisticated solution.[14]

Second language learning

Sociocultural theory had a major influence on language acquisition theory and – consequently – on (second) language teaching. An overview over these developments is provided by Gass and Selinker.[15] There is agreement in the field that it is not enough to just hear or read new language in order to acquire new concepts and structures, but learners also need to actively use and produce new linguistic elements. Put in jargon, the following three conditions for language development have to be met.

Firstly, the availability of *rich, challenging comprehensible oral and written language input* is crucial. There should be extensive exposure to new concepts and language forms, adapted to the learner in such a way that it just would exceed the actual acquisition phase.[16] The teachers talk offers opportunities to contribute to the exposure of comprehensible language but the teacher could also play an active role in making the language of the textbook comprehensible and available as a resource for learning.

As a second condition, there should be ample *opportunities for language production*. Reading and listening are not a sufficient conditions: pupils must also get the opportunity to actively produce new language. In exploratory talk they can explore new vocabulary and language functions, while focusing on both content and form.[17] This can also imply clarification requests, indicating trouble sources in the language produced.

Thirdly, language learners need *feedback* on their utterances. In order to develop correct and appropriate language, and to reach their goals in communication, learners have to get reactions on what they say or write. Teachers and class mates can react on both content and form of pupils' language and provide important information for a pupil on whether the partners have made themselves understood. If not, they can make new efforts, posing the message in another way, explaining themselves, choosing other words and expressions.

In interaction, these three elements (comprehensible input, opportunities for language production, and feedback on content and form) can not be separated: talking about new ideas and concepts creates a constant flow of listening, responding, asking clarifications, elaborating one anothers ideas, simultaneously negotiating and constructing meaning. Marton & Tsui, discussing a sociocultural view on language learning, therefore call the classroom a space for learning in interaction.[18]

Defining these keys for language acquisition in this way affected the ideas about language education. Teaching language from textbooks, focusing on language forms is not the only way to further language development and may not be the best way either. In the last two decades, extended possibilities for implicit (second) language teaching have been found in other content areas, within immersion programmes[19] and content-based instruction.[20] The need for attention to second language learners in other school subjects was also prompted by the lack of academic success of immigrant learners in education. Required cognitive academic language skills (CALP) would go beyond many pupils' capacities, especially those for whom the classroom language is not the home language and who mainly have developed basic interpersonal communicative skills (BICS).[21] The difference between these types is not a strict one, but rather a continuum along two lines: the context embeddedness of a language task and its cognitive level.

In education, the complexity of language in tasks can be influenced by a variety of teaching strategies. For example, this can be the case when the language of textbooks is of a cognitively highly demanding character, with reduced contexts, formal vocabulary and few connections to the daily world and wording. By working with these texts and promoting interaction around texts, teachers can guide pupils to connect new knowledge, concepts and vocabulary to the linguistic resources and familiar contexts of the pupils. The way in which this can be done is studied both in experimental settings and in natural teaching situations. Ellis et al. showed how active participation in interaction, discussing e.g. difficult text segments, led to more comprehension and vocabulary development in experimental studies. [22] In an experimental study, Gibbons showed how pupils develop new concepts in classroom interaction.[23] This way, a teacher who together with the pupils tries to seek for mutual understanding could bridge the gap between BICS and CALP in interaction.

In spite of these theoretical possibilities to learn language and concepts in classroom interaction, the asymmetrical roles of teacher and pupils make classrooms a less supportive context for negotiation of meaning to take place. In comparison to natural communication outside school, traditional classroom conversation does not seem to be a fertile soil for the negotiation of meaning.[24] What is more, content teachers who notice the limited language skills of their pupils tend to working in a teacher fronted way of communicating, requiring less (in stead of more) language production and to lower cognitive demands.[25] They seem to focus more on providing more comprehensible input, than on realising the other two prerequisites for language development. This phenomenon has recently been illustrated in a series of small case studies in Van Eerde et al.[26] and Hajer et al.[27]

Connecting these findings to prevailing views on Realistic Mathematics Education, we wondered how teachers would find their way in creating an interactive mathematics classroom with limited speakers of the language of instruction.

Integrating theories on teaching and learning mathematics and (second) language

In theory it seems that RME and second language teaching can be mutually symbiotic since they have some common characteristics: a focus on active participation in classroom interaction as a means of learning through language and reasoning. Thus, the integration of mathematics and language learning seems to be a fruitful approach at least theoretically. In mathematics classes based on RME one might expect to observe this integration at least to a certain extent.

Analysing classroom observational data from the perspective of mathematics and language learning requires the integration of key concepts of the above discussed theories in a model for analysis. What aspects are considered to be of major influence for the construction and negotiation of meaning in a multilingual mathematics classroom, working with text books based on the realistic view on mathematics education? We identify the following:

- The provision of mathematical problems in meaningful contexts that are explored mathematically and the language of which is made accessible and comprehensible.
- The promotion of active participation of pupils, giving them the opportunities to construct and verbalise their mathematical solutions, promoting classroom discussions, and asking for clarifications and justifications.
- Feedback of the teacher on pupils' contributions, which should not be immediate, but delayed in order to promote contributions from different pupils and horizontal interaction between pupils.

3 *Research design*

Research questions

The challenge of this study became whether we can observe this theoretically presupposed symbiosis between teaching and learning mathematics and language in our research data. Our main research question is:

> 'How do the teachers create opportunities to learn (the language of) mathematics in (whole group) classroom interaction?'

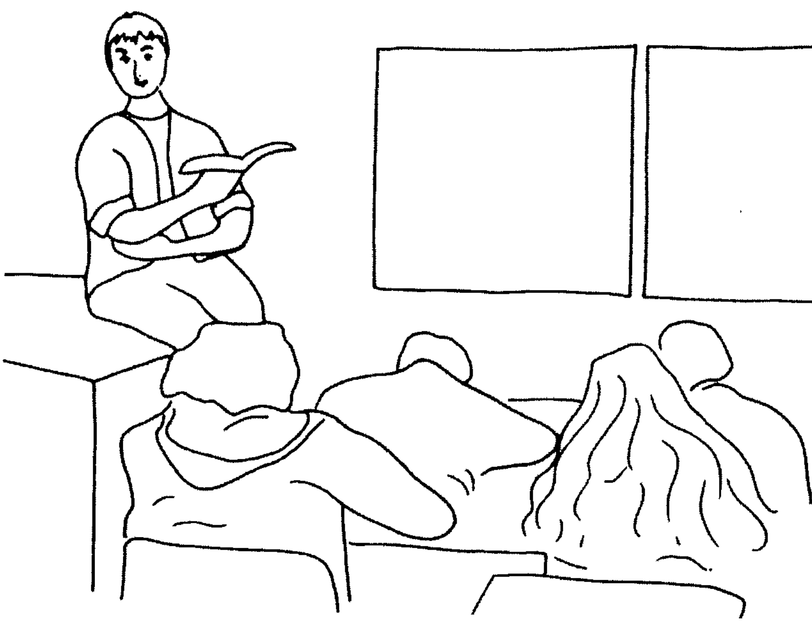
We distinguished three subquestions within this broader question:
- How do teachers stimulate pupils to explore mathematical contexts as presented in the textbooks?
- How do teachers give their pupils possibilities to verbalise, and reflect on their thinking processes?
- How do teachers give feedback on both content (mathematics) and form (language) when pupils participate in classroom interaction?
- If different interaction practices can be observed, how do these relate to teacher's views on language and mathematics teaching?

Method of analysis

In addition to the information on our methodological approach, described in the introductory chapter we will describe briefly how we went about analysing the mathematics lessons. This exploratory study was carried out in the following way. The 10 video-recorded mathematics lessons at The Sun and the 13 lessons at The Rainbow were transcribed and analysed by a mathematics education specialist as well as a second language education specialist from our team. A first analysis resulted in a description of the global interaction patterns and its thematic content.[28] The descriptions contained the overall communication pattern (teacher monologue, group discussion, question-answer series in dialogues between teacher and individual pupils etc.), and its didactic function (explanation of new subject matter, going over homework, demonstration, making assignments, etc.).

The first steps enabled us to select episodes of participation and interaction in which active involvement of pupils could be observed. We analysed these fragments from the different theoretical perspectives. Which interactional and teaching strategies did the teachers use to promote pupil participation? Did pupils refer to a lack of clarity in the oral or written language? Which mathematical concepts and strategies were subject of these moments of meaning construction? How can the use of language in textbooks,

Still Assignment: draw a graph, table with missing values



the language of the teachers and the pupils be characterised, as well as the type of teacher feedback on pupil language and mathematics production? These and other issues were analysed by researchers individually as well as in pairs. The analysis of data from both classrooms showed that in one of the classrooms hardly any extended episodes of intensive negotiation of meaning were observed. We decided to focus further analyses on that classroom (The Sun) that contained the most interaction on mathematics content.

In a second analysis several long episodes of one lesson in one classroom were selected for an in-depth analysis from different angles.

4 *Characteristics of classroom interaction at The Sun and at The Rainbow College*

In the next section we will present our results. First we will describe the global structure of the lessons and the analysis of the selected episodes in which active involvement of pupils could be observed in the two classrooms. Second, our findings on the opportunities for (second-) language learning are presented. Finally, we will account for differences between teachers, on the basis of several interviews with them.

Meaning construction as part of larger classroom discourse structures

Participation routines at The Rainbow College
The first analysis revealed the global structure of the lessons and communication patterns in the mathematics lessons at The Rainbow. The teacher starts off with checking whether the homework was made. Several pupils are requested to show their work; no interaction on the tasks is taking place. Then, a whole group discussion is started about the textbook assignments the pupils made. The interaction pattern is guided by the textbook: the teacher reads the assignment aloud, eliciting an answer from one pupil, and evaluating this answer; in case of an incorrect answer, the turn is transferred to another pupil. The interaction has roughly an Initiation – Response – Evaluation (IRE) structure.[29] Most pupils' utterances contain only a few words.

When all assignments are dealt with in this way, the teacher gives a short introduction to the new paragraph in the textbook by presenting a task, representative for the new subject matter. In a monologue he models the procedure that pupils should follow to solve this type of tasks. No discussion with the class is taking place here. At the teacher's indication, pupils then start off working at the new paragraph, some of them using the opportunity to walk to the teacher's table to get individual help; the dialogues that take place at the teacher's table are discussed extensively in Elbers et al. in chapter 5. In some lessons, the teacher starts going over these assignments as the final part of the lesson, communicating with the class in the above described IRE-sequences.

All 13 lessons that were registrated reflected this way of working. Within these data, it was difficult to identify moments of extended negotiation of mathematical concepts. Some longer exchanges in which several pupils contributed to the same mathematical question where found, though, as part of the 'going over homework' fragments. This is illustrated by the following excerpt, in which an assignment is discussed in which pupils had to match short diary fragments about the weather conditions with graphs of the temperature on different days.

Fragment 1 The Rainbow, 021299

Teacher:	Opgave b welke dag hoort bij grafiek 2 Mustafa? *Assignment b, which date matches graph 2, Mustapha?*
Mustapha:	31 maart (2.0) *March 31 (2.0)*
Teacher:	Nee (1.0) *No (1.0)*
Pupil 1:	( )
Teacher:	E:h wie heeft er wat anders? *Who has got another answer?*
Mustapha:	Het begint boven nul. *It starts above zero.*
Nordin:	1 april is boven nul. *April 1 is above zero.* ((leerlingen praten door elkaar)) *(pupils talking in each others mouth)*
Teacher:	Hoe gaat de a/ sssh ik hoor daar mensen ik hoor *How goes aah I hear people* daar weer mensen doe dat nu niet jongens. *again don't do that boys.*
Mustapha:	Bij 31 maart ( ) *At March 31 ( )*
Teacher:	Als we eenmaal gaan dalen als het koeler wordt hoe *Once we start going down,* gaat dat dan de temperatuur als die weer gaat dalen *once it starts to cool down,* dat is om een uur of drie 's middags drie vier wat *around three o'clock in the afternoon, what* gebeurt er dan gaat dat heel langzaam of gaat dat *happens there, does it go down quickly* heel snel? *or slowly?*

Mustapha:	gaat ( ) *goes ( )*
Teacher:	Gaat heel snel als ik dan het verhaaltje lees 's *It goes rather quickly. If I then read the* morgens net boven de nul 's middags meer zon het *story in the morning just above zero in the afternoon more sun,* wordt bijna tien graden ik lees daar niet in terug *it gets almost 10 degrees then I do not read* dat die temperatuur ineens heel snel daalt. *that the temperature goes down rather quickly.*
Mustapha:	Ja maar bij 31 maart. *Yes but on March 31.*
Teacher:	Nordin? *Nordin?*
Mustapha:	Zat 'ie boven de tien. *It was above ten.*
Teacher:	Ja dat klopt. *Yes that's correct.*

This sequence is typical for Mr Jager's class. He immediately reacts to the pupil's contributions. Pupils are not asked to account for their answer. Their utterances are mostly rather short and do not consist of longer sentence structures. It is only when a wrong answer is given, that more explanation is requested or short discussions arise. In conclusion: we could not find any extended moments of negotiation of mathematical concepts in whole classroom discussions. During individual help, some dialogues were identified in which lack of clarity was dealt with in interaction. Negotiation of meaning in classroom interaction was clearly not an integral part of this mathematics teacher's strategies.

Participation routines at The Sun College

When do moments of negotiating meaning occur in mr Boom's lessons at The Sun College? His lessons have a more varied and rich structure of phases, activities and interaction patterns compared to the lessons at The Rainbow, as well as to other classroom interaction studies. In general the structure is 'going over homework', 'introduction of new subject matter', 'making assignments individually with opportunities for individual help', as was the case at The Rainbow. The difference between the two teachers and classes mainly concerns the interaction patterns and given tasks. Describing the rough structures of classroom interaction, large parts of Mr Boom's mathematics lessons could not easily be described in terms of Initiation-Response-Evaluation moves. He often postpones explicit feedback moves until more than one answer has been given.

The following fragment illustrates this type of interaction. In the introduction of the series of lessons on graphs, Mr Boom puts six graphs on the blackboard, presents cards that label this contexts and then asks the group to discuss witch label matches which graph. The group discusses the label 'temperature in a house with central heating'. After several suggestions we come to the following fragment:

Fragment 2 The Sun, 081199

Teacher:	O.k.. Wie heeft er, heeft er iets anders? Jongens, *OK who has got something else. Boys,* doe eens mee. Toni? *please take part! Toni?*
Toni:	Bij de meeste verwarmingen is het zo dat als 'ie te *Most heating switch itself* warm wordt dan schakelt die zichzelf uit. En als *off when it gets too warm. And when* het kouder wordt dan twintig graden. Ja, je zet hem *it gets colder than say 20 degrees, if you put it* op twintig graden, als 'ie warm wordt dan schakelt *on twenty degrees if it gets warm it switches* 'ie zichzelf uit en als het 's avonds weer koud *itself off and if it gets colder in the evening* wordt dan schakelt 'ie zichzelf aan. *it switches on again.*
Pupil 1:	Ja. *Yes.*
Pupil 2:	Dat is het. *That's it*
Teacher:	Dat klopt wel. *That's OK*
Pupil 2:	Maar blijft toch een temperatuur. *But the temperature remains the same.*
Pupil 3:	( ) op twintig graden zetten, blijft 'ie zo *( ) put it on twenty degrees, it stays* heet. Daarom, daarom is zo aan 't schuiven. *there, therefore it changes like that.*
Teacher:	Ja, dus we hebben gehoord. Toni zegt ja, het *So what we heard Toni saying was: yes, it* schommelt een beetje want af en toe springt 'ie aan *fluctuates a bit because once in a while it switches* en weer uit.

	off and on.
Pupil:	`Maar dan blijft de winter komen, als het koud (   ).`
	But then when it is winter, if it is cold ( ).
Teacher:	`Wie is ervoor dat 'ie op de plaats komt waar`
	Who is in favour of this, what
	`Niklas gezegd. Wie is daar voor? Klopt dat?`
	Niklas told us? Who is in favour of that, is that correct?

We can notice that Mr Boom does not give feedback in terms of whether a pupil's answer is correct or false. Pupils respond to each other's contributions spontaneously. After a number of contributions Mr Boom takes the lead again and formulates a provisional conclusion, which will be discussed again later on in the conversation.

At two moments within the first series of lessons Mr Boom obviously pre-planned such a whole class discussion as an explanation of new subject matter. After these discussions the pupils could start their work, elaborating on the information discussed. Introducing a new subject, activating pupils' prior knowledge and connecting mathematical concepts to daily contexts seem to be the goals of these discussions. During other lessons, whole group negotiation of meaning occurs while going over homework assignments. The decision that one specific assignment would need special attention is often made by the teacher himself, but at other moments pupils indicate their need of additional attention to the task. The involvement of pupils in the interaction functions as a source of information for Mr Boom to locate difficulties and to take motivated action to give additional explanation that is not needed for other assignments. It also occurs that the more intensive interaction seems to develop on the spot: a longer monologic explanation by the teacher (an instruction of how to draw a graph) seems to last too long and the teacher tries to involve pupils and to keep them concentrated by eliciting contributions. This online-planning is illustrated by the lack of explicit announcements like 'we are going to talk about...' that Mr Boom gives at other, pre-planned moments.

The teacher-initiated exchanges consist of a lot of initiating and responding moves with a long delayed feedback. He recalls earlier contributions and acknowledges the pupils involved, which helps to create an atmosphere of group learning. In doing so the teacher underlines that he is not the only knowledge source or knowledge transmitter: all pupils can think as well and be mathematicians. The interaction structures in these cases become looser and are not easily fitted into give Initiation-Response-Evaluation structures that are based on a clear expert role of the teacher who evaluates responses. Previous studies have shown that a more equal status of teacher and pupils can be found in smaller settings.[30] There, both pupil and teacher are more inclined to take initiatives, respond, ask for clarification and give feedback. Older studies refer to the cognitive level of tasks: Erickson's distinction between Academic Task Structure (ATS) and Social Participation Structure (SPS),[31] suggests a mutually interdependence between variation in participation patterns and cognitive level, a hypothesis for which Hajer found evidence in multi-ethnic content classes.[32] In our study we see that Mr Boom's tasks are rather open-ended, requiring higher order thinking skills. Pupil contributions are unpredictable by content or form. Display questions would require a strict order, with the teacher conducting ritual, predictable, and often short answers to his elicitation.

In contrast to the rich negotiation of meaning this teacher aims at in whole group activities, the dyadic interactions during individual work are relatively poor. Here the teacher often communicates with the pupil in monologues, or 'teacher informs'.[33]

5 *"The missing value", an in-depth analysis of a math lesson at The Sun*

The differences between the two mathematics classes are abundant. Hardly any negotiation of meaning in classroom interaction can be observed at The Rainbow, where the teacher is the one who takes initiatives and attempts to control the interaction in short-cut Initiation-Response-Evaluation sequences. At The Sun however, whole group discussions are rather important within the mathematics lessons. It is during these moments that the more extensive negotiation of meaning can sometimes be observed in which several participants try to grasp new concepts or procedures, responding to each others utterances and together building new knowledge. The teacher is still the expert, but pupils are welcomed as persons with their own knowledge, to which their peers are made to listen. This climate seems to be related to the higher order thinking skills that Mr Boom requires from his pupils.

On the basis of the first analyses of participation and interaction patterns, we decided to focus further analyses on the lessons of Mr Boom at The Sun for 'good practice' examples of teacher strategies that can promote a desirable type of pupil participation.

Mr Boom's techniques for interactive mathematics teaching

Teachers use different strategies to stimulate pupils' interactive learning processes. We discern four different groups of techniques to stimulate the interactive learning of mathematics. The presentation and introduction of well-constructed mathematical problems, ways and activities teachers use to stimulate the construction of mathematical concepts and skills, techniques to respond to what pupils say, and techniques to come to shared meanings. These techniques are based on classifications by different authors.[34] In the following, we analyse some fragments in chronological order of one mathematics lesson of Mr Boom at The Sun that illustrate the opportunities for learning the teacher frequently offers his pupils and we also discuss limitations of Mr Boom's teaching strategies.

The mathematical problem discussed in the fifth lesson of the series on graphs is embedded in the context of an infant welfare centre where the weight of babies is monitored monthly. The task at hand concerns the construction of a graph on the basis of a table in which some values are missing. Some pupils do not know how to construct the graph because of the missing values. The teacher explains how to construct a co-ordinate system and notes down the dimensions along the axes. Then he presents the missing values as a common problem for all the pupils.

Assignment 17

The growth of many babies is regularly checked at the infant welfare centre. The data from Leonie are presented in the table underneath. Three times they have forgotten to record her weight.

Age in months	1	2	3	4	5	6	7	8	9	10	11	12
Weight in kilos	3.9	4.7	5.4	6.1	...	7.2	7.6	...	...	8.8	9.1	9.4

a Draw a co-ordinate system. Start with the axes as you see alongside. Draw the graph that goes with the table.

b How many kilos did Leonie weigh roughly when she was 5 months of age?

c And when she was 9 months of age?

The teacher almost directly starts to tell how to solve the problem of the missing value without further introduction. He pays no attention to the context of the infant welfare centre and there has been no check on the concepts 'record' and 'monitored'.

After presenting the problem Mr Boom asks for solutions, he asks them how they have approached the problem.

Fragment 3 The Sun, 151199

Teacher: En nou komen we bij het probleem: op een gegeven
And now we come to the problem: at a given
ogenblik ben je bij vijf en dan staat er geen stip.
moment you get to five and then there is no point.
Wat moet je dan doen, Hennia?
What do you have to do then, Hennia?

Hennia: Gewoon het middelste van, tussen vier maanden en zes
Just calculate the middle of, between four months
maanden uitrekenen.
and six months.

Teacher:	Je hebt het gemiddelde uitgerekend. 6.1 is de één, *You have calculated the mean, one is 6.1,* 7.2 is de ander dan, eh *the other is 7.2 then, er*
Hennia:	Gemiddelde. *Mean.*
Teacher:	Dan heb je het gemiddelde. Hoe doe je dat, dat *Then you have the mean. How do you do that,* gemiddelde? *that mean?*
Hennia:	Eh, hoe deed ik dat ook alweer? *Er, how did I do that again?*
Teacher:	Ja, hoe doe je dat? *Yes, how do you do that?*
Hennia:	Die 6.1 en die 7.2 bij elkaar optellen en dan delen *Add that 6.1 and that 7.2 and divide* door twee. *by two.*

The teacher introduces the missing numbers as a problem and asks Hennia how she solved the problem. Hennia answers: 'calculate the middle between 4 and 6 months' and the teacher rephrases Hennia's answer by saying: 'you have calculated the mean, one is 6.1, the other is 7.2'.

By this *rephrasing* he probably tries to clarify to the others what Hennia says and stimulates the others to participate. Maybe he also checks whether his interpretation of her words is the right one. This rephrasing, however, has risks: the teacher changes Hennia's answer. He changes 'the middle' into 'the mean' and mentions the values at 4 and 6 months. Hennia repeats his concept: 'mean', but it is not clear if this is meant as a confirmation or if she just repeats her teacher. In any case she orients to the teacher's reformulation of her words. In the second place the teacher refers to the measurements, weight in kilogram (6.1 and 7.2) in stead of the months 4 and 6 in which these were registered.

It is not clear what Hennia meant by the 'middle' and how she did her calculation. The teacher first could have asked her what she meant by 'the middle' and how she intended to calculate that before giving his own interpretation. When the teacher after rephrasing her answer, asks information about Hennia's strategy he again refers to the mean. Hennia does not remember directly how she did her calculation. This might indicate that she meant something different.

Mr Boom then invites the class to say what they think of Hennia's answer. But in spite of his efforts, pupils give their own answers in stead[35] (cf. Berenst & Koole in chapter 4 in this volume, who analyse this event in terms of negotiating participation frameworks). One of them is Maktoub.

Fragment 4 The Sun, 151199

Maktoub:	Ik heb gewoon pauze gemaakt. *I made a break.*
Teacher:	Jij wilde een pauze maken. *Did you want to make a break of it.*
Maktoub:	( )
Teacher:	O, dus dan woog ze gewoon effe niks... *Oh so she weighed nothing for a while...*
Class:	Ja hoor. *Yeh.*

With *make a break* Maktoub probably means that he did not draw the graph at the point of the missing value. This answer is excluded from serious consideration by Mr Boom's somewhat mocking reaction, supported by a sardonic *yeah* from the class.

A second pupil, responding to Mr Boom's elicitation is Patricia, who is followed by Karl.

Fragment 5 The Sun, 151199

Teacher:	Patricia wat heb jij. Jij zegt ik kan het niet zo *Now, we had got to you Patricia. What have you got? You say I cannot see it* overzien, want ik heb een andere oplossing. Wat heb *so clearly, because I have a different solution. What did* jij gedaan? *you do?*
Patricia:	Ik heb gewoon gekeken hoeveel ertussen elke stap *I just looked at how much there was between* staan. *each step.*
Teacher:	Toen heb je dat weer gedaan. En toen het bij 6.1 *Then you did that again. And then added it* opgeteld. *to 6.1.*
Pupil 2:	Dat wou ik ook doen, maar het is steeds *That is what I would do too, but it is always* verschillend. *different.*
Pupil 3:	Ja. *Yes.*

Teacher: Ja, het is steeds verschillend. Ja, da' s wel
Yes, it is always different. Yes, that's a bit
lastig. Als het steeds hetzelfde was, was het
awkward. If it were always the same, it would be
gemakkelijker geweest. Dus je moet wel een beetje
more convenient. Therefore you have to take a bit of
gokken dan. Wat heb jij gedaan Karl ?
a guess. What did you do Karl?

Karl: Ik heb die 3,9 en dan 0,8 is 4.7. O, nee, erbij
I did that 3.9 and then 0.8 equals 4.7. Oh, no, and
0,7, bij 0,7.
0.7, to 0.7.

Teacher: En toen weer een keer bij 0,7?
And then once more with 0.7?

Karl: Ja, en toen heb ik gewoon nog een keer erbij 0,7
Yes, and then I just added 0.7
gedaan.
once more.

When Patricia tells her solution, two other pupils recognise their own strategy and this triggers them to actively participate in the interaction. One pupil recognises the approach he wanted to choose and the limitation it has: the interval between the numbers is not constant (*it is always different*). Karl has a similar approach. He explains that he calculated the difference between 3.9 and 4.7, first this is 0.8 later on he says it is 0.7, and ads this up to 6.1. This shows how he struggles with the changing differences between the measurements.

The teacher then summarises the contributions of the pupils, still without giving his opinion.

Fragment 6 The Sun, 151199

Teacher: Wat jullie allemaal een beetje doen, is toch wel om
What all of you are tending to do, is in
op de een of andere manier zelf iets daar neer te
some way or another to put something there yourselves.
zetten. De een heeft met stapjes gedacht, steeds
Someone thinks with small steps, so much each time.
zoveel. Jij bent er meer tussen in gaan zitten.
You chose somewhere in between.

He explains that they all choose a similar approach, one approach he associates with 'steps' the other with 'somewhere in between'. He does not react to the problem Patricia has because of the different intervals, which we also recognise in Karl's explanation. Maktoub's solution is excluded from this summary. The teacher *delays his feedback*, probably to draw in other pupils' contributions. This results in Toni giving the next solution:

Fragment 7 The Sun, 151199

Teacher:	Toni heeft het nog anders. *Tony has yet another solution.*
Toni:	Pauzes, gewoon niks. *Breaks, just nothing.*
Teacher:	Pauzes, je hebt gewoon niks getekend. *Breaks, you just drew nothing.*
Toni:	Nee, gewoon streep trekken en dan effetjes niks en *No just drew the line and then nothing for a bit and* dan weer verder de streep en weer. *then the line again.*

Here Toni, who sits right behind Maktoub, repeats the solution Maktoub presented earlier. He even uses the same word ('break') to characterise his solution: the line of the graph takes a break at the missing value and continues again where we have information again on the weight of the baby. It is remarkable that Mr Boom accepts this solution from Toni while he gave Maktoub a rather negative reaction. Then, the teacher delays his feedback again to look for more solutions or maybe with the intention to *stimulate reflection*. In any case it results in the following reaction from Hennia.

Fragment 8 The Sun, 151199

Teacher:	Ja, Hennia? *Yes, Hennia?*
Hennia:	Ja, maar alles wat je schrijft, dat is toch niet *Yes, but just because you write something does not make it* waar. Want je weet niet. Een kind kan in een keer *true. Because you do not know. A child can grow all of a sudden* groeien en in keer heel, dinges, kort blijven. *and at times remain very, thingy, short. It is never right.*
Teacher:	Dit is *This is*
Hennia:	Dat klopt nooit. *It is never right.*

Teacher: Dit is zo'n waarheid Hennia. Dit is zo goed. Als
This is so true Hennia. This is really good. If
iemand op het ene moment vijf kilo weegt en een
someone weighs five kilos at some time and a
maand later weegt 'ie 6 kilo, dan is, dan weet 'ie
month later he weighs 6 kilos, then, then you can never
nooit wat 'ie daar tussenin is geweest. Maar hij
be sure what he was in between. But he
kan wel ondertussen zeven kilo geweest zijn. Dat
could have been seven kilos in the meantime. You
weet je niet. Dus eigenlijk weet je het nooit. Je
do not know. Therefore you actually never know. You
weet alleen maar die puntjes.
only know the points.

Mr Boom's silence after Toni's answer does stimulate Hennia's reflection. She makes an interesting point that you can't be sure about the missing value and gets credit for it. Hennia returns to the context of the problem, the weight of babies. The teacher elaborates Hennia's answer in the context of the baby's weight, but the example that he gives is not very realistic. The weight of a baby does not make such big fluctuations. It is highly questionable whether this will make the context come alive. With the exception of Hennia, no pupil mentions any of it.

After the inventory of different contributions of the pupils the teacher recapitulates these and says they are all a bit right.

Fragment 9 The Sun, 151199

Teacher: Weet je hoe, wat nou bij wiskunde moet? Het is een
Do you now know what you have to do in mathematics? It is a
beetje mengeling tussen wat jullie gedaan hebben,
bit of a mixture of what you have done,
Toni heeft bijna gelijk en je moet een pauze nemen.
Hennia heeft een beetje
Toni is almost right you have to take a break. Hennia is almost
gelijk. Je moet een soort gemiddelde nemen. Zoeken
right, you have to take a sort of average. Looking
naar de stapjes. Jongens hebben een beetje gelijk.
for the steps, the lads are partly right.

He also labels the solutions as 'take a break', 'a sort of average' and 'looking for the steps', thus acknowledging these contributions. Then at the end, the teacher introduces his own approach for finding the missing measurement: connect the given points in the graph, read off the missing point and note that down in the table:

Fragment 10 The Sun, 151199

Teacher:	`Wat we meestal doen is het volgende. Stel je voor`
	What we usually do is the following. Suppose
	`dat bij vier dit punt hoort. Bij vijf weet ik het`
	that this point corresponds to four. I do not know what corresponds to five.
	`niet. Weet ik niet dus daar komt geen punt. De`
	I do not know so there is no point there. The
	`volgende is bij zes. Dus hier heb je allemaal puntjes`
	next is at six. So here you have points
	`zo lopen. Wat we gewoon doen is gewoon verbinden.`
	that run like this. What we usually do is just connect them.

Toni:	`Ja, had ik ook.` *Yes, that is what I had too.*

Teacher:	`Toni, Toni, je moet niks overslaan. Je moet gewoon`
	Toni, Toni, you must not miss anything out. You just have to
	`zeggen, nou ik ga naar de volgende punt. Zo hup`
	say, now I go to the next point. Like this, jump
	`((verbindt twee punten in assenstelsel op het bord)),`
	((connects two points in coordinate system on the board)),
	`en nou moet je toevallig een heel stuk overbruggen.`
	and as it happens you now have to fill in a large piece.

The teacher does not compare his solution with those of the pupils. He presents his approach as the thing to do: 'what we usually do is....'. In doing this he does not give any justification why his approach is better than those of the pupils are. He could have given a justification for his solution in the context: the growth of a baby. After all in most cases the growth of a baby develops more or less regular, so that connecting the given values will give a good estimation of the missing measurement.

The context plays hardly any role in this interaction, neither as a source for giving meaning to the problem nor for the application of mathematical concepts. This is a lost chance because the context could have given meaning to the problem at hand and a justification of Mr Boom's solution.

During his lessons Mr Boom uses different ways to challenge and motivate his pupils. For example he labels a question as difficult but at the same time encourages his pupils by saying: 'come on, you are smart'. He uses his own body weight as context (he is not the slimmest of men) and makes jokes about it.

After the reconstruction of the pupils' contributions and the clarification of his own solution, the teacher wants to evaluate the learning process. He introduces a similar problem with a missing value in the context of his own length. When Mr Boom for example checks whether Claudia understands his explanation she says:

Fragment 11 The Sun, 151199

Teacher:	Wat moet ik nou doen, Claudia, als ik die grafiek *What do I have to do now Claudia if I make a graph* maak? *out of this?*
Claudia:	Het gemiddelde van dat meten. *Take the mean of that.*
Teacher:	Dat kan, maar wat heb ik gezegd wat de goede manier *That is possible but what what did I say was the correct* is? *way?*

In response, Claudia repeats Hennia's solution: calculate the mean. This response shows that Claudia has not picked out, from among the various solutions that have been proposed, the solution favoured by the teacher. This may very well be a result of Mr Boom's evaluation practice in this discussion. He assesses several solutions as 'good' and indeed highlights Hennia's proposal as clever. This seems to have confused Claudia into thinking that this is the solution favoured by the teacher. The teacher encourages the pupils to find a quicker and simpler solution but this is in vain, they stick to the solutions one of the other pupils gave. Only one pupil, Fadh, seems to apply the teacher's solution.

Fragment 12 The Sun, 151199

Fahd:	Je gaat van eh, 14,8 naar 16.3 en dan eh, kijken *You go from fourteen point eight to sixteen point three and then uh uh look* wat er op die lijn van vijf is en dan en zet je daar *what is on the line of five and than you put* een punt. *a dot there.*
Teacher:	Mja, ik denk, ik heb het vermoeden, je zegt het *Hmmm yes I guess I, I, I have the suspicion you say it* moeilijk maar ik heb het vermoeden dat het goed is. *in a complicated way but I have the suspicion that it is OK.* Laten we kijken of iemand het simpeler kan zeggen. *Let's see if someone can say it in a simpler way.*

Fahd is the only pupil that seems to have chosen the approach of the teacher. But according to the teacher Fadh does not put his thoughts clearly into words. The teacher does not give him the opportunity to try this again nor asks him for a justification. Maybe Mr Boom does not go into Fadh's reaction because he wants to give others the possibility to reformulate it. Apart from the question how Fadh feels after this reaction of his teacher, this is a missed opportunity because nobody else mentions Mr Boom's solution.

The uncertainty among the pupils about the right approach remains as appears from Hennia's final question, what to do with a problem like this in the text test. The teacher answers her by repeating his solution once again step by step.

In conclusion: Mr Boom's interaction promoting strategies

The analysis of this long discourse shows that Mr Boom has at his disposal *a wide variety of strategies to stimulate participation and interaction.* He stimulates the contribution of the pupils by asking them to tell their strategies, by rephrasing these, by labelling them, and delaying feedback to give everybody the opportunity to tell their solutions. Although different pupils participate in the interaction it seems that there is a series of successive vertical interactions between the teacher and one pupil. The *social norms* of the class are reflected in the interaction. Mr Boom is interested in the pupils' solutions. Different pupils tell their solutions and if requested clarify their own thinking. The pupils seem to listen to each other, there are not many interruptions and occasionally they react on each other. There is, in other words only limited horizontal interaction between pupils. Mr Boom gives the pupils much opportunity for language production in expressing their mathematical solutions. The mathematical constructions of the pupils become observable in the interaction and can become subject of discussion for the whole class. In this way he *stimulates negotiation* and *construction of meaning takes place.*

Mr Boom's challenging way of teaching also has some drawbacks. Some observations seem to indicate the inclusion and exclusion of some pupils. Mr Boom seems to appreciate Hennia's contributions very much, while Maktoub's solution is ignored more than once. Apparently not all pupils are treated alike (see also Pels, this volume, on teachers' reprimands).

Mr Boom *pays neither attention* to the context of the problem, nor to the concepts and the text in the mathematics book. Hennia refers to the context with her remark that you can never be sure about a missing value. After that Mr Boom tries to go into the context but it does not come alive. Because of this he misses the opportunity to use the context in the justification of his own solution.

There seems to be enough opportunity for pupils to give their contributions. Why do the pupils stick to their own solutions when the teacher is offering a simpler one? This may have several reasons. It is probably partly due to Mr Boom's way of delaying feedback. He says for example that all the solutions were a 'bit right', without saying why. Moreover, Mr Boom did not compare or discuss the different solutions by reflecting on socio-mathematical norms about efficiency or sophistication of strategies. He does not make explicit why a certain solution is different from another or even better. Even though this teacher tries to get pupils to discuss each others' answers, the pupils orient to the teacher as the one who evaluates answers.[36] There is hardly any horizontal interaction between pupils. Explicit negotiation of meaning between pupils does not occur. It is also possible that the pupils thought Hennia's solution (the mean) was a logical one. And as we said before, Mr Boom did not give any justification for his own strategy to convince his pupils for example by referring to the context. Mr Boom's teaching seems to reflect a view of the discovery of mathematics rather than the reinvention of mathematics.

6 Opportunities for (second-) language learning in the math classroom

In section 2 we described conditions for L2-acquisition in content classes: the availability of comprehensible written and oral language, opportunities for producing new language elements, and receiving feedback on form and content of these utterances. Here we will apply these concepts to Mr Boom's mathematics lessons to see whether he succeeds in creating these conditions within his approach, specifically within the lesson on the 'missing value'.

Estimating the comprehensibility for the pupils of the teacher's oral language and of the written language in the mathematics book is a hard enterprise, given the differences in language skills within the group.[37] Analyses of this language input reveal some potential trouble sources. Prenger shows the complexity at vocabulary level.[38] She compared the vocabulary of the textbook with Mr Boom's oral language, distinguishing specific mathematical concepts (like *table, coordinate system, graph*), general academic vocabulary (*explain, regular, overlook, omit*) and infrequent words from daily life (*infant welfare centre, stopper, fairway*). Research shows that a lot of these potential language and context problems stay unobserved in mathematics classes in general.[39] In our data, there is quite a number of infrequent words that only occurred once or twice in the chapter. This can be explained by the many different contexts in the assignments: each assignment introduces new situations from daily life. However, Prenger shows a mismatch between this written and the oral input.[40] In classroom interaction in the 'missing value' lesson, Mr Boom only uses the words 'table' and 'weighed', without explaining the meaning or the whole context of the infant welfare centre. The centre's function and way of working, the measuring of babies at (regular) intervals is not discussed. Not only in this case, but also in other assignments, contexts are not discussed, or in other words, the teacher does not check the comprehensibility of these written texts. We have evidence from thinking aloud tasks around the same subject matter,[41] that these infrequent words from daily contexts can cause problems for second language learners. A proper understanding is often essential for the solution of an assignment. Even at other text levels the comprehensibility can cause trouble: the exact order in which the assignment has to be carried out often depends on close reading of the instructions.

In whole group interaction pupils do not ask for clarification of the assignments. Koole & Berenst (chapter 4) show that the common norm obviously is during these moments to show what you understand and not so much to express non-understanding.[42] The majority of The Sun class being native speakers of Dutch, one could say that second languages learners could postpone asking Mr Boom questions about unknown vocabulary until after the whole group discussions. But even in the dyadic interaction between individual pupils and the teacher, he concentrates on procedural aspects of the tasks. He seems not to be aware of potential problem sources in the context of the assignment, including vocabulary. In combination with a lack of prompting pupils to define their problem, these questions simply do not come up[43] on dyadic interaction for a broader discussion of this aspect. Written input is not made comprehensible in classroom interaction, or in dyadic interaction.

A second important prerequisite for language development, discussed in section 2, is the opportunity to actively use new language. During whole group interaction these opportunities are relatively large in terms of the number of different pupils involved and the length of their utterances, as compared to studies in similar contexts like Hajer.[44] In the first lesson, in which they match labels and graphs, 19 out of 26 pupils got at least one turn. Interaction between pupils in pairs or small groups is not promoted, which could have given more pupils the chance to produce new mathematical language. In the case of the missing value episode, Mr Boom could have asked groups to compare their solutions and then ask every group to contribute to the whole class discussion. We also notice that no written output is required in Mr Boom's mathematics lessons except for the assignments from the mathematics book pupils make individually.

Considering the quality of their participation in linguistic terms, we see that pupils contribute with relatively long utterances that show their own production and are not merely reproductions of language from the textbook. Even here the classroom interaction is remarkable compared to studies that show utterances, limited in size and originality. Pupils seem to feel free to try out putting their thoughts into language.

The third aspect we studied is the feedback on pupils' oral language production. Several pupils, also the multilingual ones, could be observed taking the opportunity to produce new language forms. The quality of the teachers feedback is however doubtful from the perspective of L2-development. Often, feedback is limited to accepting pupils input with 'okay', 'yeah' or repetitions by the teacher. Pupils do not know whether their contribution is adequate, correct, mathematically nor in its linguistic formulation. Poor formulations or unclarities in these contributions do not get attention, such as requests to pupils to rephrase or explain what they said. The teacher's reaction on Fahd in the 'missing value'-fragment (printed above as fragment 12) can illustrate this:

Fragment 13 The Sun, 151199

Fahd:	Je gaat van eh, 14,8 naar 16.3 en dan eh, kijken *You go from fourteen point eight to sixteen point three and then uh uh look* wat er op die lijn van vijf is en dan en zet je daar *what is on the line of five and than you put* een punt. *a dot there.*
Teacher:	Mja, ik denk, ik heb het vermoeden, je zegt het *Hmmm yes I guess I, I, I have the suspicion you say it* moeilijk maar ik heb het vermoeden dat het goed is. *in a complicated way but I have the suspicion that it is OK.* Laten we kijken of iemand het simpeler kan zeggen. *Let's see if someone can say it in a simpler way.*

In the context of the discussion Fahd's contribution is perfectly clear, we think. The only thing is he says 'lay a dot' instead of 'put a dot'. By transferring the turn to another pupil, the teacher does not provide Fahd with the chance to rephrase his formulation, which could have been very useful for him as a L2-learner. The importance of rephrasing pupils 'interlanguage' (second language forms that could be improved in content and form), is missed frequently in our data, as in the next example. The pupils have to read a graph and give their reading in their own words. As usual, several pupils get the chance to give their solution:

Fragment 14 The Sun, 101199

Teacher:	Wat heb jij staan? *What have you got?*
Maktoub:	Dat ie niet snel groeit. *That he is not growing fast.*
Teacher:	Dat ie niet snel groeit in gewicht. Hennia wat heb *That he is not growing fast in weight. Hennia what* jij staan? *have you got?*
Hennia:	Dat hij minder dikker wordt. *That he gets less fatter.*
Teacher:	Minder dikker, maar hij wordt nog wel dikker he? *Less fatter, but he still gets fatter huh?*
Hennia:	Ja dat zeg ik, minder dikker. *Yes that's what I say less fatter.*

Pupil 1:	Minder. *Less.*
Teacher:	Minder dikker, ja je zegt het keurig, klopt. *Less fatter yes you say it nicely, correct.*

The different wordings that are used by the pupils form additional input for L2-learners. But is the quality of all phrasings equal? Maktoub 'growing' is elaborated by the teacher as 'growing in weight', an unusual way of saying 'gaining weight'. In our view the unusual phrasing of Hennia 'less fatter' should have been corrected or related to the more formal 'gaining less weight'. The teacher's explicit reinforcement of this incorrect form (you say it nicely) is not appropriate. It is all right for Hennia to feel confident in her L2, but here the teacher misses a chance of contributing to her L2-development. In general, the teacher does not systematically build academic language skills on the daily communicative skills that children produce. Only once, during individual help of a Dutch girl, we heard him paying notice to the distinction between describing the context of a graph (in daily words) and the description of the graph in mathematical terms (lines that rise, remain stable, or fall). In this aspect there is much to gain in strengthening the lessons as language learning context.

Finally, we will discuss an interesting finding concerning the language of the assessment test at the end of the series of lessons on graphs. Like the textbook, the assignments contain much infrequent vocabulary due to the contexts chosen. Had the teacher paid attention to this specific vocabulary during his lessons, some pupils could have understood the test items better. But pupils are also confronted with new contexts and vocabulary. From the active participation of Hennia in the lesson on 'the missing value' we have evidence that she understands the (cognitively rather complex) fact that you will never be sure about the missing value, even though you can draw a graph. In the assessment test a similar issue comes up concerning the graph related to a family's trip in a cable car in the mountain. There is a missing value here too:

> Fred says "I am sure that they were at 750 meters height after 9 minutes". Is Fred right? Explain clearly why he does or does not.

Hennia misses this item. We know that she understands the issue, so something must be wrong in her understanding of this particular context. This might be the text type: it starts with the quote of an unknown speaker (who is this Fred?), then changes into a question and an instruction. It can also have to do with the context of the cable lift: Hennia also misses two earlier items related to the cable lift context. Another hypothesis for her failure is that the mathematical language 'the missing value' is absent and that Hennia can not relate this item to the issue that was raised in the lesson and that she understood so well. The crucial words of Fred that have to be picked up by Hennia are "I am sure", but the instruction "Explain..." does not explicitly refer to this part of his statement. The right interpretation would have required specific academic reading skills in this type of mathematics test items.

In conclusion: although the oral participation of pupils in whole group interaction at The Sun is a positive feature for language development, other crucial conditions are not fulfilled. The written input in textbook and tests is not made comprehensible, nor at a vocabulary level, nor at a more general level of academic reading skills. The example of the fully participating Hennia, who nevertheless does not succeed in passing the test raises questions on the value of participation per se: there are more aspects of language learning in interaction that need to be fulfilled to guarantee L2-learners to develop mathematics skills through their second language. We could not observe a systematic organisation of tasks leading pupils from receptive to productive use of the goal concepts and formulations.

7 *Comparing two teachers: behaviour and accounts*

Two multiethnic classrooms have been observed in which teachers and pupils work with the same textbook. Does that automatically lead to comparable teaching approaches? Clearly not. Not only does the overall organisation of teaching in The Rainbow differ from that in The Sun, but also do we see a fair amount of variation in the quality of Mr Boom's teaching at The Sun. In this section we first focus on *differences between what the teachers do* in the classroom observations and evaluate these in terms of creating opportunities for learning. For the sake of the question governing this chapter ('Is good maths teaching compatible with good L2-teaching?') we will focus on examples of better and worse teaching. The following aspects will be discussed: the mathematical content, teacher strategies for stimulating interaction, the social norms for interaction, the socio-mathematical norms, the language development, and the pedagogical climate. In the subsequent section, we focus on differences between what the teachers say in the interviews about their view on mathematics learning and on language development in mathematics lessons, and their roles in the class. Conclusions will relate data from the classroom to the interviews. The point of this section is not to make out who is the better teacher, Mr Boom or Mr Jager, but to establish what good maths and language teaching may look like. Having said that, we will argue that it is not a coincidence that we find more examples of good practice with Mr Boom than with Mr Jager.

Comparing classroom observations: differences in what teachers do

The mathematical content

As stated earlier in this section, we could hardly discover any moments of negotiation of mathematical concepts in classroom interaction in Mr Jager's class at The Rainbow, whereas Mr Boom frequently challenges his class to think aloud about mathematical concepts. These teachers work with the same textbook on the same mathematical content, so one can wonder how the differences in interaction relate to mathematics content in the two classes. There are some interesting differences in the way both teachers treat the subject of graphs using almost the same textbook. We look at the role of the textbook, of contexts and of conceptual and procedural knowledge.

Mr Jager strictly connects his lessons to the mathematics textbook. He often reads the questions from the textbook out loud or asks a pupil to do so. This gives the 'going over homework' fragments a clear and predictable structure, which seems to function as a means of managing the classroom order. This does not mean that the text itself is being discussed or explained, but it limits the discrepancy between oral and written input. It takes Mr Jager 9 lessons to complete the chapter, even though he did not elaborate on it with additional assignments, contexts and classroom discussions. In contrast, Mr Boom hardly pays any attention to the textbook. He elaborates on the content matter in the textbook by adding his own problems to introduce the new subject, by presenting variations of problems in the book, and by interspersing them with contexts from his own personal life (for example, his own weight) to which the pupils react very enthusiastically. In this way he increases the pupils' motivation as well as their opportunities to learn.

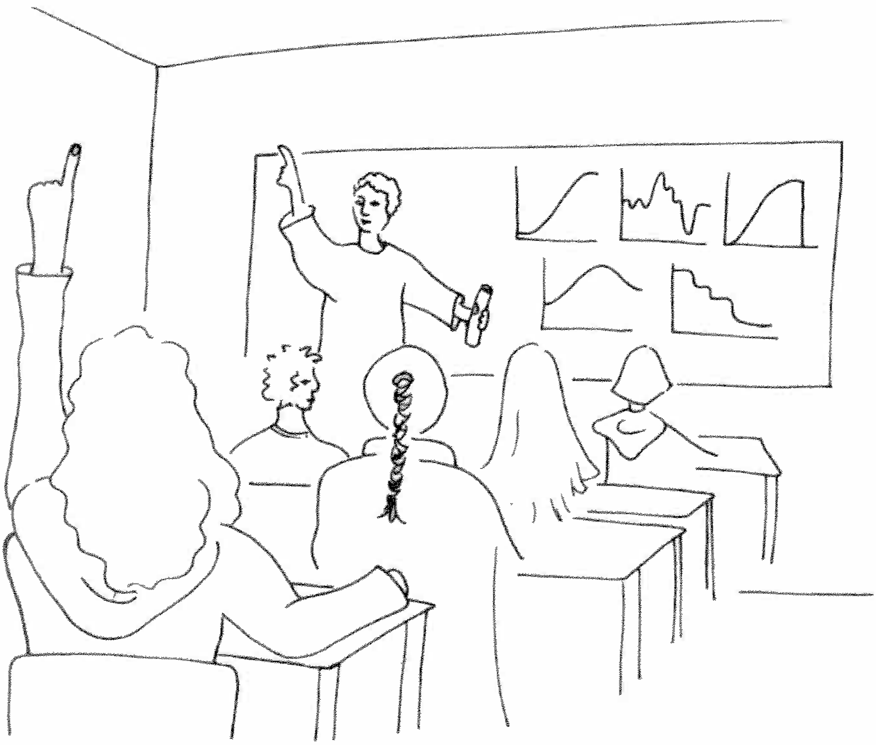
Both teachers hardly pay any attention to the introduction of contexts. They do not check whether pupils are familiar with the contexts and the concepts that are used. The assignments from the textbook are 'solved' without relating the answers to the context. In other words, no attention is paid to the process of 'horizontal mathematisation': transforming everyday problems into mathematical problems.

In mathematics lessons, procedural and conceptual knowledge is at stake. Procedures are in the centre of attention for both teachers, in the whole class discussions as well as during individual help. The teachers do explain the meaning of concepts such as rising, descending and staying constant on graphs. But neither of them explicitly negotiates the meaning of these concepts. They never ask pupils to express a concept in their own words. Moreover, when a concept does get topicalised, it is not necessarily the most prominent concept. In one lesson Mr Boom spends much time explaining the mathematical concept of 'sawtooth' *(zaagtand)* and gives about five different labels for it varying from a 'paper saver' to a 'thunderbolt'. Pupils keep asking questions about this relatively unimportant concept while, at the same time, we observed that some pupils do not know the difference between a 'co-ordinate system' and a 'graph'.

Teacher strategies for promoting interaction and participation

The two teachers differ radically in their attempts to stimulate interaction and in the quality of the interaction. Most of the interaction we observed in Mr Jager's class had the character of the IRE pattern, maintaining his central role as director of the classroom

Still Plenary discussion of the different graphs



interaction. The exchanges are short and directed towards individual pupils rather than to the class as a group. By his questions, Mr Jager seems not to challenge the class cognitively. At The Sun, we observed several lessons in which Mr Boom puts the solutions of children in the centre of the interaction in stead of his own solution. Mr Boom has a variety of techniques to stimulate the interaction with and participation of pupils, as we showed earlier. He elaborates on the problems in the textbook, offering challenging and engaging contexts, he stimulates the interactive construction of meaning, has a repertoire of responding to what pupils say, and techniques to make common knowledge explicit. Most of the interaction in both classes is still vertical between the teacher and pupils, also when there is a whole class discussion, and Mr Boom thus misses opportunities to use horizontal interaction between pupils as a source for learning.

Social norms and socio-mathematical norms

Social norms refer to ways of behaving in classroom interaction. As stated in section 2, realistic mathematics education requires a change of these norms. In Mr Jager's class the traditional rules of knowledge transmission still seem to be valid: he hardly ever pays any attention to the thinking process of the pupils, it is as if there still is only one correct answer that should be delivered by the teacher. Mr Boom has succeeded in creating a climate with different social norms, stimulating the pupils by asking for solutions, explanations and justifications, and challenging them to participate in the discussion. Several pupils seem confident in playing this new role, sometimes they even judge their own solution as the best although the teacher offers his one as a better approach to the problem (see 'the missing value'). Not all pupils have accommodated to this new role of active participant. They do give their solutions and listen to each other, but they hardly ever ask each other a question or react to each others' solutions. We noted that Mr Boom did make clear that he is interested in the mathematical thinking of his pupils but never explicitly discussed their role in the rest of the discussion.

Socio-mathematical norms are in part concerned with the quality of mathematical solutions. In Mr Jager's lessons this aspect is missing because he hardly pays any attention to the solutions given by his pupils other than by judging them as right or wrong. In Mr Boom's class this aspect seems to play a role because in different lessons we observed Mr Boom asking for 'more simple' solutions than a certain given one. But he does not succeed in making this clear to his pupils. This became very clear in the long observation about finding missing values where he tried to convince the pupils that his solution was simpler than their 'own', which was in vain. Mr Boom did not convince the pupils why his solution was better. We have the impression that Mr Boom presumes the pupils understand what he means by 'a more simple solution'. The socio-mathematical norms just as the social norms are more or less implicit instead of explicitly discussed.

Language development

Although there are many pupils with poor language skills at both schools, neither of the two teachers pays attention to language development. The text in the mathematics book is not made accessible for the pupils; the teachers do not check comprehension of the assignments. There is a big difference in the opportunities both teachers create for language production. Mr Boom stimulates productive language by asking open questions requiring higher order thinking skills. In Mr Jager's class, pupils' language production is limited by his focus on eliciting correct answers without interest in the mental processes. Neither of the two teachers gives feedback on what pupils say as a result of which pupils do not get opportunities for (second) language learning.

Pedagogical climate

The pedagogical attitude of both teachers differs completely. Mr Boom's approach to the pupils is quite personal. He gives individual help at the pupils' tables, he varies his whole class and individual explanations with informal talk and he makes jokes, but he also makes personal appeals to pupils who do not behave. Mr Boom requires pupils to take on their own responsibility, for example, for correcting their homework and to know what subject is under discussion. The pupils seem to have great respect for him. Mr Jager does not make personal contact with the pupils and stays behind his desk most of the time. Because of his disciplinary problem he tries to keep good control of the pupils. To keep control, he does not give them room for contributions in the discussion, which seems to be related to the short IRE-sequences. Some of his pupils do not show any respect for Mr Jager. This climate is an essential restriction for the learning process. The relation between the pedagogical climate and the cognitive climate would need additional study: one could wonder whether lower cognitive demands create the opportunities for pupils to concentrate on other things than mathematics. This subject is extensively discussed in the chapter on problems of disengagement.[45]

Comparing interviews: differences in what teachers say

In addition to our observations of the two mathematics classes, which showed significant differences in participation and negotiation of meaning, we will now listen to what the teachers have to say. Changing teachers' behaviour from traditional mathematics teaching towards realistic mathematics teaching is not guaranteed by offering new textbooks. We have observed two teachers working with the same textbook in quite different ways. We will need to pay attention to their beliefs or cognitions[46] to find additional data which can explain this finding. How do they present themselves in the series of stimulated recall interviews?

Some of the teacher practices can be explained from the *roles* in which they obviously see themselves within the class. In the interviews they comment on their work from these different roles, or rather they express different identities in their talk. Mr Boom seems to be the more experienced teacher, who has no problems with classroom management and can concentrate on content, while also observing individual pupils' contributions. He talks as a self-confident planner and conductor of the class orchestra, triggering their thinking. Mr Jager mostly talks as a classroom manager. He seems to have less energy left to pay attention to individual pupils and the development of their mathematical thinking. Typical for the two positions is the way in which the teachers answer the opening questions of the SR-interviews "How did this lesson go?" Some answers Mr Jager gave in different interviews:

> "I again found the start extremely turbulent. I got angry straightaway with a number of unco-operative pupils."

> "I was in a very angry mood myself. Afterwards I thought about why this happens; I could not find the answer. I think that I have a low threshold with those children when it comes to continuously chattering and disrupting the lesson."

Mr Jager's focus on his role as classroom manager also influences his interaction strategies. He repeatedly accounts for his turn allocation to specific pupils as a matter of classroom management:

> "The pupils I asked at the time had not done their homework. (...) Now I wasn't going to transfer the turn to someone else who wants to answer, because clearly you just didn't do your homework, so then I let you sweat for a while."

As an observer of learning processes, Mr Jager talks about the class as 'the people', 'the group', 'they' and less by using names of pupils. It seems as if he does not have the mental space or time to note individual differences or contributions. When asked to give his opinions on pupils' learning processes he often expresses insecurity. "This Chantal seems to understand". "I got the feeling that it is too difficult". "Hopefully they can do it". "Maybe he doesn't understand much".

Mr Boom, however, often recalls and quotes individual pupil's utterances and comments on them. He also expresses the need to get a picture of pupils' thinking processes. As a result, he seems to be a better observer of individual pupils. Being the more experienced classroom manager, Mr Boom talks much more than Mr Jager about the content of his lessons and would typically say things like:

> "As such I am satisfied that the desired outcome was actually achieved, the discussion went well, it was important for me that the combination of tables and graphs..."

To conclude, the teachers underline different roles in class. Roughly, Mr Jager presents himself as a manager of discipline and Mr Boom as a manager of learning processes.

Boom and Jager clearly hold *different views on* what *mathematics learning and teaching* is. Mr Jager expresses the view that mathematics, at least the subject of graphs, requires systematic steps through assignments.

> "As stupid as you might be, to put it that way, you can always solve it if you just follow the steps. Like 'now I have to do this and then I'll take the next step' and whether you understand it or not, you will find the solution anyway. The smarter ones will go from step 1 to step 3 to step 6 and they will just skip the steps in between."

Mr Jager expresses a rather traditional, algorithmic view on mathematics. In line with this view he talks in terms of telling pupils new subject matter. Mr Jager holds the view that pupils should be able to work autonomously with the textbook without much help. Whole group introductions to new subject matter seem to have the function of lowering anxiety and giving some grip ("They like me to give them something to hold on to"). After his brief introductions they should be able to 'start their work quickly,' and 'start discovering on your own'. Mr Jager expects, or hopes, that his short explanations will enable more gifted pupils to work on their own, and others can come and ask for additional assistance.

> "I hope that the smarter pupils will have sufficient explanation if you explain to them as a whole group. You just show them and give a small example at the blackboard, and this is the way to do it. And as soon as everybody gets to work you give the chance to those pupils who do not understand it so well to tell you in private, not in the whole group, that they do not understand it, so you lower the threshold for them."

Even while going over assignments he does not express the need for active pupil participation.

> "I read the assignment and elicit the answers from the pupils in the class. I try to explain why the correct answer is the correct one and with an answer I say 'take a closer look'."

Mr Jager sees learning as an individual process, not as a group process. In principle all turns are directed to individual pupils. Mr Boom expresses a different view on (mathematics) learning that – again – is connected to his interaction strategies:

> "This thing with graphs is hard to grasp, every graph is different and needs interpretation. Earlier, you got sums and the answer was correct but now the answer is not the point, but the problem solving process. That is a skill that you do not develop by writing correct answers in your notebook, you have to have thought along with me."

Mr Boom stresses the process character of RME and the active participation of pupils. He talks in terms of 'to participate' and 'to think with me' and 'to grasp'. He is well aware of the demands he puts on pupils' active listening skills, while working his way. He sees it as his duty to promote the development of these listening skills, also because it is a crucial skill for his specific content area:

> "I had to discuss the fact that you can read something from a graph and from the form (...) But I think I could also have said that in three words, I could also have said it myself (...) Three words, everyone gets it. But I did not want to do that. I wanted them to find it out for themselves because next time they will have to find it out for themselves. (...) And it is also nice to show that you know something."

He comments on the change from a more traditional way into a way of teaching mathematics according to the ideas of RME:

> "It is a total change. The fact that they first have to think it out themselves and do not have to follow algorithms is sometimes very threatening for pupils. As teacher you very quickly fall back on the method; you have to do that if you do that you achieve a good percentage."

He suggests that you would need a certain self-confidence as a teacher before being able to teach realistic mathematics in whole group interaction.

Neither of the two teachers expressed clear views on language development in mathematics lessons. Even though 'language issues' were considered to be one of the project's main focuses, teachers did not come up with remarks and observations on this aspect in the interviews. One exception is Mr Jager's observation of a pupil's question on the meaning of the word 'regular' in the first lesson, adding the comment "that was a nice one for you", not mentioning it as relevant to his own behaviour or teacher strategies. It could of course be that this type of question is absolutely normal for his daily work.

At The Sun, we see inconsistency in Mr Boom's comments on language aspects of the lesson. In line with Mr Boom's stress on participation in whole group discussion, he explicitly sees the development of listening skills as part of his task. But despite the frequently required reading of assignments (see section 3), it is remarkable that he does not include the development of reading skills in his lessons as well. He seems aware of the potential trouble source and the need for understanding the exact wording of e.g. test items, but the responsibility for indicating this type of trouble does not lie with him. After the interviewer has quoted a sentence from the assessment test, Mr Boom remarks:

Fragment 15 The Sun, SRD 171199

Teacher:	Nee ik kan wel alle zinnen altijd gaan *No, I could rephrase all sentences* herformuleren maar dan kom je nooit ergens. (..) Op *but that leads you nowhere. (...) At* een gegeven moment staan er in boeken dat soort *a certain moment the book contains these kinds of* zinnen en op het examen is het zo gesteld, omdat *sentences and also at the exams you will find them, because* geen enkele vraag op twee manieren uitgelegd moet *no single question may be interpreted in two ways.* kunnen worden, dus die zijn heel erg ingewikkeld *So they are very complicated,*

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dus dat moet dan maar.
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that's just the way it is.

Even if the exams contain complicated language, there are four years in which the teacher can promote the specific reading skills that the pupils will need in the end, so one could wonder why pupils need to stumble over language thresholds in the first year.

We can conclude that the views of both teachers on learning and teaching mathematics are consistent with their observed classroom behaviour. What they say about the teaching and learning of mathematics accounts for the differences in what they do in class. Mr Boom's outspoken view on modern mathematics teaching and the need for changes in traditional classroom interaction is an example. Mr Jager's occupation with classroom management seems to restrict his efforts to communicate with pupils more freely in whole group interaction. Language issues are hardly a theme in the interviews with both teachers.

8 *Conclusion*

This chapter has addressed the partly normative question whether Realistic Mathematics Education can contribute to a stimulating climate for learning both mathematics and language. How can we see teachers create opportunities to learn (the language of) mathematics in (whole group) classroom interaction?

First, how do the teachers stimulate pupils to explore contexts, and can minority pupils in the two multiethnic classrooms be observed to explore contexts in such a way that they contributed to meaningful learning? In Mr Jager's class the subject of 'graphs' was treated in a traditional way, without efforts to promote active thinking processes in the group of learners. Our analyses show that Mr Boom pays some attention to this point as far as it concerns contexts of his own life, like the development of his body weight. Mr Boom nor Mr Jager pay any attention to the introduction or comprehensibility of the contexts in the mathematics book. This means that a lot of opportunities for learning mathematics and language are missed, especially for minority pupils for whom many infrequent words have proven to cause problems. Remarkably, we did not observe many comprehension questions from the pupils either, but this is not a guarantee for their understanding. Given the limited opportunities to ask questions both in whole group phases[47] and in the phase of individual help,[48] one must conclude that it is to a large extent up to the pupils to find their way and overcome difficulties in class or elsewhere.

The second question addressed was how the two teachers give their pupils possibilities to verbalise and reflect on their thinking processes. Mr Jager at The Rainbow was observed to hardly ever pay any attention to solutions, but Mr Boom did indeed at least in some lessons. Pupils' contributions could however be more extensive if group-work or dyadic interaction between pupils were part of the mathematics lessons.

The third subquestion was whether the two teachers provided feedback on both content (mathematics) and form (language) when pupils participate in classroom interaction. Given the restricted participation at The Rainbow, this question was mainly relevant for pupils at The Sun and their language use. Mr Boom was observed as a teacher who focused on stimulating pupils to talk, but extending his feedback until after a number of contributions. This resulted in the fact that pupils get no clues whether what they say is clear and correct. This implies a negative answer to the third question.

What then can be concluded on the question *to what extent do the teachers create opportunities to learn (the language) of mathematics?*

Working with the same textbook, on the same chapter in a multilingual, multiethnic class, can lead to very different interaction patterns. The role of the teacher's personality, views on mathematics learning and his/her choice of teaching strategies is of major importance. One teacher, Mr Jager, created quite limited opportunities to learn the language of a limited body of mathematics knowledge. The other, Mr Boom, created an atmosphere in which pupils were actively involved in classroom interaction. He deliberately promoted active participation in interaction. Mr Boom showed a wide repertoire of strategies for stimulating thinking and active participation of pupils in the interactive process of meaning construction. The strength of Mr Boom seems to be his view on mathematics learning as a constructive process, his personality as an experienced teacher and his challenging approach towards pupils. This leads to a relatively high amount of pupil involvement and negotiating content. This combination creates the varied interaction climate in which pupil contributions are valued and used in the process of building new insights and skills. There is thus a link between the cognitive climate with high expectations of pupils thinking abilities, the interaction patterns in which they can show these skills and the pedagogical climate in the group in which they feel safe and motivated to do so.

Though some prerequisites for realistic mathematics education seem to be present, we have to conclude that even this teacher misses a number of opportunities to really put the symbiosis of mathematics and language learning theory into practice. The interactive teaching of mathematics concerns a very subtle process in which much depends on the teacher's view, knowledge, skills and last but not least on his or her 'teaching sensitivity'. The poor maths results of Hennia, a Moroccan girl and one of the most involved pupils in whole group interaction, indicate that active participation is not a guarantee for mathematics learning. There is more at stake.

The opportunities to include all pupils, regardless of their linguistic skills, are not optimal in Mr Boom's class. From what we know of mathematics and language teaching, learning opportunities for minority pupils can be increased in different ways. For example more attention can be paid to the selection of contexts that are meaningful to all pupils, and to the introduction of and orientation on these contexts and the text in the mathematics book. Through working in groups or dyads, more pupils could be prompted to talk about mathematics than in whole group discussions. Feedback on pupil contributions could be improved as well. However, here we observe a contradiction between the teaching and learning of mathematics and language. In Realistic Mathematics Education feedback is often delayed to give all the pupils the possibility to participate in the interactive learning process. But for the learning of (second) language direct feedback on what a pupil says is necessary to have an optimal effect. Given the different views of the two teachers on language, learning and mathematics, the relation should be studied between teacher cognitions and their ways of combining content and language teaching into language sensitive content instruction. Perhaps textbooks that put high demands on pupils' language proficiency can be enriched with more explicit language focused tasks, if teachers do not create these tasks in classroom interaction in multiethnic groups of learners.

The next chapter by Prenger will build on these findings and explore how pupils deal with assignments in the textbooks which are a crucial source for the learning of mathematics.

Notes

1 Andriessen & Phalet 2003; WODC 2006.
2 Hajer 1996; 2000.
3 Hajer et al. 2000.
4 Van den Boer 2003.
5 Van den Boer 2003.
6 Hajer 1996; 2000.
7 Freudenthal 1973.
8 Treffers 1987.
9 Edwards & Mercer 1987.
10 Mercer 1995.
11 Cobb et al. 1993; Gravemeijer 1995.
12 Yackel & Cobb 1996.
13 Gravemeijer 1995.
14 Yackel & Cobb 1996.
15 Gass & Selinker 2001.
16 Krashen 1982; 1985.
17 Swain 1993.
18 Marton & Tsui 2004.
19 Cloud, Genesee & Hamayan 2000.
20 Brinton, Snow & Wesche 2003.
21 Cummins 1984.
22 Ellis et al. 1994.

23 Gibbons 2002.
24 Van den Branden 1995.
25 Hajer 1996; 2000.
26 Van Eerde et al. 2006a.
27 Hajer et al. 2006.
28 Cf. Lemke 1990.
29 Mehan 1979; Mercer 1995.
30 See e.g. Hajer 1996.
31 Erickson & Schultz 1982, elaborated by Van Lier 1988.
32 Hajer 1996.
33 See Elbers et al., chapter 5 in this volume.
34 Mercer 1995; Van Eerde 1996; Nelissen & Van Oers 2000.
35 Cf. Koole & Berenst, chapter 4 in this volume.
36 Cf. Koole & Berenst, chapter 4 in this volume.
37 See Prenger in this volume, for a description of our data on these skills.
38 Prenger 2001.
39 Van den Boer 2003.
40 Prenger 2001.
41 Prenger, chapter 3 in this volume.
42 Koole & Berenst, chapter 4 in this volume.
43 See Elbers et al., chapter 5 in this volume.
44 Hajer 1996.
45 Pels, chapter 7 in this volume.
46 Woods 1996; Verloop 1999.
47 Koole & Berenst, chapter 4 in this volume.
48 Elbers et al., chapter 5 in this volume.